Introduction to Electromagnetism

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Lecture - 58
Electromagnetic Waves in Free Space –I
Qualitative Picture

With the wave equations set up in the previous lecture, we are now ready to see how electromagnetic waves actually come into being.

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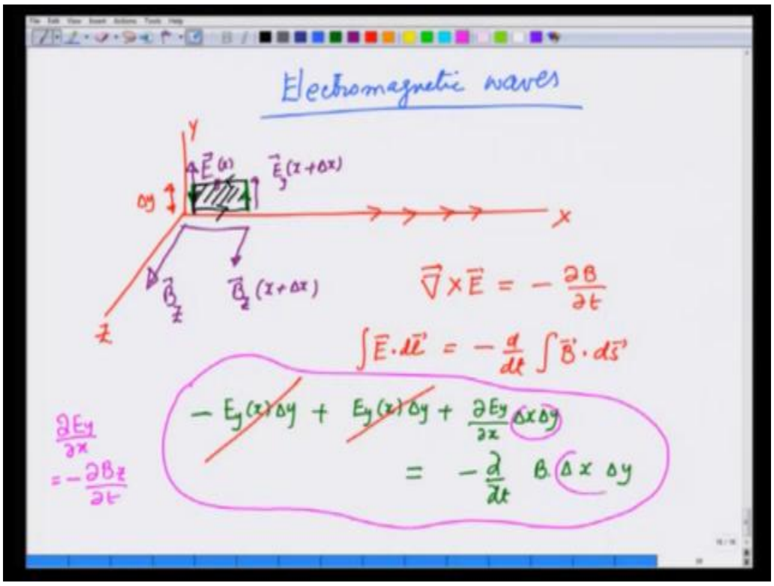

We are going to keep our discussion very simple and consider plane waves; that means, the amplitude is going to be same over a plane travelling in one direction. In particular, I am going to take a wave travelling in the x direction. So, I am going to motivate you by taking E in the y direction and B in the z direction; and show you that, if they vary with time and space, they can sustain each other. So, let us say E is at x; and, as you go little farther out, E changes to E x plus delta x; E is in the y direction. Similarly, B is in the z direction. So, let us put z here and, as you go little farther out, it changes to B z x plus delta x and it also is changing with time.

Now, I know from Faraday's law that, curl of E is equal to minus dB dt. In particular in this case, if I take a square like this or a rectangle like this, travelling or traversing it counter clockwise as shown these by black arrows, I know that, by Faraday's law, I am going to have integral E dot dl is equal to minus d by dt of B dot ds; where, ds is the area ((Refer Slide Time: 02:10)). Now, since E is in only y direction, therefore, the horizontal lines in this do not contribute to the line integral. Let the width of this rectangle be delta y. And then, I am going to get by previous – whenever we proved this Stokes' theorem, I am going to get E y at x times delta y in the negative direction from the left-hand side black line. And, on the right-hand side, I am going to get plus E y at x delta y plus partial E y partial x delta x delta y on the right-hand side line going up. This is the line integral, which is going to be equal to minus d by dt of B times delta x delta y. Now, I am going to put a partial derivative here, because now this thing is not under the integral sign anymore. Let us cancel a few terms. This cancels with this and I end up getting this whole thing; then, gives me partial E y partial x is equal to minus partial B, which is in z direction partial t, because these two terms also cancels.

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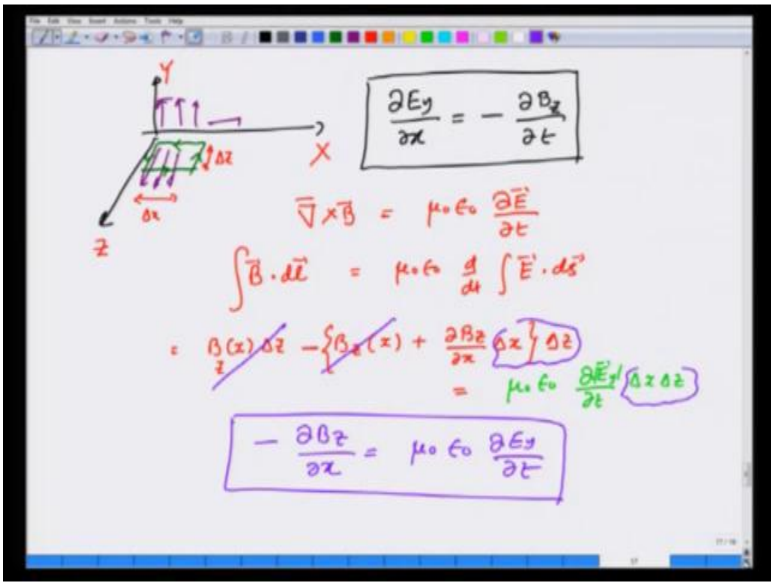

So, what we get from this is what I am taking is x direction, y direction, z direction. We have taken E going this way and B going this way; wave travelling to the right. We will again see how it travels to the right. And, I get partial E y over partial x is equal to minus partial B z over partial t. That is one equation. Let us now apply the other equation, which gives me curl of B is equal to mu 0 epsilon 0 dE dt. Remember this is coming from the displacement current. For this, I will take a rectangle in the xz plane here and traverse it counter clockwise. Again, remember this is my x direction; this is my y direction; this is my z direction; and, this distance is delta x; this is going to be delta z.

Now, Stokes theorem gives me B dot dl integration is equal to mu 0 epsilon 0 d by d t of E dot ds. B dot dl like in the previous argument, is going to be B at x. And, we are going again in the same direction as z. So, this is going to be delta z. The lines parallel to x-axis do not contribute, because B is perpendicular to that line minus B of z at x plus delta x, which I can write as B x plus partial B z over partial x delta x multiplied by delta z. And, this is going to be equal to mu 0 epsilon 0 dE dt; E is in y direction; let us write the magnitude – times delta s, which is delta x, delta z. Again we are going to cancel terms; this term will cancel with this; delta x, delta z cancels with this. And, I end up getting the equation minus d B z dx is equal to mu 0 epsilon 0 d E y dt.

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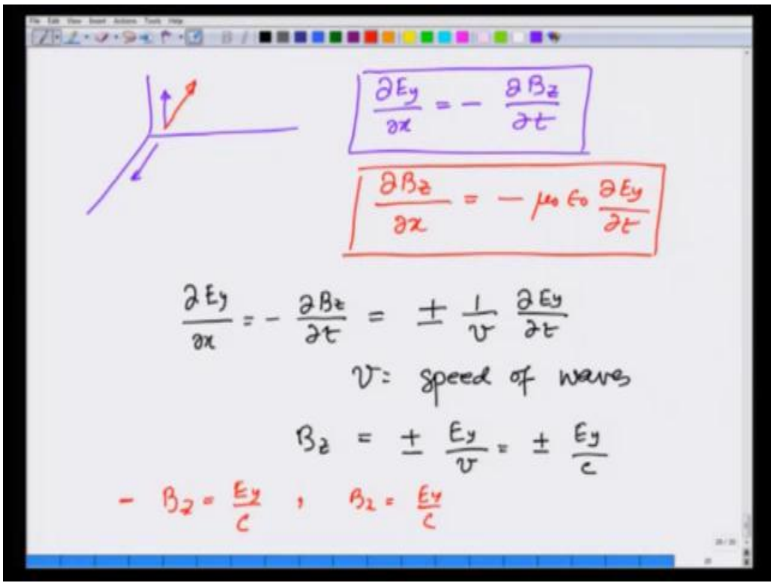

So, the two equations that we have obtained are for this E y going in this direction B z. We have obtained are partial E y partial x is equal to minus partial B z partial t. That is one equation and, the other equation is partial B z over partial x is equal to minus mu 0 epsilon 0 partial E y partial t. Let us see what we can learn about these fields from these two equations. Now, from the equation, d E y d x is equal to minus partial B z partial t. We see if the wave is travelling this should be equal to plus or minus 1 over v partial E y partial t; where, v is the speed of wave. And, this immediately tells you that, B z should be equal to plus or minus E y over v. I know electromagnetic waves travel with speed c. So, let us write this or we denote it by c – E y over c. The plus sign is for the wave travelling to the left; and, the minus sign is for the wave travelling to… And therefore, if the wave is traveling to the left, minus B z would be equal to E y over c. And therefore, if the wave is travelling to the left, B will be pointing in the negative z direction. On the other hand, if I have B z equals E y over c, a wave would be traveling to the right. What you can see is that, E cross B gives me the direction of the wave. Let us see how we can combine these two waves – these two equations.

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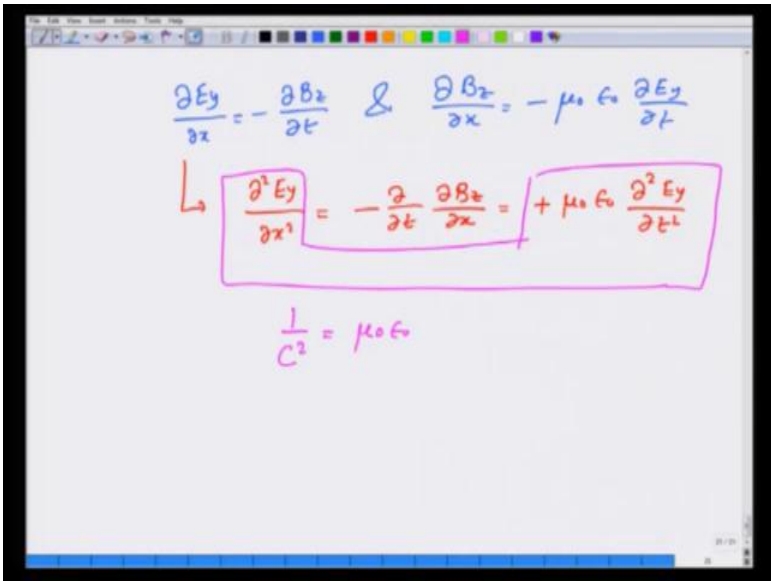

So, the equations I have is partial E y over partial x is equal to minus d B z over dt and partial B z over partial x is equal to minus mu 0 epsilon 0 d E y by dt. If I take this first equation; differentiate it once more with respect to x, I get d 2 E y over dx square is equal to minus d by dt of d B z by dx. I have switched x and t, which is equal to plus mu 0 epsilon 0 d 2 E y over d t square remember this is the wave equation with the speed 1 over c square being equal to mu 0 epsilon 0. So, what we have argued so far is that, if I have two perpendicular fields E and B, which vary in both in time and space, then it is possible for them to propagate and sustain each other and they propagate with a speed c, which is equal to 1 over square root of mu 0 epsilon 0.

In the next lecture, we will be obtaining this wave equation more rigorously directly from Maxwell's equations.