Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Module - 07
Lecture - 60
Energy and intensity and momentum carried by electromagnetic waves

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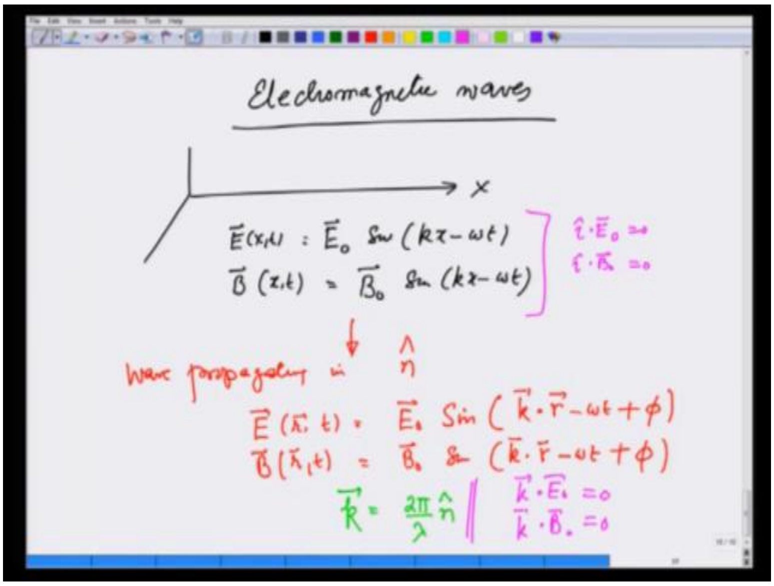

In the previous two lectures we talked about electromagnetic waves that come naturally out of Maxwell's equations. In particular we focused on wave propagating in the x direction and wrote this as e x t is equal to e 0 sine of k x minus omega t and be x t is equal to b 0 sine of a x minus omega t. If you generalize this I can in general write for wave propagating in the direction of unit vector n I can write e r vector t is equal to e 0 sine or cosine of some other face. So, I can write this as k dot r minus omega t and b as a function of r and p as equal to b 0 sin of k dot r minus omega t. And if I want two more general I will add a phases also some pi that can make a cosine or whatever where the vector k is equal to 2 pi over lambda in the direction of the propagation.

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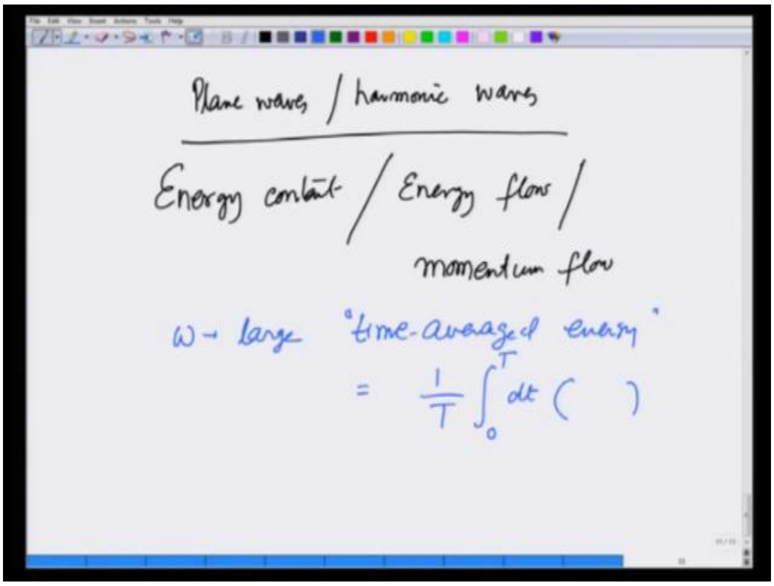

In the previous lecture we saw the waves for propagating I direction I dot e 0 was 0 I dot b 0 was 0. In the same manner in general I am going to have k dot e 0 is equal to 0. That means the electric field is perpendicular to the direction of propagation and similarly a dot b is going to be 0 and e cross b is going to be in the direction of propagation. With this background what you want to focus on is again waves from this time which are harmonic and plane waves. So, plane waves, harmonic, mono chromatic or moving with frequency omega waves and see what is the energy content of these waves, how much energy do they carry and how much momentum do they contain. Remember, these are the quantity we have had already calculated for electro static field.

We will also focus on that omega is going to be large. So, we are talking about something like the ten raise fifteen, ten raise ten, ten raise to fourteen. That means, in one second this so many oscillation we are talking about light waves or microwaves. In that case, it does not make sense of energy variation but will be talking about time averaged energy and what we mean by this will be taking one over the time period and integrate whatever quantity we get from 0 to t over time. Whatever quantity energy density or the energy flow the momentum flow. So at such high frequency what we see is for example if I am looking at this light I am not seeing it varying I am seeing a constant light because it varies ten raise to fifteen times per second and therefore I see only the average light my eyes somehow averages it out I will

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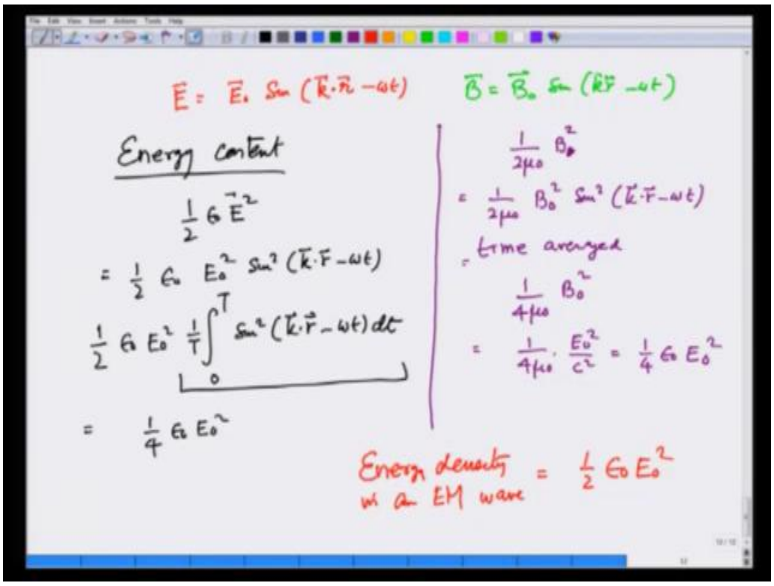

So, we have e equal to e 0 sine k dot r minus omega t and b is equal to b 0 sine of k of a dot r minus a omega t if you want you can add pi also doesn't really make a difference. Let’s calculate the energy content recall that energy in the electric field is one half epsilon 0 e square and therefore in this case it is going to become one half epsilon 0 e 0 square that is the amplitude square sine square k dot r minus omega t. Recall I said that this is varying in time very fast so I will be talking about average energy. Average energy is going to be one half epsilon 0 e 0 square 0 to t one over t sine square k dot r at a given point r is fix minus omega t d t and this you know from your ordinary mathematics is one half.

So, this becomes one fourth of epsilon 0 t 0 square. Let us now calculate what the energy in the magnetic field is magnetic is. Magnetic field you recall that it carries energy one over two mu 0 b 0 square sine square k dot r minus omega t which when time average will give me another factor of two. So, this becomes one over four mu 0 b 0 square recall from previous lecture that b 0 is nothing but e 0 over c. So, this becomes one over four mu 0 e 0 square c square but one over c square is mu 0 epsilon 0. So, this becomes one fourth epsilon 0 e 0 square therefore the energy contained in the electric field and magnetic field component of this electromagnetic wave is exactly the same which is the one fourth epsilon 0 e 0 square and you add the two and you get the energy density in an electromagnetic e m wave to be one half epsilon 0 e 0 square. So, when this wave exist for example this light is coming to me then in between average energy per unit volume is one half epsilon 0 e 0 square.

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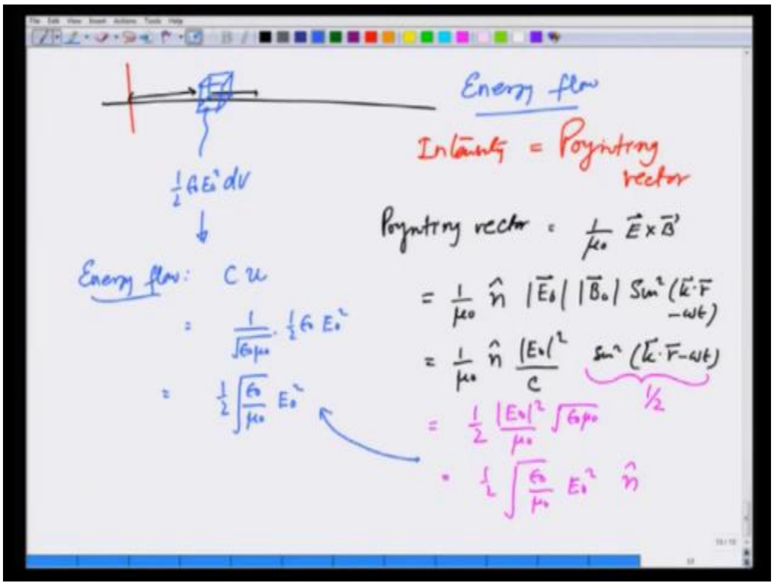

So, this wave let us again take it as it is propagating in x direction if I take a small box here of unit volume for small volume d v then we content this is going to be one half epsilon 0 e 0 square d v how about the energy flow which is related to intensity, which is related to pointing vector. The pointing vector is given as one over mu 0 e cross b and we all ready seen that e cross b is in the direction of propagation. So, I can write this as one mu 0 direction n mode e 0 and modulus of b 0 sine square a dot r minus omega t b 0 we recall that is again e 0 by c. So, I can write this as one over mu 0 in the direction of propagation e 0 square over c sine square k dot r minus omega t when i take the time average sin square k dot r of minus omega t is going to give me a factor of two one half and therefore this becomes one half e 0 square over mu 0 and c is nothing but square root of epsilon 0 mu 0 one. So, this become one half square root of epsilon 0 over mu 0 e 0 square in the direction of propagation.

That, means if there is an electromagnetic, plane electromagnetic wave going in some direction in the same direction per unit area per unit time the energy that flows is one half square root of epsilon 0 and mu 0 e 0 square. Let us also see from the energy density point of view the energy flow if you recall form one or previous lecture is going to be nothing but c time energy density u which is going to be one over square root of epsilon 0 mu 0 times one half epsilon 0 e 0 square which is same as one half square root of epsilon 0 over mu 0 e 0 square same answer.

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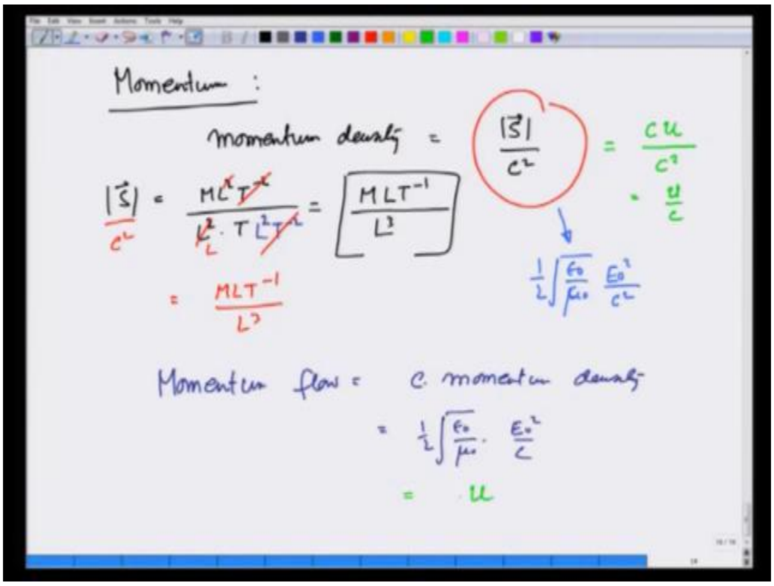

So, that is energy that is carried by the electromagnetic wave or that is the intensity that we received. This is also related to the momentum carried by electromagnetic wave. Recall that the momentum density is nothing but s over c square let us check that modulus of s is nothing but energy m l square t minus two per unit area per unit time which then becomes equal to momentum density m l t minus one over l cube. So, if I divide by C Square here C Square is nothing but l square t minus two. This cancels from doing c square one of the l I cancel and make it l by get m l t minus one over l cube. So, this is the momentum density and therefore if there is a plane electromagnetic wave going the momentum density is given as one half square root of epsilon 0 over mu 0 e 0 square over c square. Given this momentum density what is the movement flow.

The momentum flow is nothing but c times of momentum density which is going to be equal to one half epsilon 0 over mu 0 e 0 square over c. We, write them in terms of u so s is nothing but c times u, u is the energy density in the electromagnetic field over c square which is mu over c and the momentum flow is nothing but u itself. If, there is momentum flow that means is there is an electromagnetic wave going this way you carry the momentum which is equal to u per unit area per unit time.

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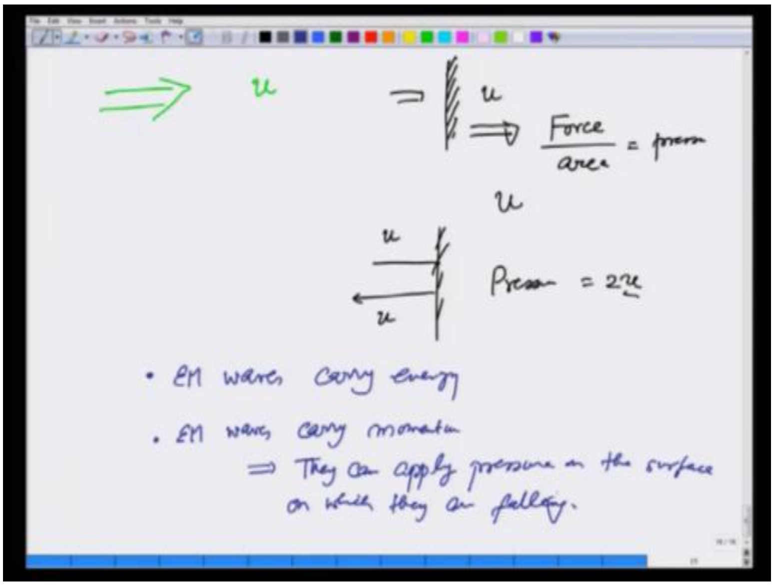

What does that momentum do? Suppose if i put a screen here which absorbs all this radiation fowling if all the radiation fowling is that absorbed that means all the moment u per unit are per unit time which is coming gets absorb in this. So, this thing should get momentum in this direction which is momentum per unit time per unit area areas nothing but force per unit area which is equal to pressure. So, an electromagnetic wave falling on steam in where get absorbs apply the pressure equal to energy density.

If, the wave reflected perfectly then the momentum coming in per unit area, per unit time would be u. Momentum going out with u again under pressure would be equal to two. So, I conclude this lecture by restating again that e m waves carry energy which we see in the form of light or heat. Which comes from towards from e m waves carry momentum and this implies they can apply pressure on the surface on which they are formed.