Introduction to Electromagnetism

Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 63
Reflection of waves at a boundary – II
Electromagnetic waves reflecting from a dielectric surface

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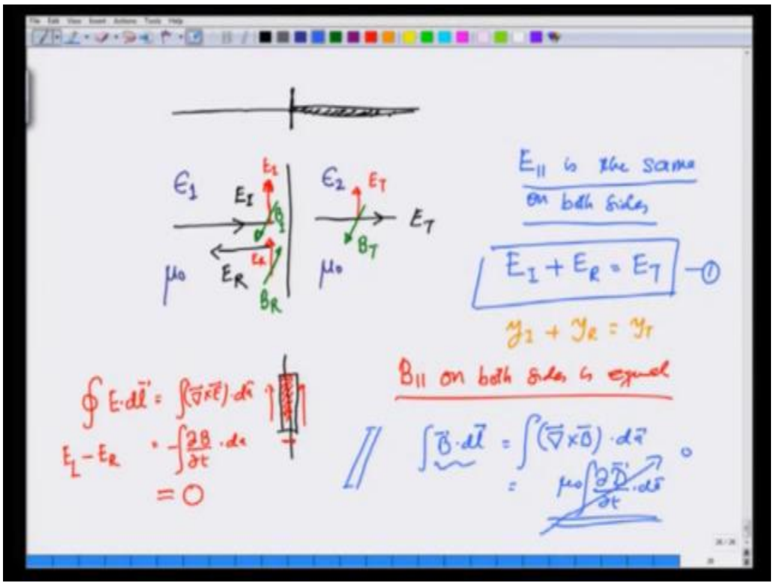

In the previous lecture we saw how when a mechanical wave, coming on a string goes from one medium to the other, the other media being another string, so that there is a boundary, there is a reflection. In a similar manner, we are now going to see that when electromagnetic waves fall from on a surface from one medium to the other, there is going to be reflection. We are going to assume right in the beginning there is a reflected wave, there is an incident wave, and there is a transmitted wave. Let us see one medium on one side has permittivity epsilon 1, on the other side has the permittivity epsilon 2, and let us take them to be nonmagnetic, so that there permeability is same mu 0, when the wave come from the left hand side.

Let us take its e field to be shown by red arrow, and because e cross b should give me the direction of propagation. I am going to show you the b filed by green colour is going to be pointing this way. So, this is the b field and e field is shown on that .On the other hand when it gets reflected. Suppose I take e to be in the same direction as the incoming wave.

Then e cross b should give me the direction of propagation, and therefore, now the reflected be filled is going to be the other way. So, this is b reflected, let us call the first one be incident, this is e incident and e reflected. At the same time in the transmitted wave, there is going to be e transmitted, and it is going in the same direction as the incident wave I am going to have b incident, b transmitted as shown here in the same direction as b incident. So, we have set up what the electric field and magnetic fields are at the boundary, and now we need to apply the conditions. The boundary conditions are; e parallel is the same on both sides. This arises simply from stokes theorem as follows. If I look at this boundary, and make a very small loop here, the e field on this the side like this, on the side like this.

Then e dot d l is going to be equal to integration curl of e dot d a. Curl of e is nothing but d b d t it with a minus sign dot d a, and this is e on the left hand side. Let me write left minus e r, because we are traversing the loop in two different directions; however, as we shrink the loop, we make the size, the red shaded region almost zero width. The right hand side becomes zero, and therefore, e on the left should be equal to e on the right side, so e parallel is same on both side. And this gives me the first equation, which is e r I plus e r is equal to e transmitted. This is equation number one. I referred back to the earlier equation that we had derived in the string which was y incident plus y reflected y e is equal to y transmitted, this similar to that. In a similar manner b parallel on both sides, is equal, why is that, that again follows from stokes theorem, because if I now take a loop like this parallel to b. Then I have b dot d l across this loop is equal to curl of b dot d a, which is equal to mu 0 displacement over d t dot d a. As the area second to 0, this y goes to 0, and left hand side becomes zero, which again gives like the argument for electric field that b on two sides must be equal.

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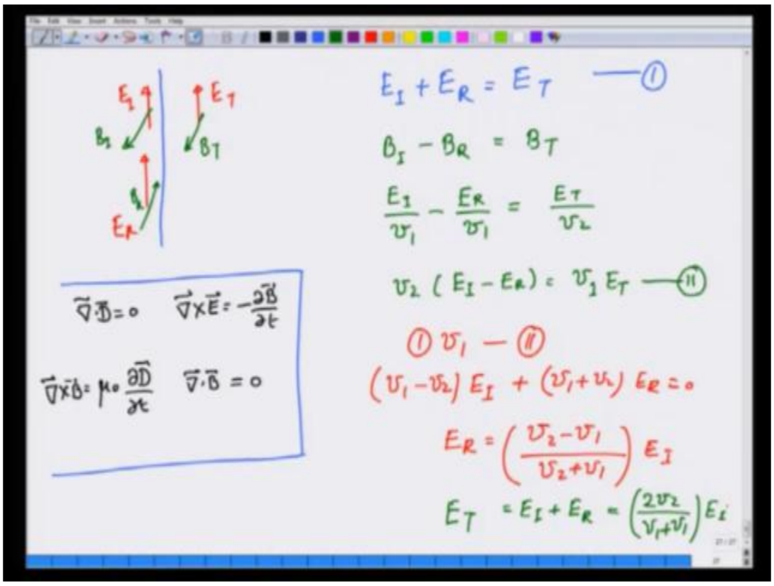

So, on this boundary, I have e e transmitted and e reflected. And I have b incident, b reflected, and b transmitted; the two equations are e incident plus e reflected e is equal to e transmitted; that is my equation one. And the other equation is, b incident b reflected in the other direction b reflected is equal to b transmitted; that is equation two. But now I know from electromagnetic theory that b incident is nothing but e incident divided by v 1 in that medium minus b reflected is nothing but e reflected over v 1 in that medium. This should be equal to e transmitted over v 2 in that medium. You may be little un comfortable, because I did not do this equation for a wave in the medium, but the relationships are the same, and this can be obtained very easily by writing maxis equation, the medium which are.

Let me just write the among the side for completeness, which are divergence of d is equal to 0 and an absence of any charge, curl of e remain the same as the minus d b d t. Curl of b is equal to, again there is no free current the everything comes from displacement current is going to be mu 0, mu 0 is the same d d by d t and divergence of b is equal to 0. So, this again, if manipulate the relationship that b is equal to e by v. And this immediately needs to v 2 e I minus e r is equal to v 1 e transmitted; that is my equation two. I have got two equations, to unknowns; e transmitted and e reflected and I can solve for them. Let us multiply equation one by v 1, and subtract equation two. This gives me v 1 minus v 2 e incident plus v 1 plus v 2 e reflected is equal to 0, and this gives e reflected is equal to v 2 minus v 1 over v 2 plus v 1 e incident. Exactly similar relationship in terms of velocity is in the two medium, as it was for string. As similarly e transmitted will be given by nothing but e incident plus e reflected which will give me 2 v 2 over v 2 plus v 1 e incident.

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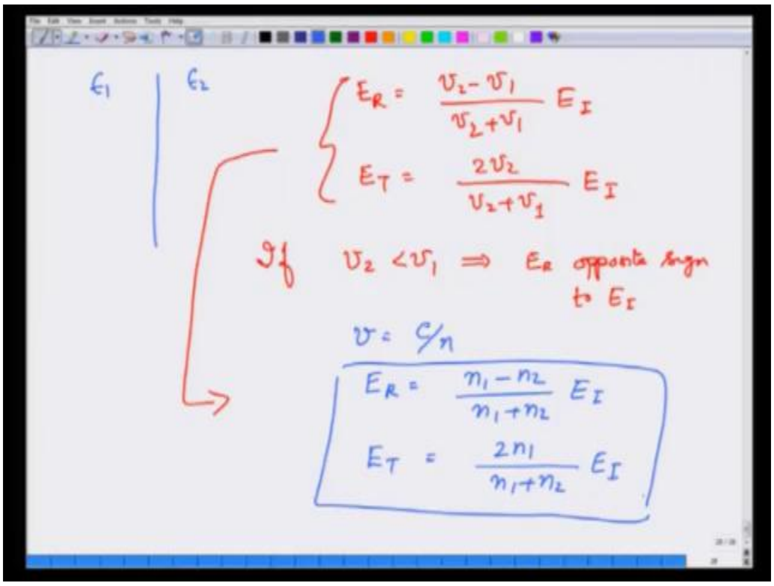

So, we have got in this medium, which is epsilon 1 on one side and epsilon 2 on the other side, and therefore, there is some reflection and transmission, which are incident wave does not transmitted completely and we have got e reflected, it was v 2 minus v 1 over v 2 plus v 1 e incident, and e transmitted is equal to 2 v 2 over 2 plus v 1 e incident. These two relationships are very similar to the relationship obtained for waves on a string. Notice that if v 2 is less then v 1, this implies e r has opposite sign to e incident; that means, it is reflects direction on or there is a pie face change. Let us translate these equations, to equations in terms of the refractive index.

Now I know v is c y n, and therefore, e reflected is going to be n 1 minus n 2 over n 1 plus n 2 e incident and e transmitted, is going to be 2 n 1 over 1 plus n 2 e incident. Again you notice that if n 2; that is a medium two, has larger refractive index then medium 1, the reflected wave has a face change. This you have been taught in you eleventh twelfth grade, but now we see that this arise basically from the boundary condition on electric field and magnetic field.

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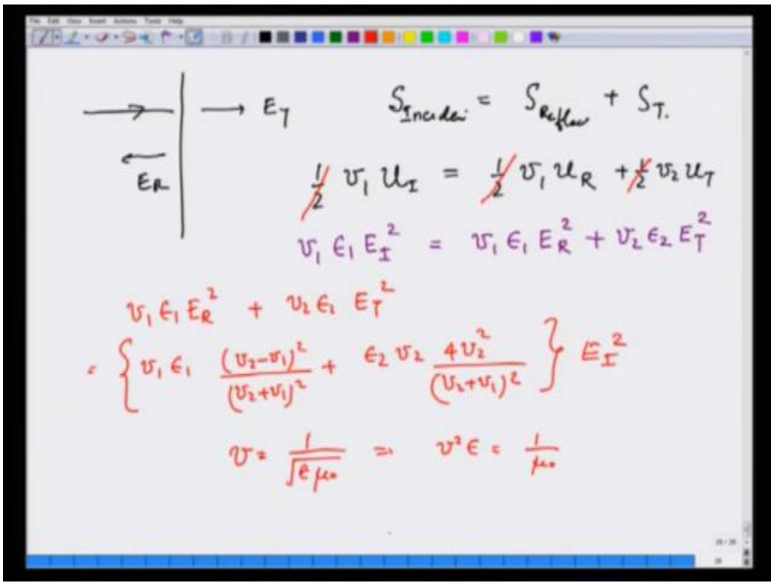

What about energy transmission. So, this waves come this wave is coming gets transmitted, gets reflected. I should have the pointing vector is coming in incident, should be equal to the energy which is transmitted plus the energy which is reflected, s reflected plus s transmitted. Incident 1 is going to be v 1 times u energy density incident one-half, this should be equal to one half v 1 u reflected plus one half v 2 u transmitted, which is indeed what the energy flow is. This half cancels. This is going to be equal to v 1 times epsilon 1 e incident square and the right hand side is going to v 1 epsilon 1 times e reflected square plus v 2 epsilon 2 times e transmitted square. Let us see if this is true. Let us calculate the right hand side. Right hand side is v 1 epsilon 1 e reflected square plus v 2 epsilon 2 e transmitted square which is equal to v 1 epsilon 1 times v 2 minus v 1 square over v 2 plus v 1 square plus epsilon 2 v 2 times 4 v 2 square over v 2 plus v 1 square times e I square. Now I know that v is equal to 1 over square root of epsilon mu 0. Keep in mind that mu is the same, and therefore v square epsilon is 1 over mu 0 for both the medium.

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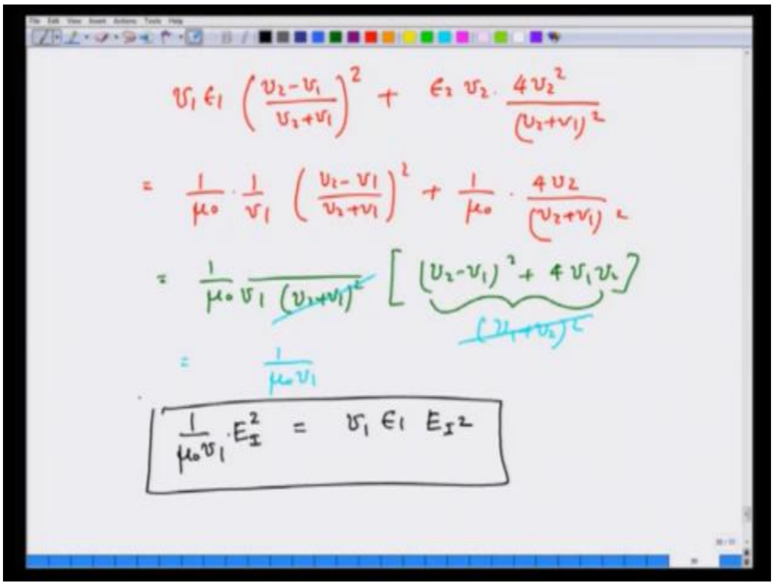

And therefore, I can write this whole thing as; v 1 epsilon 1 that is v 2 minus v 1 over v 2 plus v 1 whole square plus epsilon 2 v 2 4 v 2 square over v 2 plus v 1 whole square as 1 over mu 0 1 over v 1 v 2 minus v 1 over 2 plus v 1 whole square plus epsilon 2 v 2 square is 1 over mu 0 4 v 2 over v 2 plus v 1 whole square. This is equal to 1 over mu 0 v 1 v 2 plus v 1 whole square, and inside I get v 2 minus v 1 whole square plus 4 v 1 v 2. This term is nothing but v 1 plus v 2 whole square in this cancels with this term, and therefore, I get 1 over mu 0 v 1. So, this whole things add up to 1 over mu 0 v 1 e incident square, which is nothing but equal to v 1 epsilon 1 e incident square as cab be easily seen. So, you see that our amplitude, are such that they also satisfy, energy conservation. So, what I have shown you in this lecture is, through the boundary condition, when an incident electromagnetic wave comes, it gets both reflected as well as transmitted. The ratios of transmitted and reflected electric field have been calculated, and you also shown through those that energy is conserved.