**Introduction to Electromagnetism**
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**Lecture - 06**
**Helmholtz' Theorem for Electric Field**

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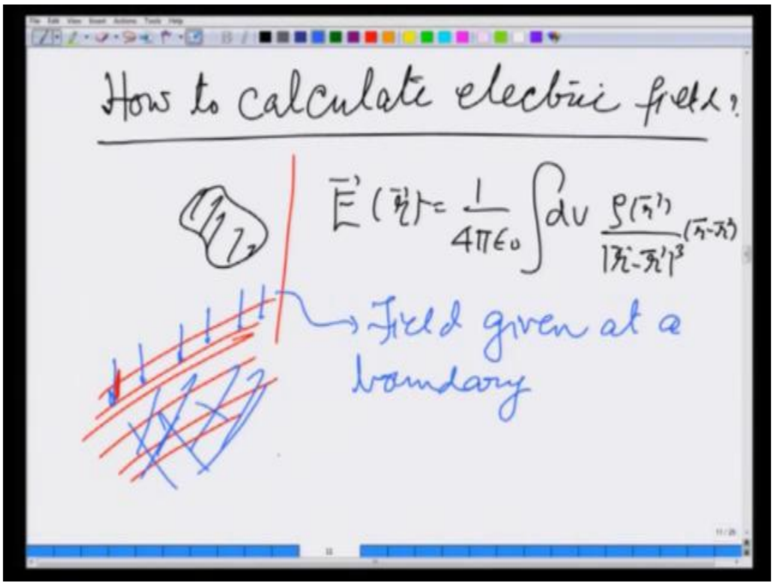


One of the jobs we are going to do is how to calculate electric field. One is obviously going to say that given a charge distribution, I am just going to do this E at r is going to be 1 over 4 pi Epsilon 0 integration d v rho r prime over r minus r prime r minus r prime cubed here. Is this how I am always going to calculate it, suppose I ask you different question. Suppose, I go to a region of space here, where I do not know about this charge, but I know field at these points, let me make it fill different color, suppose I know field at this point. So, what I will say is, field given at a boundary I do not know about the charge distribution but I do know field about a boundary. Can I calculate field out here in the rest of the space?

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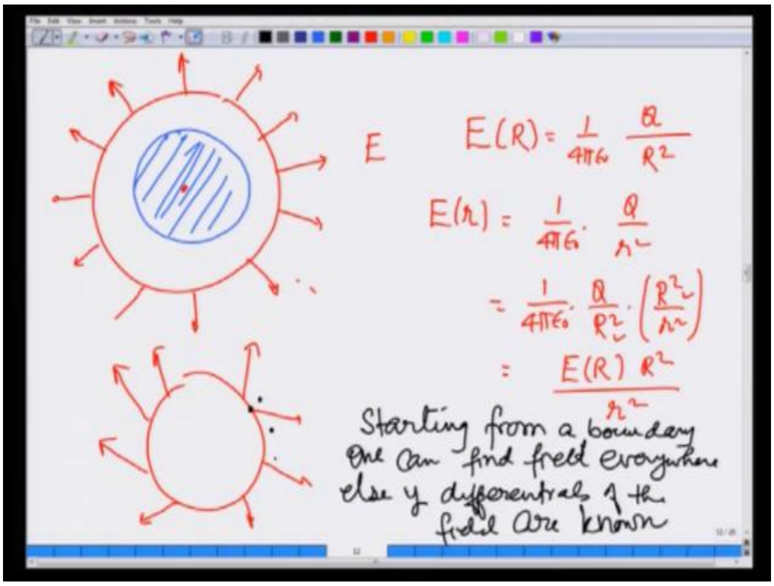


Let us look at an example, so where you do not specify the charge, how much charge the spherical body carries. But, what I give you is I hide that is spherical body, I give you that on this surface the field is all the same on this spherical surface and it is magnitude is given by some constant. Can I calculate the field in the rest of the space? In this case, it is very easy, because you going to take the center of the red sphere, you are going to say that E at the surface is; obviously, 1 over 4 pi Epsilon 0, because all is spherical some charge inside divided by R square that is the magnitude.

And therefore, E further out is going to be 1 over 4 pi Epsilon 0 Q over r square which I can write as 1 over 4 pi Epsilon 0 Q over R square times R square over r square. I have multiplied and divided by capital R square, which I am going to say E R times R square over r square. So, here I could argue, because of the nature of the field, because all spherically symmetric about a certain point. So, I could sense that this is due to a point charge, but what about if I am given the same surface, but field is you know has arbitrary direction, arbitrary magnitude on the surface. What would I do then?

Then; obviously, what I need to do is, take the field out here and using some sort of a differential equation, find out what it is next point, what it is at next point and then keep integrating. So, starting from a boundary, one can find field everywhere else if differentials of the field are known. Now, I already said why do we want to do that is, that not necessarily every time I will be knowing what the charge is somewhere. I may be given a boundary and where I know the field. How do I build it up from here?

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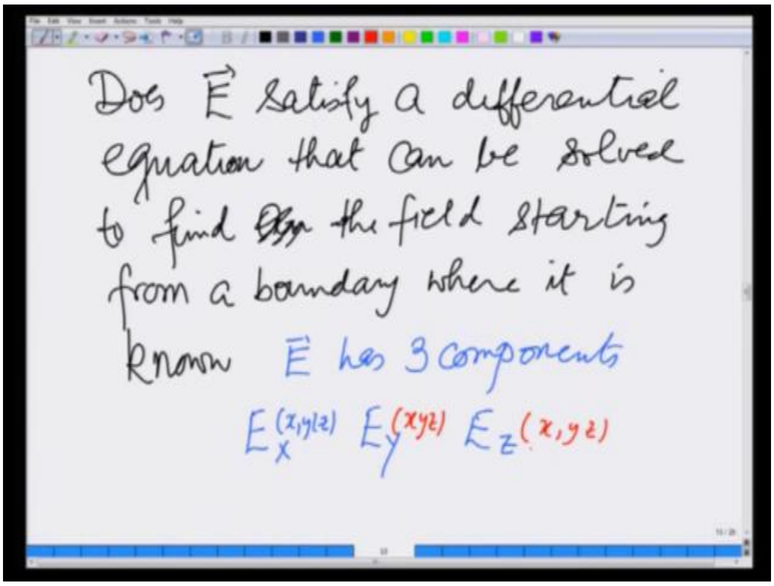


So, if I know the differentials I can do that, in other words what I am saying is does E satisfy a differential equation that can be solved to find the field starting from a boundary where it is known? Is the point clear? So, what we are doing is we are trying to find a differential equation, so that if I know field on a certain boundary, in a certain region I can build it up from there. Now, let us see how many definitions are there. E has 3 components, because it is a vector quantity, we have E x in x direction is y component and E z and these are all functions of x y and z, these are all functions of x y and z, x y and z.

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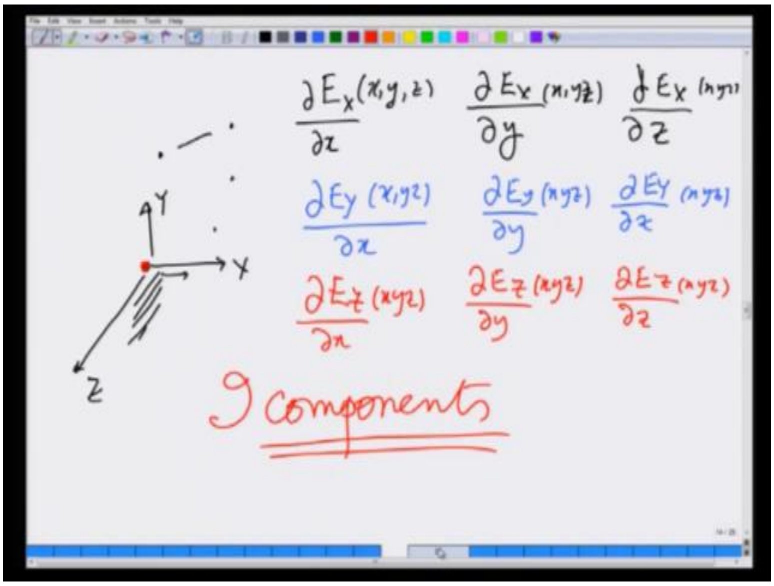


So, if I am going from point to point, this point to this point, this point this point all three components E x changes as I change x, but keep y and z fixed. That means, I am taking a partial derivative with respect to x, E x also changes if I change y or E x also changes, if I change z and so do all the other components. So, if I am at this point let us say this is x, this is y, this is z if I go from here in the x z plane, the E x component may changes as move here.

If I go in the y direction E x component may again change, so that is described by these three differential partial derivatives. Similarly, y component will change with all the three coordinates and so will the z component. So, in principle I need all these three 9 differentials, three for each component in order to calculate field at some other point starting from it is value from a given point 9 components and I should be listing all of them. But, we are fortunate there is a theorem and I am going to say without proving it the theorem called Helmholtz's theorem.

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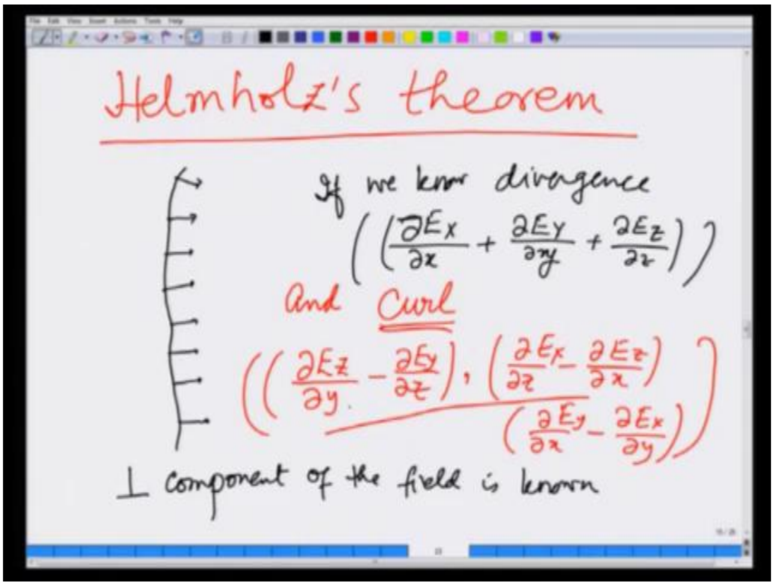


And what it states is given a boundary, suppose I know the perpendicular component off a field any vector field at the boundary. So, perpendicular component all the field is no, then I am going to define and explain those quantities later, if we know the divergence I am I will explain it is meaning in a minute which is the combination of derivatives and this from. The divergence that is some of the x component derivatives with respect to x plus the y component derivative with respect to y and z component z derivatives with respect to z.

If we know the divergence and another quantity called curl, if we know these two quantities, let me write it is components. For example, the x component of curl would be partial E z by partial y minus partial E y by partial z, y component will be partial z of x minus partial E z by partial x and the z component is going to be partial E y with respect to E x minus partial E x with respect to y. If I know this quantity the curl and if I know the divergence I can calculate field everywhere.

So, let us this is Helmholtz's theorem and if I know the perpendicular component on the boundary, then I need to know only curl and the diversions. Let us understand the meaning of these and that is what the rest of the lecture is going to be about today I am going to focus on divergence of a vector field.

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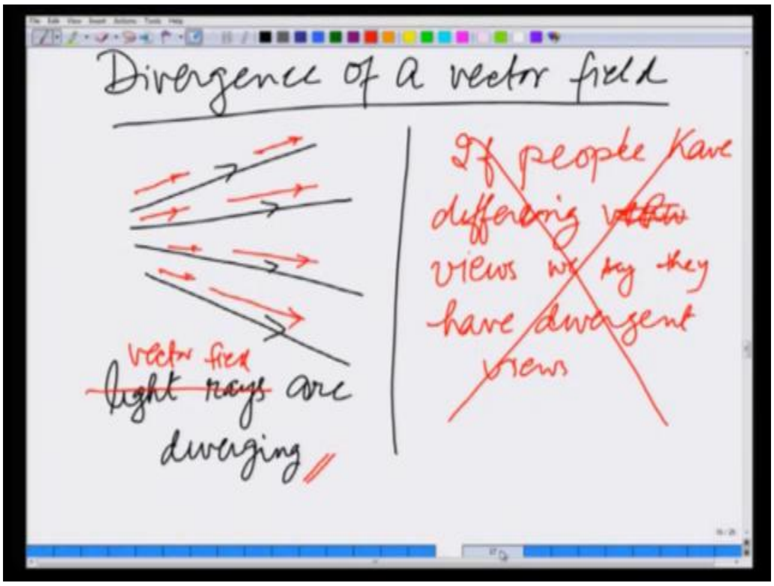


You heard the word divergent, divergence means we say views are diverging, there is diverging light rays coming. So, what basically it means is particularly, because you may have seen it in your 12th grad book, if light rays are going away from each other, we say light rays are diverging, if people have differing views we will say they have divergent views.

But, for electric field we are not going to be interested in this, what we are interested is this diverging mathematical quantities, light rays are diversion I could very well placed light rays by vector field and say these are the lines representing vectors of this field we are considering. What we want to get a view of what a field for what this divergence means and define it and in a very, very precise manner. When we say something as diverging an example to define it would be a fluid flow which I talk about earlier.

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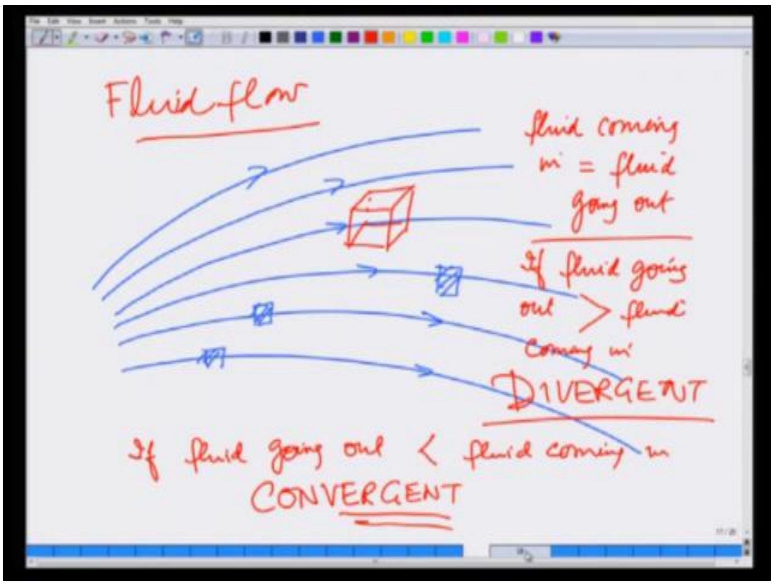


So, let us say this water or a fluid is flowing and all over the place it has a different current at different points and looks like light rays diverging that this velocity also diverging can we understand what this divergence means, can I give it definite mining. So, let us see if I take a volume small volume here, if whatever that fluid is coming in whatever is leaving is equal. So, suppose fluid coming in is equal to fluid going out in that case I would say it is not really diverging whatever comes in goes out.

On the other hand if fluid going out is greater than fluid coming in I was this divergent the fluid flow, the velocities of the current divergent. Because, they take away much more than they are bringing in or the other hand, if fluid going out is less then fluid coming in I will say this convergent. Because, that is like negative divergence, let us see make sense.

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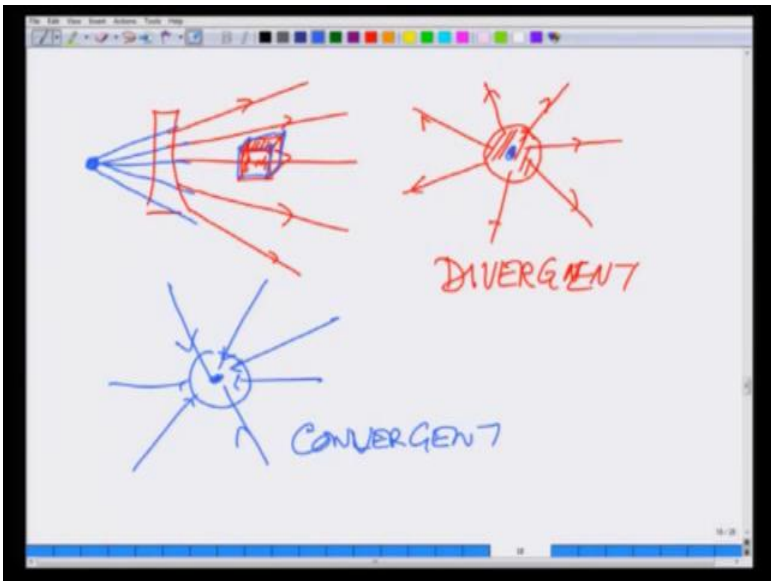


Let see light rays from a lens, because I say or diverging and if I take small volume here, in that case whatever light comes in is also going out, it maybe may not be diversion. But, on the other hand, if I take a source of like a point source of light and light is going out from here all around. If I take a small volume around it and see the light going out of this volume all over the light is going out, so this is really divergent.

On the other hand, if I take a point here we are all the light is coming in, then I will say this is convergent as converging to this point is diverging from this point, here at this point there is really no convergence and divergence they could be as we just discussed divergence of this point, because everything is coming out of here. So, this kind of gives you a field for what diversion behaviour is, divergence behaviour is going to be when something everything is going out or there is a net outflow or there is a net inflow this negative divergence or something going in.