Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 55
The Poynting vector; Solved Examples

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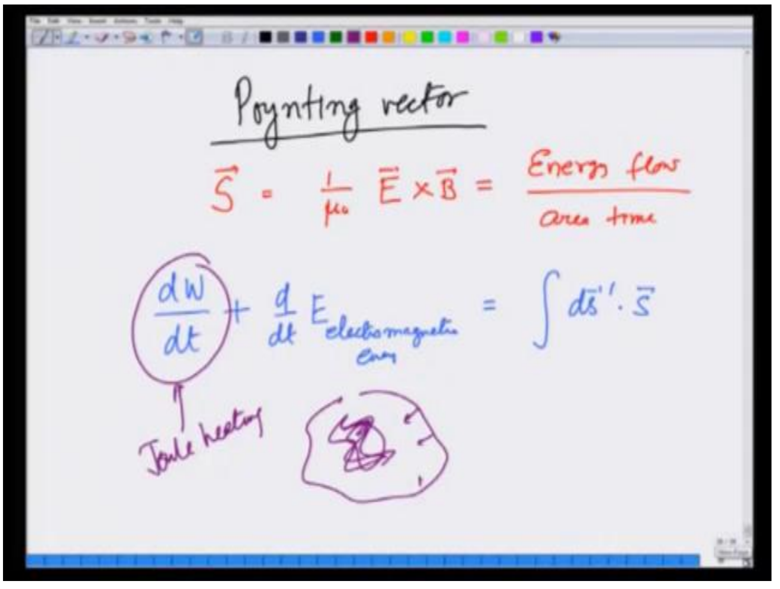

In the previous lecture, we have introduced Poynting vector for electromagnetic fields, and shown that this is equal to 1 over mu 0 E cross B and is equal to the energy flow per unit area, per unit time or power per unit area. What does this energy do? We saw in the previous lecture that this either increases, the mechanical energy of the charges inside or the net electromagnetic energy.

So, that this comes out to be equal to d s prime dot S, where let me remind you again, I am talking about a volume like this, d s prime is pointing in and then we are talking about energy content and mechanical energy content inside this volume. If you increase the mechanical energy, it may finally, come out as joule heating, because these charges start moving around, when gain kinetic energy and then lose it as heat.

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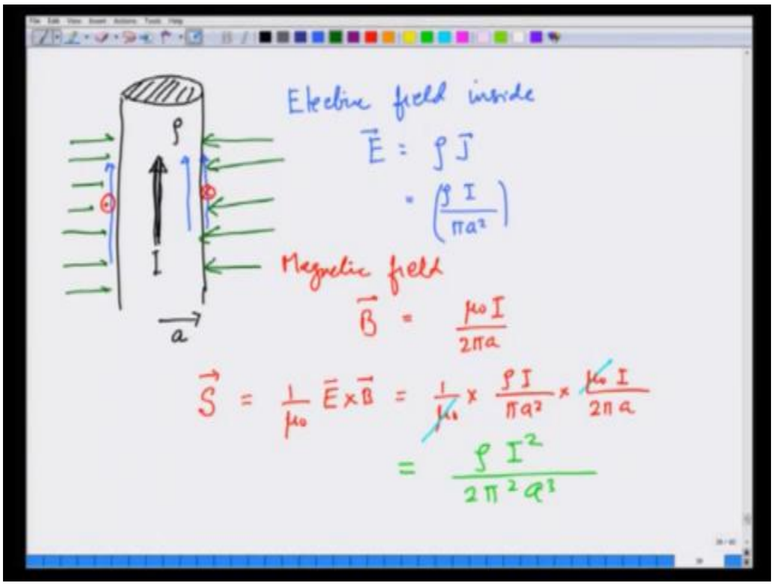

In this lecture, I am going to do two examples to show this is true. As the first example, let us take a thick current carrying wire, this is a very standard example that is solved of radius a and it is carrying a current I. Suppose, this resistivity is rho, then if I look at electric field inside E is rho J, this is a well known Ohms law result, which is nothing but rho I over pi a square, this is the electric field.

An electric field direction is in the direction of the current and by boundary condition right outside, the surface also E is going to be the same. So, this is E, how about magnetic field B, it is equal to mu naught over 2 pi a I right at the surface and since, the current is going up, the magnetic field is going to be going in here and will be coming out here. We make the electric field also it is going up, so this is B, I have shown the directions.

So, the poynting vector S is going to be 1 over mu 0 E cross B and you can see by looking at the direction that it is going to be pointing into the volume and it is value is going to be 1 over mu 0 times rho I over pi a square times mu 0 I over 2 pi a. Let us cancel a few terms, this mu 0 cancels and you end up getting rho I square over 2 pi square a cubed and the direction of the poynting vector is into the volume as you can easily see by taking E cross B.

So, we have this energy flowing into the volume of this wire. In reality, there are also surface charges that give you an E field perpendicular to the wire that makes the energy flow along the wire, but that is takes us beyond, what we want to show. Here, we just want to show that the mechanical energy or the heat that is being produced inside is actually brought in by poynting vector and this is sufficient to show that.

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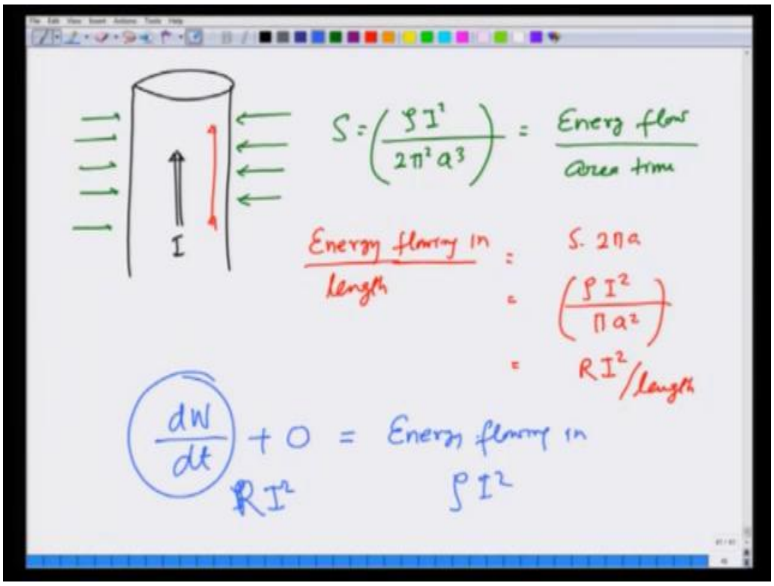

So, now what we are doing is, we are taking this wire in which there is a current I flowing and we have shown that, there is this poynting vector; that is taking the energy into the volume of this wire and S is nothing but rho I square over 2 pi square a cubed. Keep in mind that this is energy flow per unit area per unit time. So, if I take a unit length of this wire and see, how much energy is flowing into that unit length is going to be. So, energy flowing in per unit length, let us write this flowing in is going to be S times 2 pi a times the unit length, which is 1.

So, this will be rho I square over 2 cancels pi a square, this is a energy flowing in per unit length, which I can write as the resistance rho over pi a square is rho divided by the area of the wire times the length is 1. So, this is resistance times I square per unit length, this is the energy flowing in. So, if I go back to that energy balance equation d w by d t, since the current is steady, there is no change in the electromagnetic energy. So, this second term is 0 is equal to energy flowing in.

From joule heating, I know this term is nothing but R I square right hand side also comes out to be R I square. So, we have shown that this energy, which is being spent or which is being the heat which is being produced in the wire is actually brought in through these electromagnetic fields.

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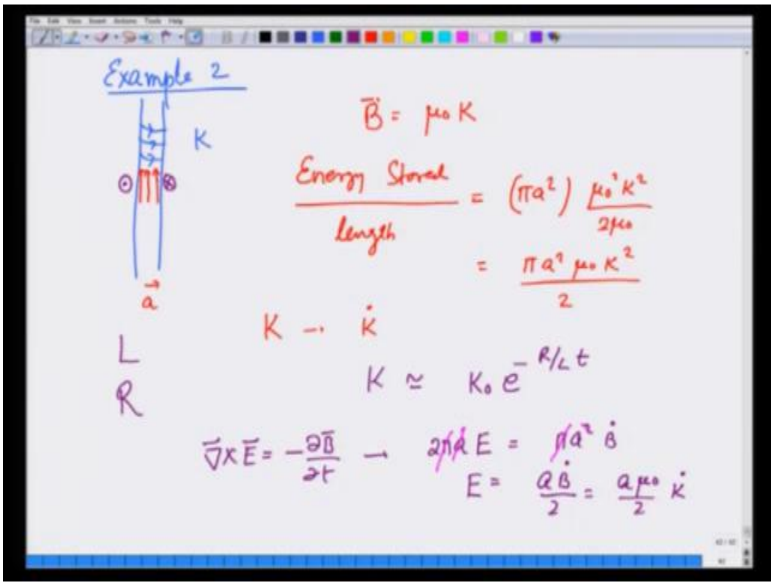

As a second demonstration of this concept, let me take a solenoid, this is example 2, let me take a solenoid, which has the surface current K. So, that the field inside B is mu 0 K, the energy stored per unit length is going to be the cross section let us again take the radius to be a. So, it is going to be pi a square times length 1 is the volume times B square mu 0 square K square over 2 mu 0. So, this is pi a square mu 0 K square over 2; that is the energy stored.

Let us now switch off the current K at a rate K dot, so that it slowly goes to 0 in sometime. Now, if I switch it off, I know if I can modulate in many different ways, but suppose this inductance is L and the resistance of coil is R, then K is going to go down like K 0, E raise to minus R over L t. But, that is not our concern; our concern is that K is being switched off.

If K is being switched off, the magnetic field also changes and therefore, the magnetic field is going down is becoming smaller and it will produce by Faradays law del cross B del cross E equals minus d B d t, it will produce an electric field. And such that, the field it will try to oppose the change in the field. So, it will try to produce a field in the same direction and therefore, the electric field produced will be such that, it goes in on the right side, I am making it on the solenoid and comes out on the left side.

Using Stokes theorem, I can write 2 pi a times E at the surface is going to be equal to pi a square the flux change times B dot with a minus sign. So, minus sign we have already taken care of by looking at the directions. So, E comes out to be pi cancels on both sides, one of the a's cancels, E comes out to be a B dot over 2, which is nothing but a mu 0 over 2 K dot; that is the electric field going in. So, right at the surface I made it outside just inside also the field would be the same, because parallel component electric field is always the same.

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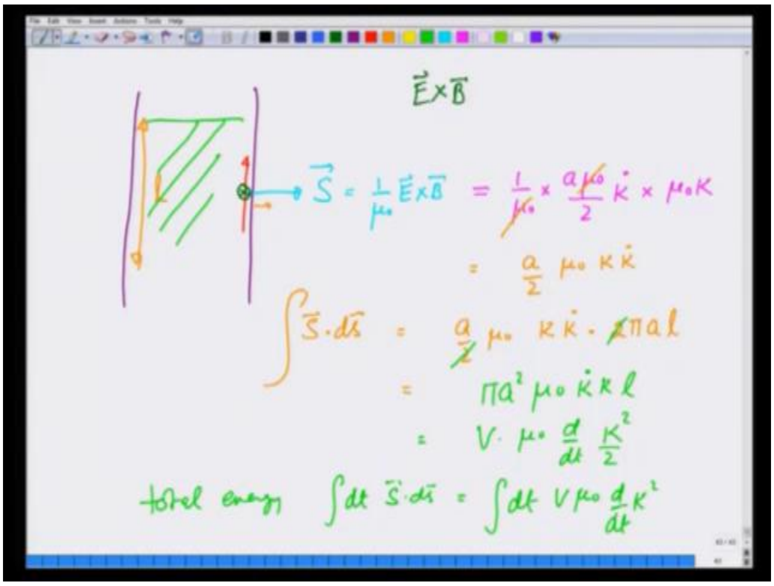

So, what I have now is that in the solenoid, let me make it slightly bigger on the surface here is the B field and here is the E field going in. So, E cross B is pointing out, this is S vector which is equal to 1 over mu 0 E cross B. That means, now the energy will be flowing out of this volume and it is value is 1 over mu 0, E we have just calculated is nothing, but a mu 0 by 2 K dot and B we have already seen it is mu 0 K. Now, again I cancel this mu 0 and this comes out to be a by 2 mu 0, K, K dot; that is the energy flow, energy which is going out of the system.

How about if I take again a length L and see how much energy has flown out by the time K has become 0. So, energy flow out will be S dot d s, now d s will be pointing out that is the energy flow outside which is going to be a by 2 mu 0, K, K dot, dot d s which is going to be 2 pi a L; that is the area of this cylindrical surface, which gives me cancels 2 pi a square mu 0, K dot K L, which is nothing but pi a square L is this entire volume. So, this volume times mu 0 d by d t of K dot square by 2; that is the energy flowing out.

So, in total energy that would have flown out by the time, I finish and this K becomes 0 is going to be integrated d t of S dot d s, which is nothing but integrated of d t v mu 0 d by d t, this is only K square, because d by d t I am already taken K square. And this comes out to be equal to v mu 0 K square by 2; that is the energy that has flown out by the time the current density came from K to 0.

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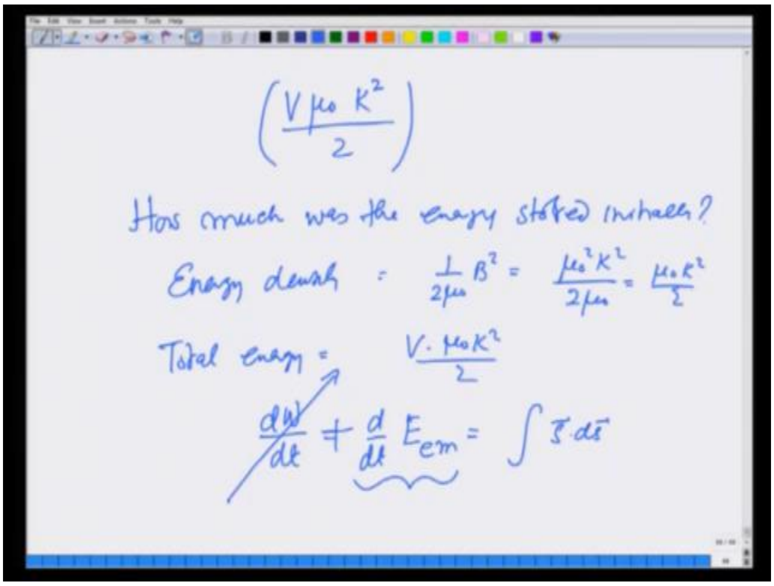

How much was the energy stored initially? Energy density in the magnetic field is 1 over 2 mu 0 times B square, which is going to be mu 0 square K square over 2 mu 0, which is mu 0 K square by 2. So, total energy in volume v was equal to v K square by 2, which has now dissipated and gone out through the surface through poynting vector. So, d w t by d t plus d by d t of E electromagnetic is equal to integral S dot d s is seen to be satisfied again. In this case d w by d t is 0, it is this term that finally, becomes 0 and that energy flows out by poynting vector.

So, what I have shown you in this lecture is that electromagnetic field, when they exist together make an energy to flow and I have taken two examples, where in one case mechanical energy was brought in by the poynting vector. In the other example, the energy was taken away from a volume and that energy initially was stored as electromagnetic energy.