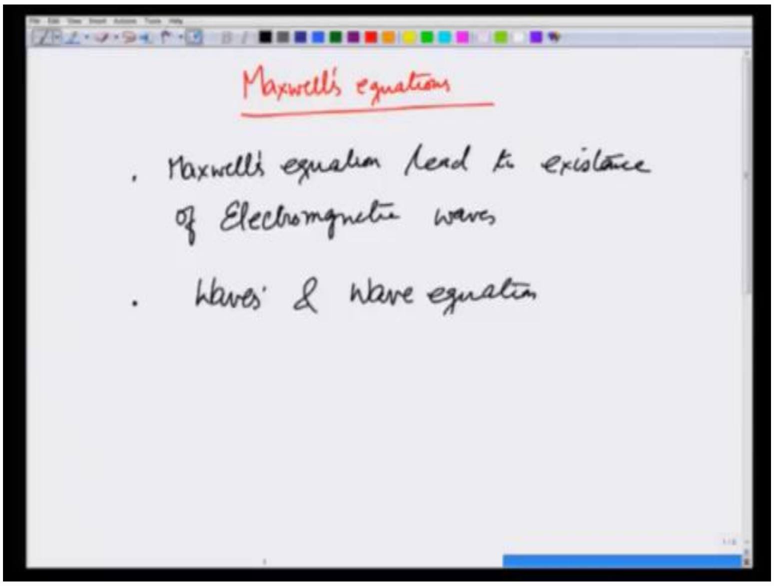
# Introduction to Electromagnetism

**Prof. Manoj K. Harbola**
**Department of Physics**
**Indian Institute of Technology, Kanpur**

## Lecture - 57
## Waves and Wave Equation

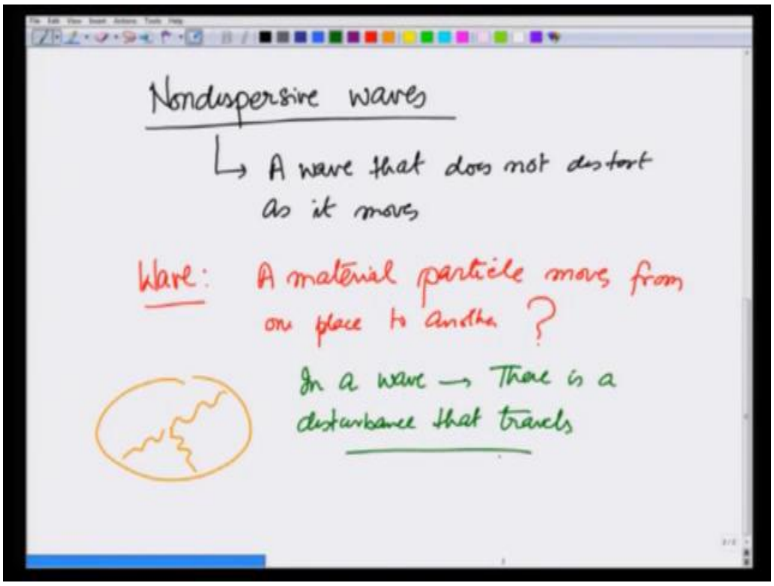
We have been talking about Maxwell’s equations.

(Refer Slide Time: 00:11)



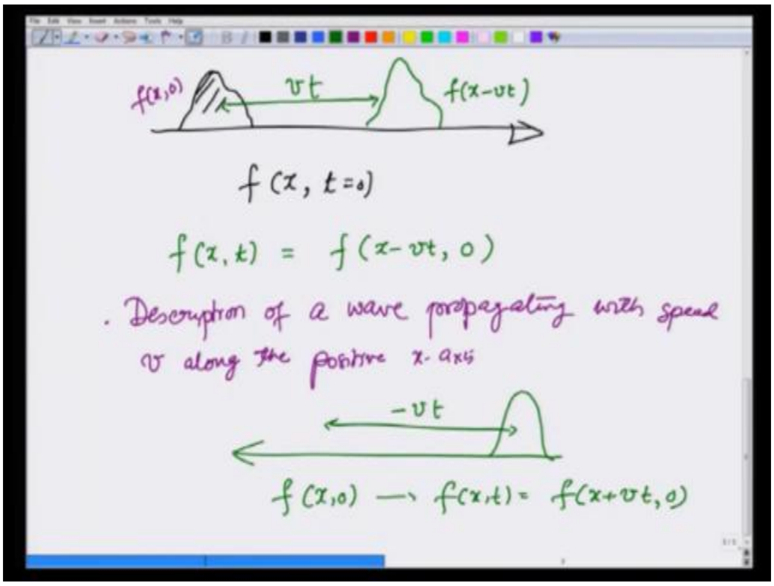

And, what we are going to do today is talk about how Maxwell's equations lead to existence of electromagnetic waves. What we mean by electromagnetic waves are these electrical and magnetic disturbances that sustained each other and propagating space. To understand it fully, we should actually first understand what waves are – waves and wave equation. So, in this lecture today, we are going to spend some time to understand what wave equation is, where it comes from, what wave phenomenon is.

(Refer Slide Time: 01:24)



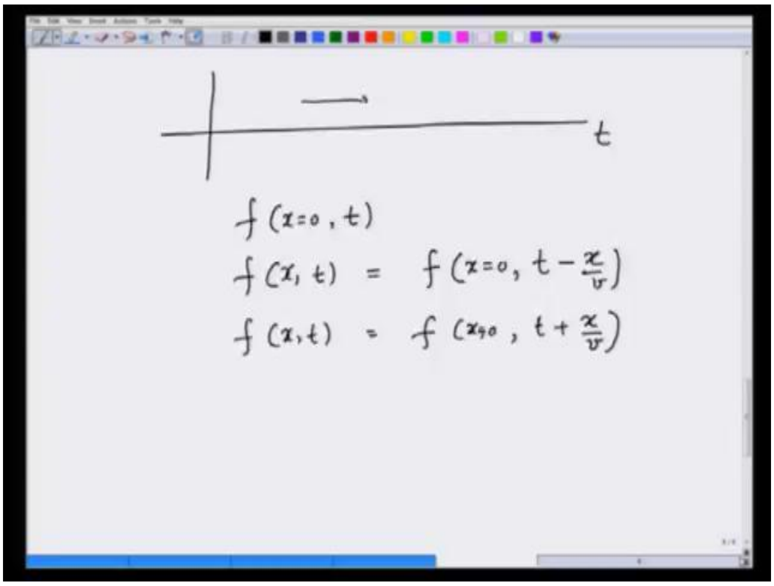

I am going to talk about nondispersive waves. By nondispersive I mean a wave that does not distort as it moves. Simply put this is what it is. So, let us understand what a wave is. In particular, we want to understand – does a wave mean that, a material particle moves from one place to another? And, the answer is no. For example, you have seen water waves, where if I make a disturbance, it travels out, but we do not see water going out. Similarly, when I am speaking, there is a pressure wave that is going, but it does not mean that, air is moving from one side to the other with this special wave. What happens is – in a wave, there is a disturbance that travels. So, when I am speaking, I am creating a disturbance in the air out here; in that, I am creating a pressure difference. That pressure difference travels, but the material particle does not go from one place to the other. Similarly, in water waves, I create a disturbance that travels from one place to the other, but water does not move from one place to other.

(Refer Slide Time: 03:23)



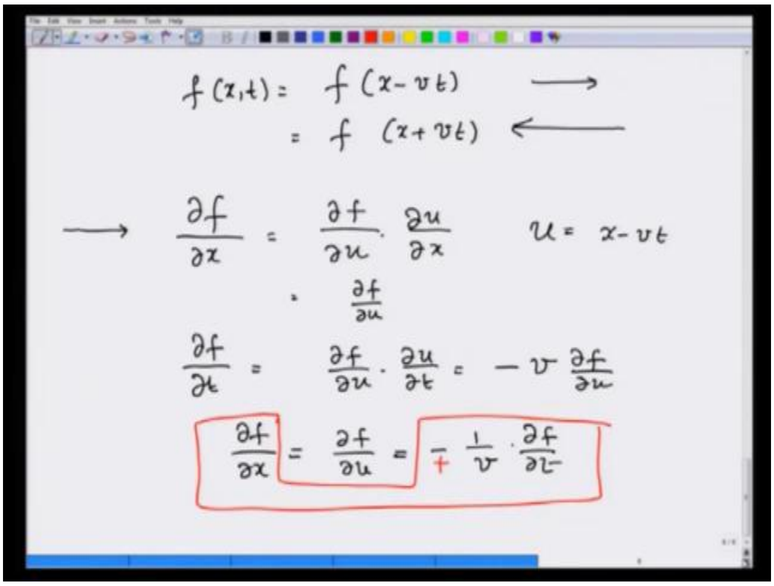

So, what we are talking about in a wave is suppose I am thinking of a wave travelling towards the x-axis; if I have a disturbance and let us call it or denote it by disturbance x at time t equal to 0; it travels undistorted. What it means is that, as it moves, it remains pretty much the same. So, let me try to make it pretty much the same as well as possible except that it has moved by a distance v t. The function has remained the same. How do I describe this function as a function of x and t? Since the function has changed only the position, it is nothing but f – the origin has shifted by v t. So, I can write this as f x minus v t time t equal to 0. This is f x minus v t and this is f x at time t equal to 0. So, what we are saying is that, the disturbance remains the same and it has shifted by distance v t. So, this is what I will call description of a wave propagating with speed v along the positive x-axis. In a similar manner, I can think of this wave propagating in the opposite direction; then, it would have moved by a distance minus v t and I would write this function, which was x, 0 going to f x, t, which is f x plus v t, 0. That is how the function changes as it moves along.

(Refer Slide Time: 05:35)



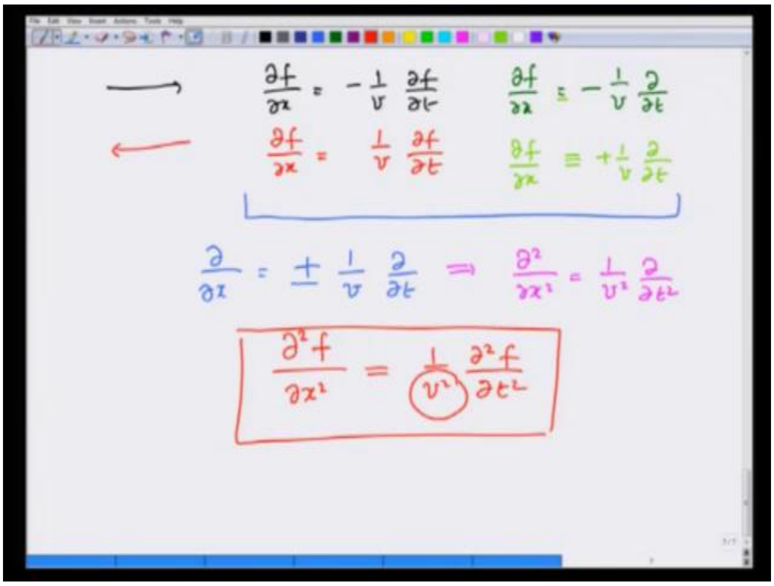

There is another way of looking at it. Suppose I plot this function with respect to time. At x is equal to 0, let us say this is at f x is equal to 0 and it is given as a function f at x is equal to 0 to the function of time t here. Then, if I am looking at function at x, at the same time, it would be f x t is equal to function at x equal to 0 at a previous time t minus x over v. That is another way of representing a wave traveling to the right. If the wave was traveling to the left, I would have had f x, t equals f at x is equal to 0 at time x over v. So, this is another way of representing a wave. What does this mean in terms of the equation?

(Refer Slide Time: 06:37)



Let me now work with the form of f x, t is equal to f x minus v t for right traveling wave. The second term is 0. So, I do not even write it; or, f x plus v t for the left traveling wave. Let us differentiate for the right traveling, that is, with respect to x, which will be df by du du by dx; where, u is equal to x minus vt. This gives me df by du itself. Similarly, df by dt is going to be df by du times du by dt, which is minus v df by du. This immediately makes it clear that, for the right traveling wave, I have df by dx is equal to df by du which is equal to minus 1 over v df by dt. And this is the wave equation for wave that is traveling to the right. For the wave which is traveling to the left, this sign will become plus.

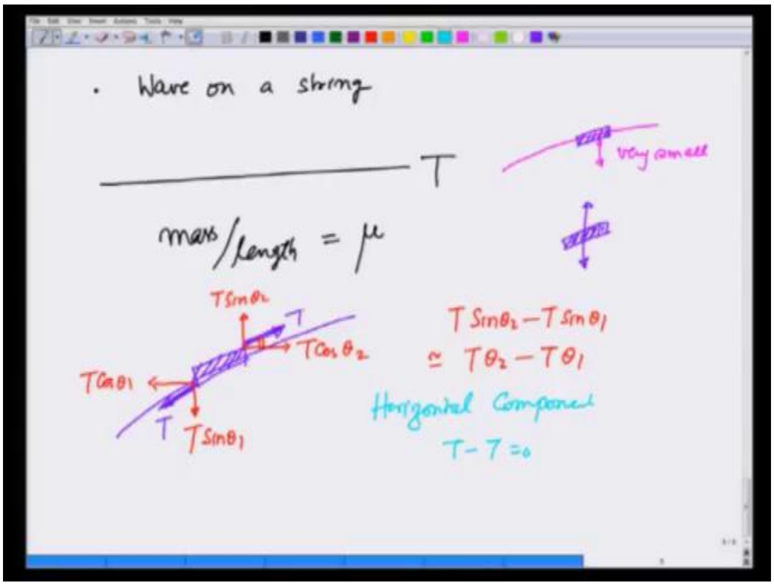
(Refer Slide Time: 07:59)




So, I have got an equation for the wave, which is traveling to the right, which is df by dx is equal to minus 1 over v df dt or for the wave which is travelling to the left – df by d x is equal to 1 over v df dt. In both, I can say that, df dx – the upper one is equivalent to taking a derivative with respect to t and multiplying by 1 over v. In the second case, I can say that, df dx is equivalent to – I should put an equivalent sign – plus 1 over v d by dt. These are valid wave equation for wave travelling to the left or travelling to the right. What about a wave, which is a superposition of the two? What about a wave, which does not travel and do I have to always specify whether it is traveling to the left or traveling to the right? We do not do that, instead we combine these two equations, write a general equation for any way, which may be travelling to the left or travelling to the right or could be stationary wave – not travelling at all like a string vibrating.

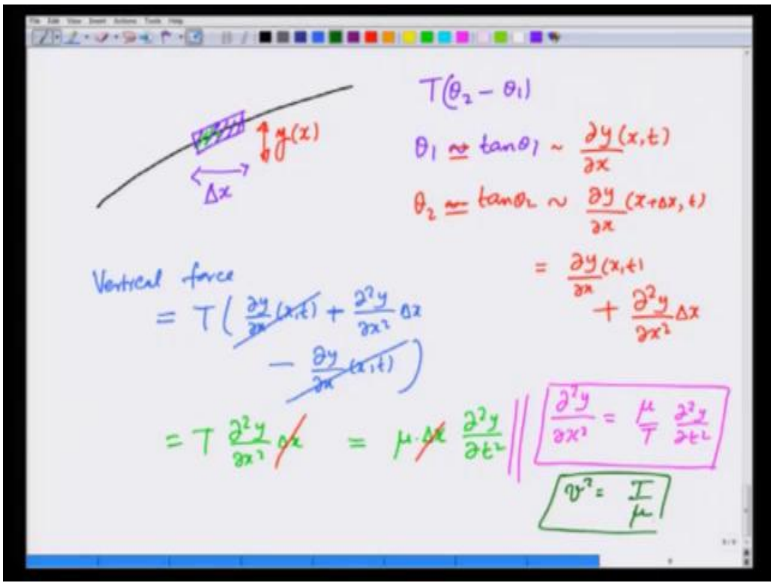
In that case, as you notice that, since we wrote d by dx is equivalent to plus or minus 1 over v d by dt, this immediately implies that, the secondary wave d 2 by dx square would be nothing but 1 over v square d by dt square. And therefore, I can write for any wave traveling to the left or traveling to the right that, d 2 f by dx square is equal to 1 over v square d 2 f by dt square. This describes a wave traveling to the left, traveling to the right or a superposition of the two or a stationary wave or whatever. If I can get for the motion of a disturbance in a system, an equation like this; then, I automatically get v also. Let us now see this through examples.

(Refer Slide Time: 10:07)




As example 1, I take wave on a string; that is, there is a string, which has tension T in it; and, its mass per unit length is equal to mu. If I distort it assuming that, the amplitude is very small, its amplitude very small; then, if I take a small portion in it right here; if I look at this portion, this portion moves up and down as the wave progresses. And, let us get the equation for that. This moves up and down because there is a restoring force because of the tension. Now, let us look at this part here. There is tension T pulling it this way; there is tension T pulling it this way. This tension on right-hand side has two components – T cosine of – let us call it theta 2 and vertical component T sin of theta 2. This angel theta 2 is going to be slightly different from the angle. On the left-hand side, this is going to be T cosine of theta 1; and, this component T sin of theta 1. And therefore, the net vertical force is T sin theta 2 minus T sin theta 1. And, if the amplitude is very small, I can roughly write this as T theta 2 minus T theta 1. And, the horizontal component is T minus T, which is 0. So, this balance in the two vertical components makes the string go up and down. Let us calculate theta 2 and theta 1.

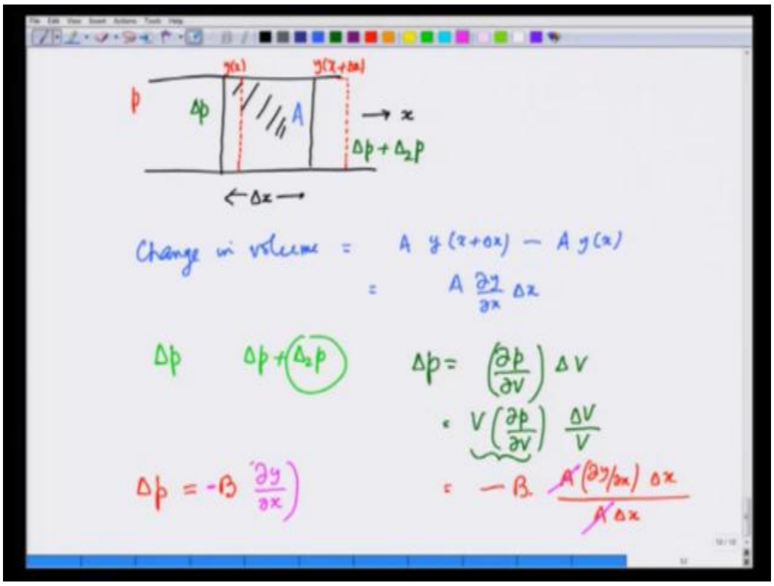
(Refer Slide Time: 12:18)




So, we are looking at this string and a small portion here; the net forces T theta 2 minus theta 1. If I take the length here to be delta x, theta 1 is roughly tangent theta 1; and, if we take this displacement to be y as a function of x, this is partial derivative dy by dx. I am writing partial because y is a function of x and t both. Theta 2 on the other hand is going to be tangent; theta 2 – I should not write an approximate sign. This is exactly equal; which is equal to dy by dx at x plus delta x at the same time t; which I can write as equal to dy by dx at x and t plus the second derivative d 2 y by d x square delta x ((Refer Slide Time: 13:24)) it.

So, the vertical force comes out to be T – theta 2 is dy by dx at x, t minus or plus d 2 y by d x square delta x minus dy by dx at x and t. This cancels and this is what I get. This force is equal to T d 2 y by d x square delta x. And, this should be equal to the acceleration of this small piece here. Its mass is going to be mu times delta x d 2 y by d t square. That is the acceleration. You combine the two; delta x cancels from both sides and you get d 2 y by dx square is equal to mu over T d 2 y by dt square. This is precisely the wave equation we had obtained earlier. And therefore, there is going to be a wave setup in this string. It could be a progressive travelling wave; it could be a stationary wave. But, this is the wave, which will be going to be set up with the speed square – v square being equal to T over mu. So, this one example how wave equation is obtained in a system and that automatically gives you the speed of wave also.

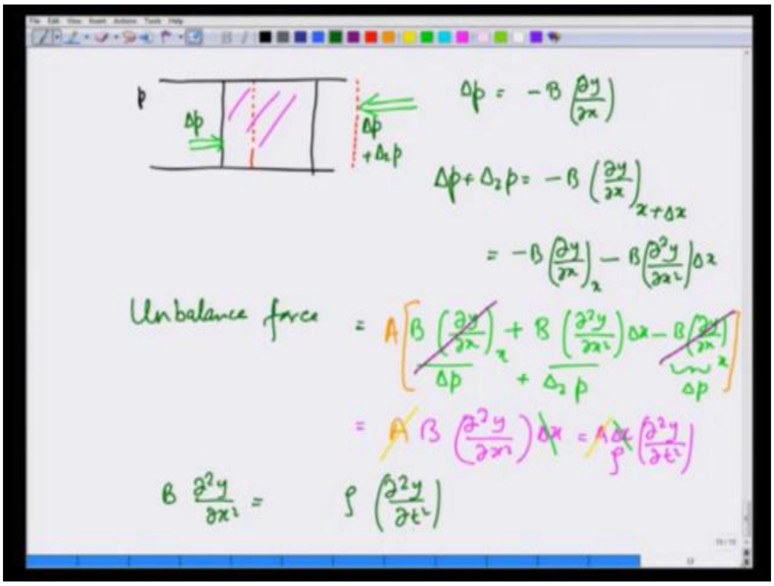
(Refer Slide Time: 15:01)



As a second example, I will take pressure wave travelling down a tube in the direction x. For this, I consider a portion of gas or liquid in this, which is here of length delta x. This is at some pressure p. And, we create an extra pressure here – delta p, so that the left-hand side moves by a distance y x. This pressure propagates and it also change its position, so that on the right-hand side, this line moves by y x plus delta x. And, pressure out here changes by delta p plus some delta – let me call it 2 p, which is a second order. What we want to do is see this pressure difference what force does it create; relate that to the acceleration of this small portion that we have taken and see what is the equation that we get.

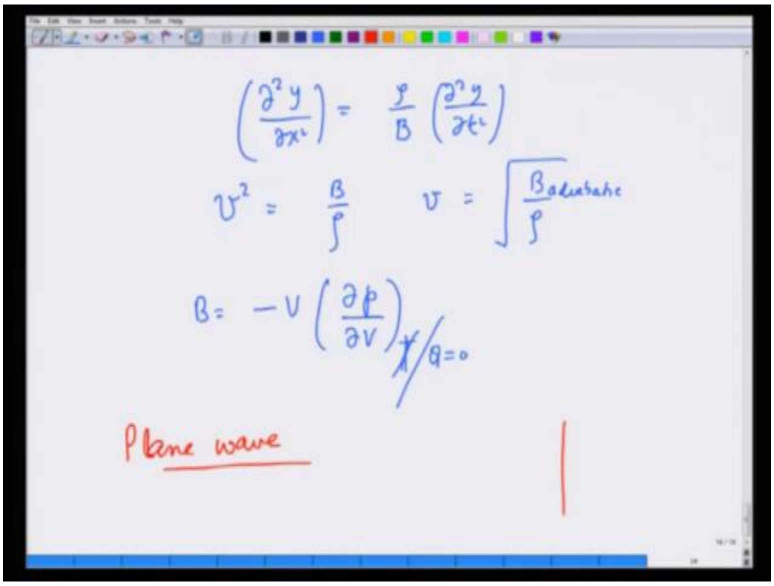
First, what this delta p does? It changes the volume. Let us first calculate the change in the volume assuming that this cross section out here is A. So, change in volume is going to be A y at x plus delta x minus A y x. This is the change in the volume of this portion that we took between two vertical lines. And, this is nothing but A dy by dx delta x. This change in volume is caused by this pressure delta p on the left-hand side – delta p plus delta 2 p on the right-hand side. This delta 2 p is the unbalanced force that actually gives rise to the acceleration of this portion. So, the compression comes from this delta p. By definition, delta p is equal to dp dv times delta v; which I can write it as v dp dv delta v over v. Notice that, this quantity v dp dv is the bulk modulus. And therefore, I can write this as minus B; delta v is A dy by dx delta x over the volume of this element is A delta x. So, delta p is given as minus B – A cancels here – dy by dx.

(Refer Slide Time: 18:07)



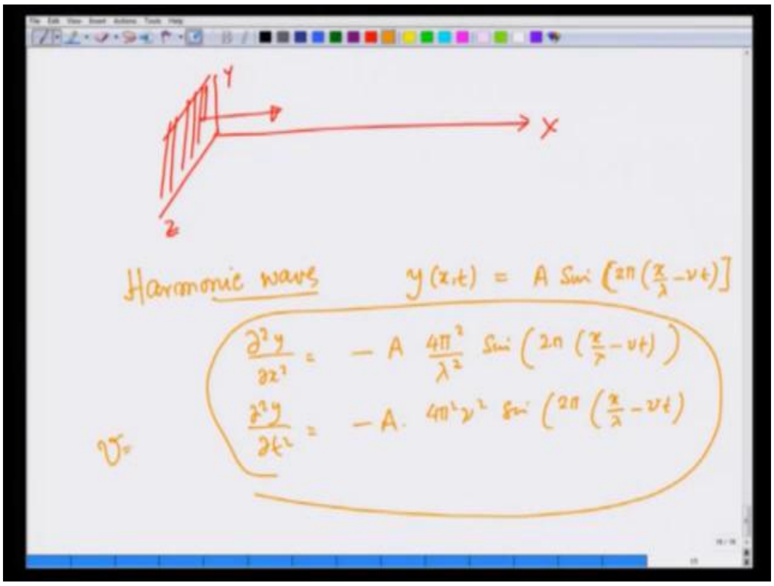
So, what we have calculated is that, in this tube, which is at pressure p; when I apply a delta p here, the small volume here changes and this delta p is given as minus B dy by dx. The pressure on right-hand side is delta p plus delta 2 p. So, delta p plus delta 2 p is going to be minus B dy by dx at x plus delta x. And, this is going to be minus B dy by dx at x minus B d 2 y over dx square times delta x. So, unbalanced force is going to be this pressure here acts to the left; delta p out here acts to the right. And therefore, unbalanced force is going to be B dy by dx x plus B d 2 y by dx square delta x minus B dy by dx at x. This is nothing but delta p; this is delta p plus delta 2 p. This term cancels and I get the unbalanced force to be B d 2 y by dx square delta x; which is going to give an acceleration to this portion out here; and, that acceleration is going to be d 2 y by dt square; and, the force should be equal to mass times this acceleration. And, mass of this portion is going to be this volume – A delta x times the density rho. And, I should also apply for the force by area out here – area out here. Let us cancel things on two sides; I cancel this A; we cancel this delta x. And, we end up getting the equation that, B d 2 y over dx square is equal to rho d 2 y by dt square.

(Refer Slide Time: 20:50)



And therefore, the equation is d 2 y by d x square is equal to rho over B d 2 y by d 2 square. This immediately gives me that, velocity square is B over rho. You know this formula that, velocity of sound in a medium is square root of B over rho. When we wrote B, which was equal to minus v dp by dv, we did not specify whether there it was a constant temperature or adiabatic. You know very well that, what is argued is that, when the changes are taking place in air, hardly any heat transfer. Therefore, we take ((Refer Slide Time: 21:39)) to be B at Q equal to 0 or adiabatic B. So, let me write this adiabatic B. So, I have shown you two examples, where by considering the equation of motion, we got an equation, which is the wave equation from which we could determine what the speed of sound or speed of that disturbance or a speed of wave in that medium is going to be. And finally, this wave equation – solution gives me how this disturbance is going travel. Now, a particular example of this is the plane wave; whereby it depends on x and the amplitude is all over is independent of y and z.

(Refer Slide Time: 22:24)




So, for example, if I have xy and z, a plane wave would have this wavefront, which has the same amplitude all over the yz plane and this disturbance travels in the x direction. We also considered harmonic waves, which have a disturbance by y x, t is equal to A sin 2 pi x over lambda minus nu t. You can see this satisfies the equation d 2 y over d x square. In this case, it is going to be minus A by 4 pi square over lambda square sin of 2 pi x over lambda minus nu t. And, d 2 y over d t square is going to be minus A minus 4 pi square nu square sin of 2 pi x over lambda minus v t. And, you can check that, this then satisfies the wave equation with velocity being equal to nu times lambda.