Introduction to Electromagnetism
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Lecture - 62
Reflection of waves at a boundary – I wave on a tense string

So far we have talked about electromagnetic waves, transporting energy, transporting momentum, and all these things. What we want to focus on this, what happens when in this lecture is, what happens when electromagnetic waves come at a boundary between two different medium. We all know they get reflected or transmitted, what exactly those coefficients are, we want to understand that, and this we should be able to derive from Maxwell's equations and related boundary conditions.

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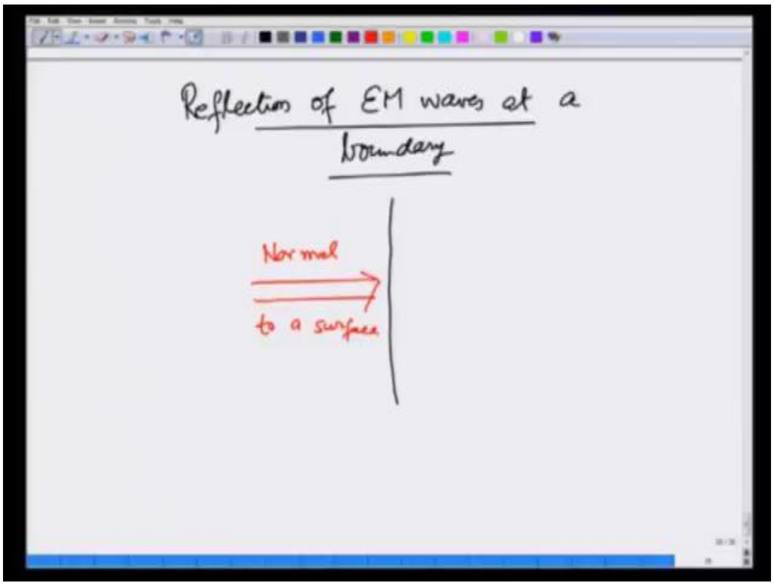

So, first what I will do is, motivate why reflection should take place at a boundary through mechanical waves, and then go over to electromagnetic waves. So, what we are going to focus in this lecture is, reflection of e m waves at a boundary. And we are going to restrict ourselves, because we want to understand the phenomena two waves that fall normal to a surface, but first you understand that it is the boundary that gives rise to reflection.

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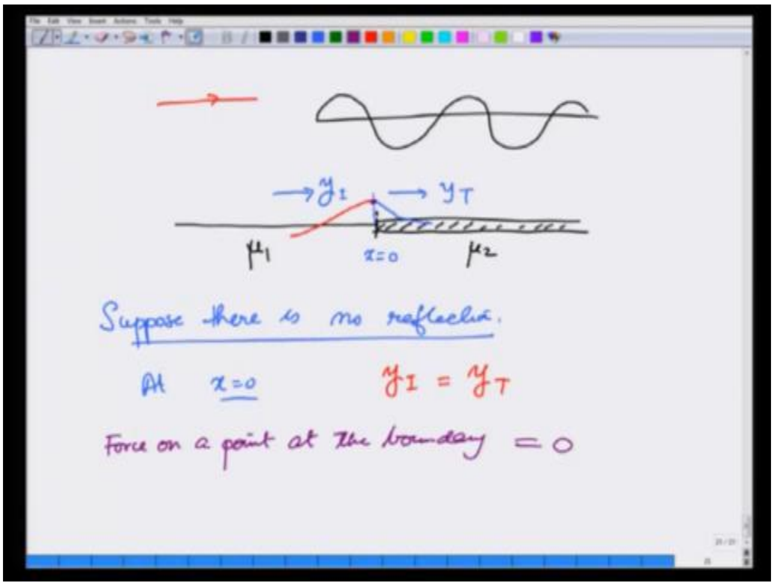

Let us first understand that, if there is a mechanical wave. For example, I could take a wave on a string. We could take a wave on a string that is going, and suppose we take two different strings; one of mass mu 1 per meter square, other one of a slightly different mass mu 2 per metre square, then what happens at this boundary. If there is a wave coming, let us say there is a wave that meets here, and then on the other side a same wave, let us say goes like this what happens? Now at this point the displacement is y. Let us take this to be x equal to 0, without any loss of generality.

The wave incident is y incident, and wave transmitted is y transmitted. Let us say there is no reflection. Suppose there is no reflection, then what would happen? I know the displacement look should look the same if I come from the two sides. So, at x equal to 0 I should have y i is equal to y t. What about one more equation, if I look at this point out here, this is a zero mass. You can imagine this to be a very small point mass with zero mass, and therefore, force on a point at the boundary must be zero, otherwise it will move with infinite acceleration. And what is force given as, let us see that in the next slide.

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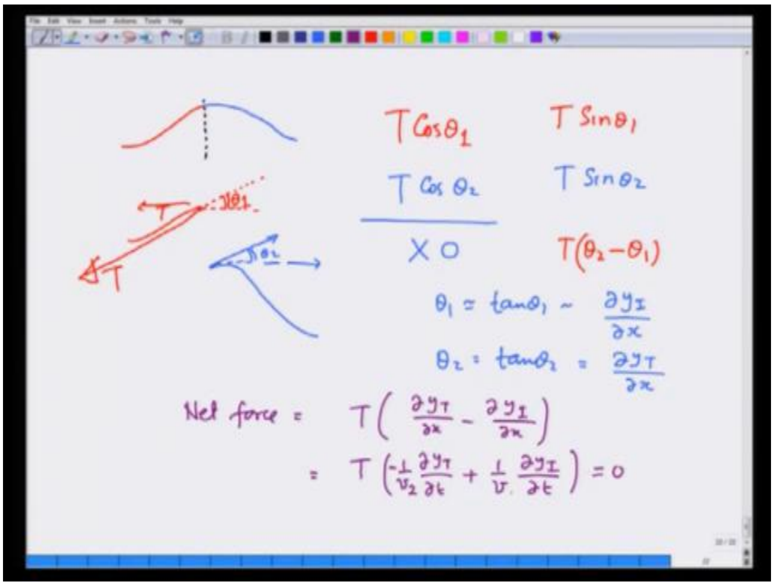

If I have this boundary, where on the left hand side I have this displacement. On the right hand side I have another displacement. Then at the boundary, if I look at from the left hand side, there is tension t in the string. Recall that I am taking a very small amplitude, so t remains the same. And therefore, at this point, this point feels a force period of t at this angle, which is at an angle theta. This provides a horizontal component t cosine of theta. Let us call it theta 1, because I am coming from the left side, and vertical component t sine theta 1. On the right hand side I have same tension, but it is making angle theta 2.

So, the horizontal force is going to be t cosine theta 2, and the vertical force is going to be t sine theta. For a small angle approximation or a small angle amplitude, I can take cosine theta 1 and cosine theta 2 to be 1, and therefore, the horizontal components cancel, and they give you zero, because the component for the blue one is towards a right, and component for the red one is towards the left. However the vertical components; since I am taking t cosine theta sine theta 2 to be going up. So, it will be t theta 2 minus theta 1. Theta 1 is roughly same as tan theta 1 which is same as d y incident by d x. Similarly, theta 2 is tangent, theta 2 which is d y transmitted over d x, and therefore, the net force is going to be t d y transmitted over d x minus d y incident over d x. And from the wave equation I can write this both the waves are travelling to the right; therefore, t d y t over d t 1 over v 2, where v 2 is the velocity in the right string plus 1 over v 1 d y t over d t d y I over d t and this should be zero.

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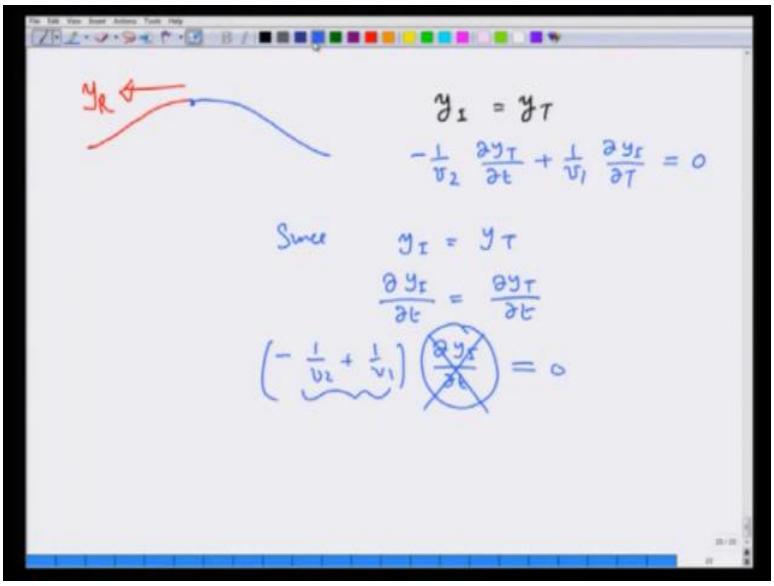

So, the two equations that I have, are this string from the left hand side, the string on the right hand side. I have y incident is equal to y transmitted. And I have minus 1 over v 2 d y transmitted over d t plus 1 over v 1 d y incident over d t is equal to 0. Since y incident is y t, all the time I have d y incident d t as d y t over d t, because they remain in phase. This condition is satisfied all for all the times. And therefore, I have minus 1 over v 2 plus 1 over v 1 d y i d t is equal to 0.

This means either this slope is zero; that means, there is no velocity for the midpoint. There the point at the connecting point, or velocities are the same. If the velocities are the same, two mediums are the same, and therefore, the entire wave transmits, but this certainly cannot be zero. How do I then satisfy the equations right; either I get that d y I by d t is 0, which is not possible or velocities are the same. To satisfy all the equations all the time therefore, I have to then take at their exits a y reflected also. So, whenever there is a boundary, so that the speeds of the two sides of that boundary are different, there has to be a reflected wave also; otherwise arise into problems of satisfying these equations, which are true nature given equations.

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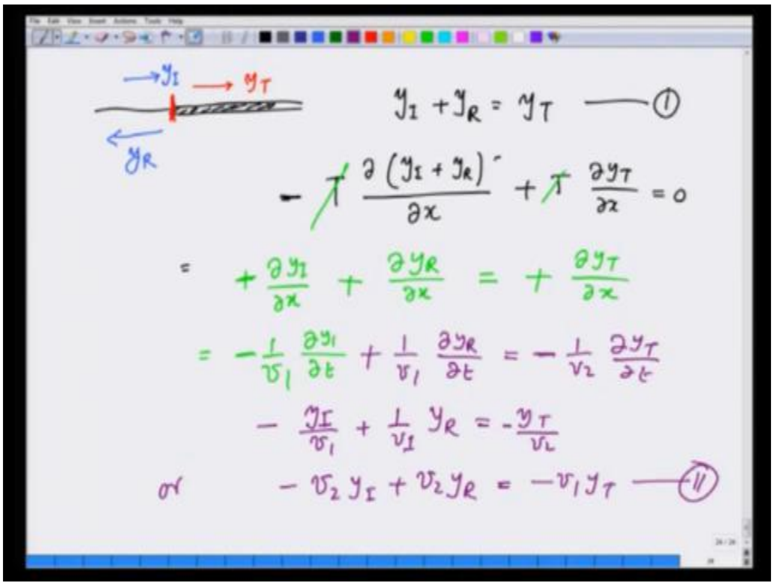

So, now what we are going to say, is that given the string of a different mass on this side, and different mass on the right side. There is going to be a incident wave, a reflected wave with y r displacement y incident displacement, and a transmitted wave with y t displacement. Again since that the boundary we should have the displacement is same all the time, I should have y incident plus y reflected is equal to y transmitted; that is equation number one. I should also have the net force zero, net displacement on the left hand side is y I plus y r. And its vertical component is going to be theta again, which is d by d x of this times the tension; that is the net force, on the left hand side. From the right hand side, I am going to have this will be the minus sign plus, from the right hand side I am going to have t d y transmitted over d x is equal to 0; that is my equation two.

Let us convert them into the time equation. So, I am going to have t I can drop, because this is 0 on the right hand side, I am going to have d y I over d x with the minus sign minus partial y r over partial x is equal to minus partial y transmitted over partial x, all this can be made to be plus. Incident wave is coming from left to right and therefore, this is, minus 1 over v 1 d y I by d t. The reflected ray goes to the left; therefore, this plus 1 over v 1 d y r over d t, and this should be equal to minus 1 over v 2 d y transmitted over d t. Again the displacements are same all the time and therefore, I can take the time derivative out, and I can write minus y incident over v 1 plus 1 over v 1 y reflected is equal to y transmitted over v 2 with the minus sign, or minus v 2 y incident plus v 2 y reflected is equal to minus v 1 y transmitted; this is my equation two. I solve the two equations and I will get results for y transmitted and y reflected in terms of y incident. Let us do that.

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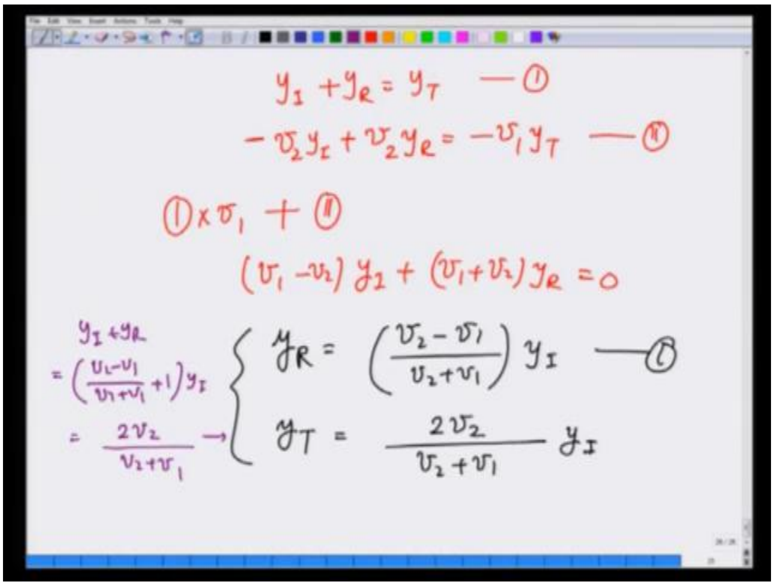

So, let me rewrite the equations on the next slide. I have y incident plus y reflected is equal to y transmitted equation one. Minus v 2 y incident plus v 2 y reflected is equal to minus v 1 y transmitted equation two. Multiply equation one by v 1 and add to equation two, I get v 1 minus v 2 y incident plus v 1 plus v 2 y reflected is equal to 0, and this gives me y reflected is equal to v 2 minus v 1 over v 2 plus v 1 y incident. So, you have got know what y reflected is. Again do the similar manipulation and you your going to get y transmitted is equal to 2 v 2 divided by v 2 plus v 1 y incident. Let us check the boundary conditions quickly. If I take y I plus y r which is equal to v 2 minus v 1 over v 2 plus v 1 plus 1 y I, this indeed gives me 2 v 2 over v 2 plus v 1, which is my answer here. Notice in this that if v 2 is smaller than v 1 y reflected is in opposite sign to y i; that means, if I have a heavier string on the right hand side, which makes me too smaller I am going to have a phase change for the reflected wave.

Now this is the answer that we get for wave on a string. You can also show with these amplitudes that the energy coming in, is equal to the energy reflected plus energy transmitted, which is required by energy conservation. Exactly similar equations are now going to be obtained for electromagnetic waves. For electromagnetic waves the boundary conditions are in terms of electric and magnetic fields, right and those boundary conditions are going to give us, what amplitude of the incoming electric field is going to, what fraction of the incoming electric field is going to be reflected, what fraction is going to be transmitted, that we will do in the next lecture.