Introduction to Electromagnetism
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Lecture - 65
Radiation from an accelerating charge – II

So far we have established that one a charged accelerates the region or corresponding to acceleration, propagates out with speed c, and it has an electric field; that is, has a tangential component, all the components which perpendicular to direction of propagation of that region. In this lecture we are going to show that this field indeed has a 1 over r dependence and carries out power; that is 1 over r square, and that means, this is indeed radiation field.

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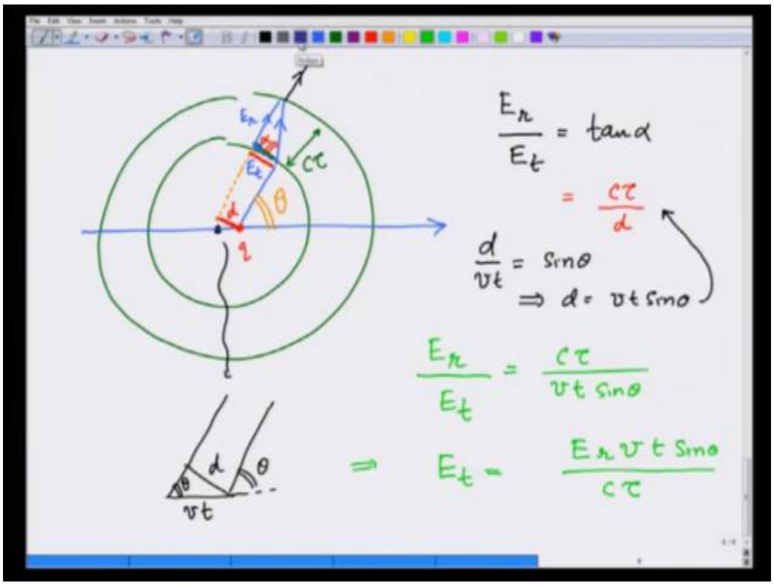

So, let me make a neat picture now concentrating only on one line of force. So, here is my inner circle, here is my outer circle, here is my charge later, here is my charge initially. This is the field line initially, this is the field line later, and I will connect them. And this charge is moving this way; charge q, and this has developed a redial component, and there is of course, component which is tangential component and the redial component. This is e plus calls it tangential, and the other forces e redial. Let this angle where I am observing the field, and the direction of acceleration be theta. Let me again connect the lines, this other line is like this. I have e and this distance c tau, and I need this distance also.

So, let us see now I have e tangential or e redial divided by e tangential is equal to tangent of alpha, where alpha is this angel; e radial divided by e tangential is tangent of alpha. This tangent of alpha is also the same, as the distance c tau divided by the distance here, which I am showing by red right next to it, which is the same distance is this. Let us call a d; c tau over d. Now, I will calculate d. let me take this picture to the bottom and show that, this is d, this angel is theta, this distance d t at the lower distance. If this is theta, this is also theta. So, I have d over v t e equals sin of theta, which implies that d is equal to v t sin theta. Substituting this here I get e redial divided by e tangential is equal to c tau over v t sin theta, which give me e tangential is equal to e radial v t sin theta divided by c tau. So, now we got a working formula with which we now start working.

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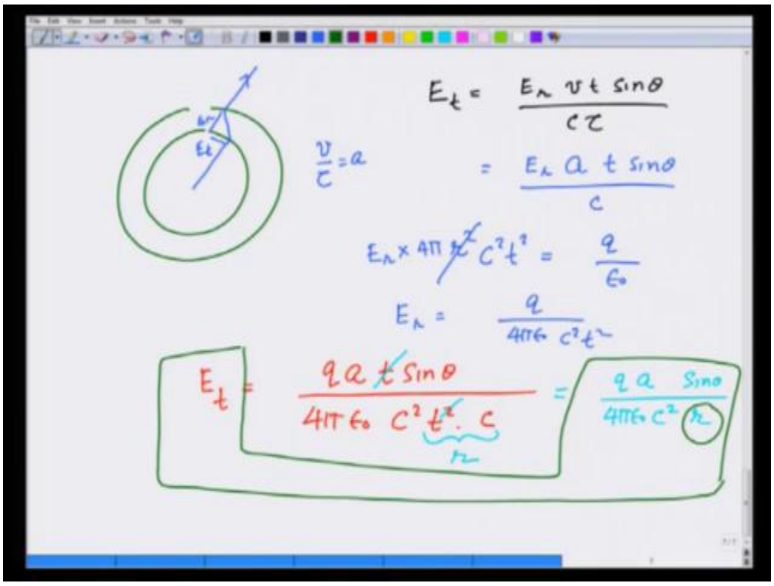

So, we have got e tangential e equals e r v t sin theta over c tau. Where again I make this picture inner circle, outer circle, the field line is very much like this. This is e tangential, this is e radial. Now, v over tau is the acceleration, and therefore, I can write this formula as e r acceleration times t sin theta over c. By Gauss's I have e r times 4 pi r squared; r is nothing, but c tau c square t square, so I can cut this and write this as c square t square is equal to total charge q divide by epsilon 0; and therefore, e radial is q over 4 pi epsilon 0 c square t square. And this immediately gives me e tangential is equal to q a t sin theta divided by 4 pi epsilon 0 c square p square time c. Again I am going to cancel one of the t's, and write c t as r again, because r is the distance up to which, this tangential component of electric field has moved, and I can write this as; q a sin theta divided by 4 pi epsilon 0 c square r. Notice, we accomplished 4 p for looking for. We have found that the tangential electric field is proportional to 1 over r, how far this tangential component has moved, and rest of the constants.

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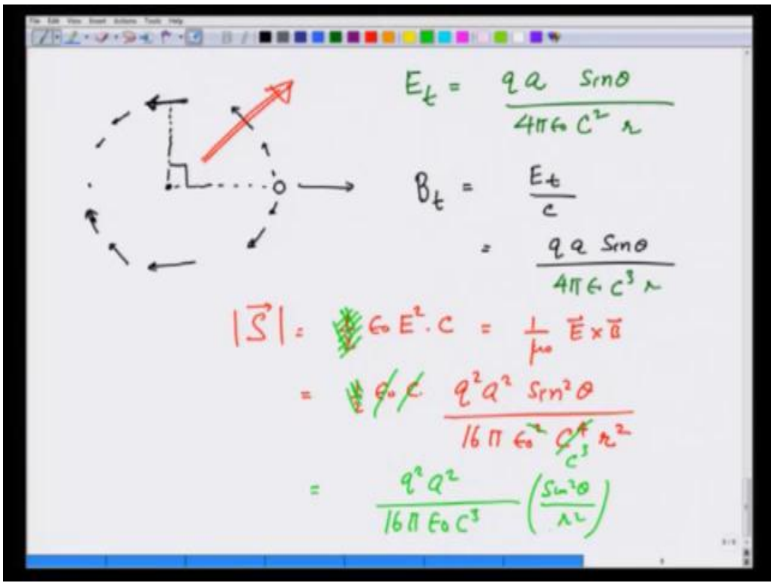

So, what we have gotten in this case is, that e tangential is equal to q a sin theta over 4 pi epsilon 0 c cube r. If I look at this charge which was given acceleration in this direction, it develops tangential field which is zero here, sin theta is maximum here at pi by 2. It is very large here, opposite in direction to the acceleration. Smaller here, even smaller here, even smaller here, becomes zero at theta is equal to zero. The other way on the other side, very large and elimination again zero.

Now, how about the pointing vector or b tangential; b tangential I have already seen as e tangential c, and therefore, it is going to be q a sin theta over this is c square; 4 pi epsilon 0 c cube r. The pointing vector s, which is going to be radially out, is going to be equal to one half epsilon 0 e square times c, which is the same as 1 over mu 0 e cross b. So, this is equal to one half of epsilon 0 c times e square, which is q square a square sin square theta divided by 16 pi epsilon 0 square c raise to 4 r square. So, the pointing vector, this is magnitude and direction is readily outward is, I will cancel one of the epsilon zeros. I will cancels one of the c's becomes c cubed. So, this is equal to, this half is also not there right now, because I am not taken the average. Half you remember rows, because of the harmonic natures, this half is not there, is equal to 2 q square a square over 16 pi epsilon 0 c cubed sin squared theta over r square.

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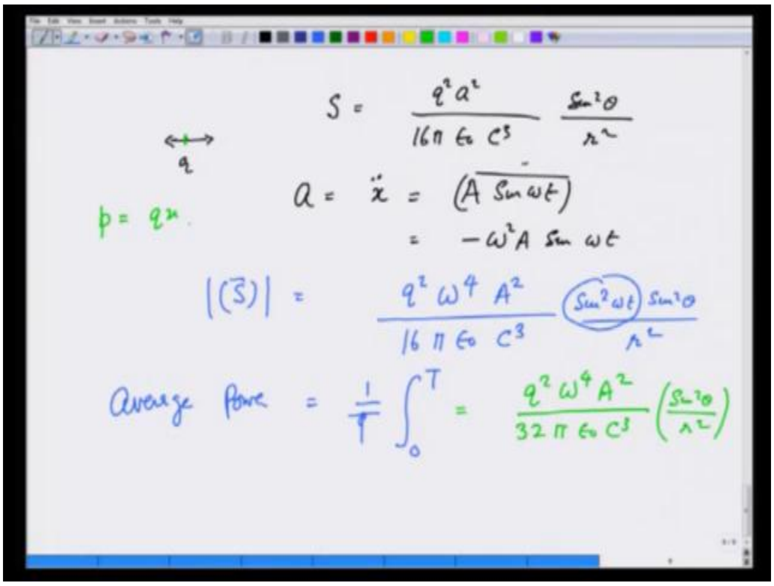

So, you see the power radiates is also goes as 1 over r square, which is very satisfying, because that how power from a point source grows. Let us see now what happens if I take this point charge, and start moving back and forth q, which is making a oscillating motion, harmonic oscillating motion. So, we already seen that s is equal to q square a square over 16 pi epsilon 0 c cube sin square theta over r square. In this case, a is equal to x double dot, where x is nothing, but some amplitude sin omega t double dot. So, which is going to be minus omega square a sin square omega t.

And therefore, the power radiated by an osculating charge would be equal to q square omega raise to 4 a square. This is sin omega t not sin square, over 16 pi epsilon 0 c cube sin square omega t sin squared theta over r square; the average power. By average power mean, we average it over cycle so that we only observe that the frequency is very large, give me a factor of half, because of this. This was a mistake was making earlier when I wrote the factor of half, so this is give me q square omega raise to 4 a square over 32 pi epsilon 0 c cubed sin square theta over r square. This is the average power that is given out by oscillating charge. Oscillating charge, since this moves away from the centre also makes a dipole which is equal to q x. So, I can even think of this as an oscillating dipole, which is giving out radiation. Radiation is maximum in the direction perpendicular to the dipole, and it is zero in the direction of the dipole. How about the total power radiated.

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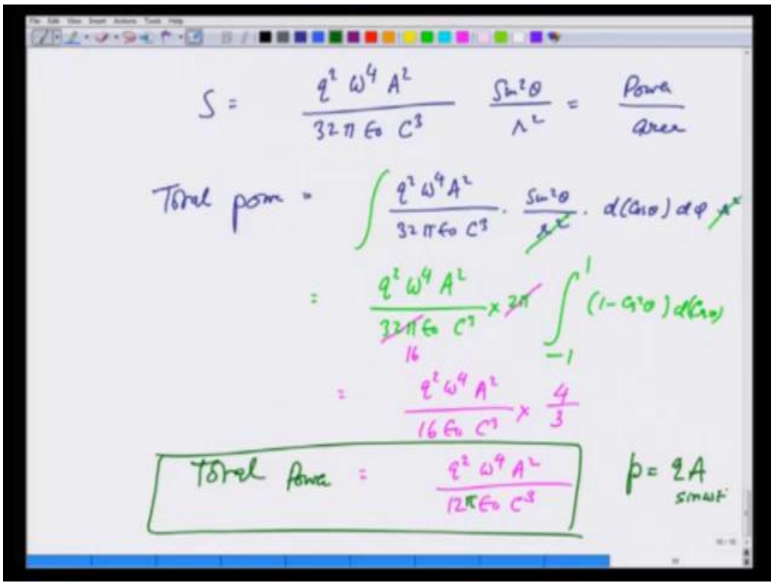

The total power radiated. So, this s is q square omega raise to 4 a square over thirty 2 pi epsilon 0 c cube sin square theta over r square, which is nothing but power per unit area. So, total power is going to be equal to q square omega raise to 4 a square over 32 pi epsilon 0 c cubed sin square theta over r square times d cosine theta d phi times r square. Remember this is the area in direction r. Now, r square cancel, there is no phi dependence so that 2 pi will come out, so this is integrated over. This becomes q square omega raise to 4 a square over 32 pi epsilon 0 c cube times 2 pi integration sin square theta, I can write as 1 minus cosine square theta b cosine theta going from minus 1 to 1, 2 pi 32 pi gives me 16.

So, this is going to be q square omega raise to 4 a square over 16 epsilon 0 c cube times 1 minus cosine squared d theta cos theta gives me four thirds. So, this is equal to q square omega raise to 4 a square over 12 epsilon 0 c cubed. Yes, I am missing a factor of pi, if you go back to the formula this was pi square here, and therefore, I have pi square here, and pi square here, pi square here. So, finally, pi squared here, and therefore, finally, I have a 12 pi. This is total power radiated by a charge oscillating back and forth or a dipole of dipole moment p e equals q a. If it is oscillating back and forth you have a time dependence sin omega t that this dipole gives out; this is known as lambda formula for radiation. So, what I have shown in this lecture is very qualitative, drawing field lines, and things like those, that an accelerating charge develops a field, which is perpendicular to direction of propagation; it goes as 1 over r and therefore, is the radiation field.

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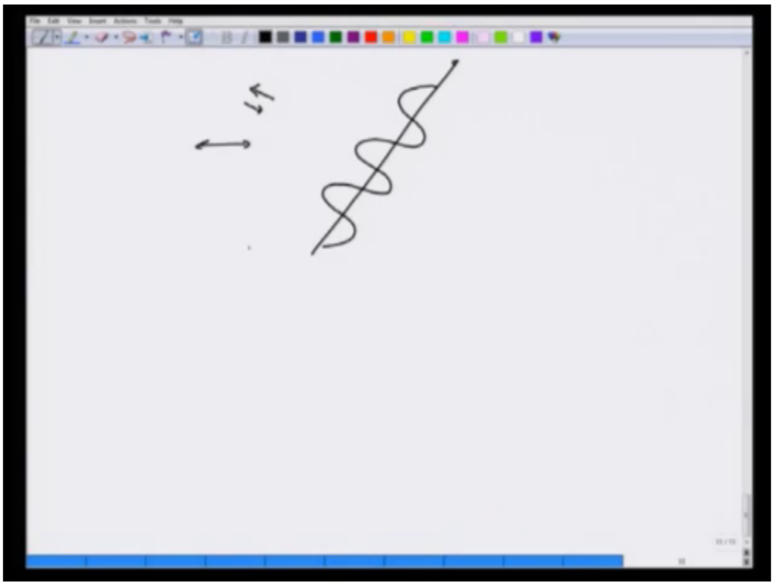

Now, imagine what will happen when this charge is moving back and forth. If it is moving back and forth, and therefore sometimes the field will be in one direction, sometimes is in the other direction, as the way propagates the field goes up and down like this, and this is the harmonic way of going out.