Introduction to Electromagnetism
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Lecture - 54
Energy Transport by Electromagnetic Fields
The Poynting vector

We have learnt about electromagnetic fields and learnt that they are governed by Maxwell's equations.

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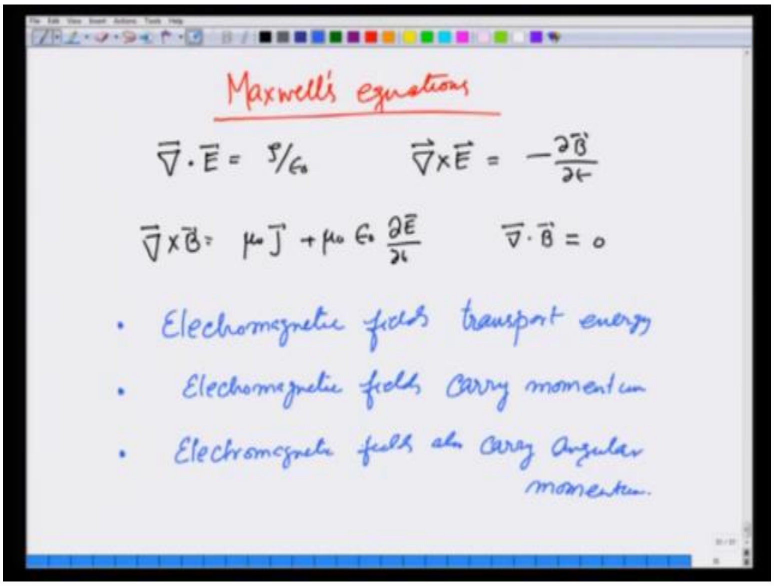

And, let me write them because I will keep using them. They are divergence of E is rho over epsilon 0. Curl of E is minus change in the magnetic field. This is the first one is the Gauss's law; second one is the Faraday's law. Third is curl of B is equal to mu 0 j plus mu 0 times the displacement current that we have already talked about – partial E partial t and divergence of B is 0. These are sufficient to describe any electromagnetic phenomena. I must point out right now that I am talking about these fields in free space. What we want to use these now for is to show that, number 1 – the electromagnetic field, that means, electric and magnetic field together transport energy. So, if I have magnetic and electric field together, they transport energy from one point to the other; two – electromagnetic field also carry momentum; and, if they carry linear momentum, then r cross p is angular momentum; and therefore, electromagnetic fields also carry angular momentum.

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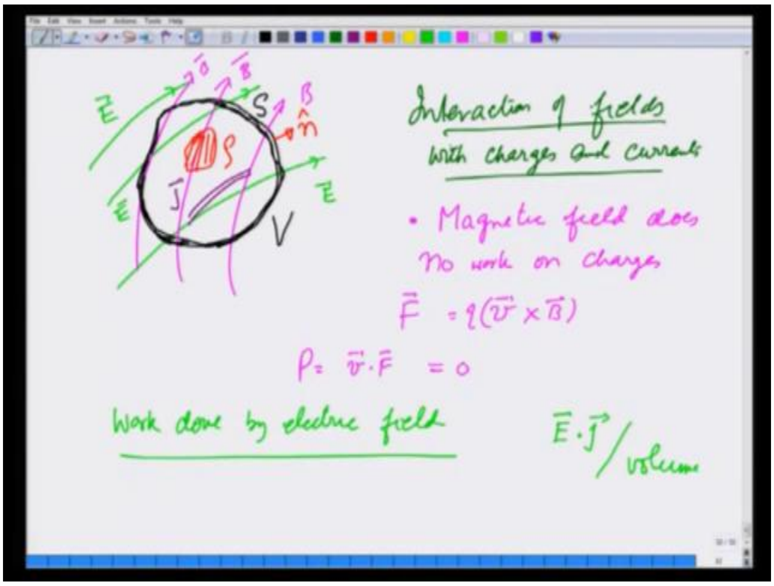

In this lecture, I want to focus on the energy. So, for this, let us consider an enclosed volume V with surface S; I will need both these. So, I am specifying these. On the surface, there is this unit vector n at each point and inside this is the charge density rho and there are currents. Let an electromagnetic field exist around it, so that we have E field and B field in this region. What I want to consider now; I will make this volume little thicker, the walls little thicker, so that you see it clearly. What I want to know now is how much energy is being pumped in by these fields into this volume or how much energy is being taken away. The only way the energy can go in and come out is through interaction of fields with charges and currents. In this, we must keep in mind that, number 1 – magnetic field does no work on charges, because the force due to magnetic field is v cross B times the charge q and the power which is v dot F therefore, is 0. So, the only work that is done on these charges and currents is by the electric field. How much work does the electric field do? Work done by electric field is E dot j per unit volume. That is easy to see – per unit volume.

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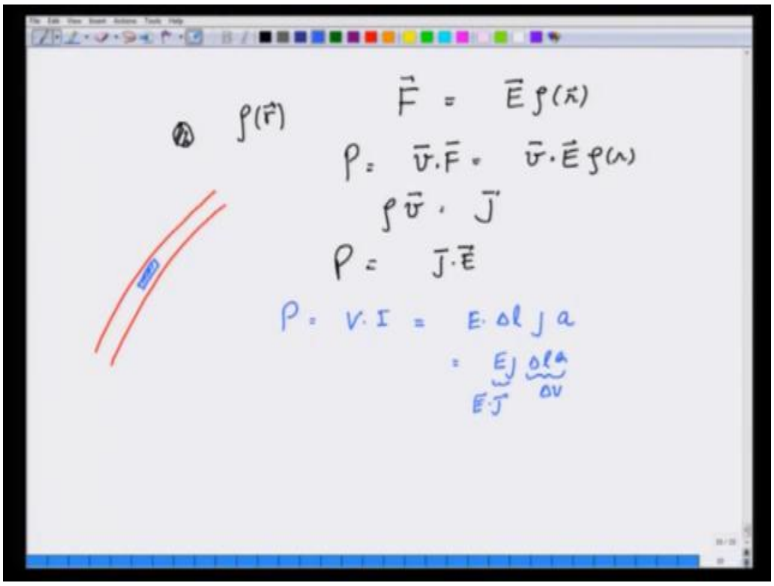

If I have certain charge and density is rho r; then, the force on this due to the electric field is going to be E times rho r; and, the power delivered is going to be v dot F, which is v dot E times rho r; and, rho v is nothing but j. So, power or the work done per unit time is going to be j dot E. Another way to look at it is if I have a wire; if I take a very small element in this; then, the power in this is going to be v times I. This is joules heating – joule heating; and, this v is nothing but E times this small length l; I is nothing but j times a. And, you can write this as E j delta l a. This is nothing but that small volume v; E j is e dot j, because the only thing that comes into delivering power is the component of E in the direction of j. So, two ways we have seen that the power delivered is j dot E.

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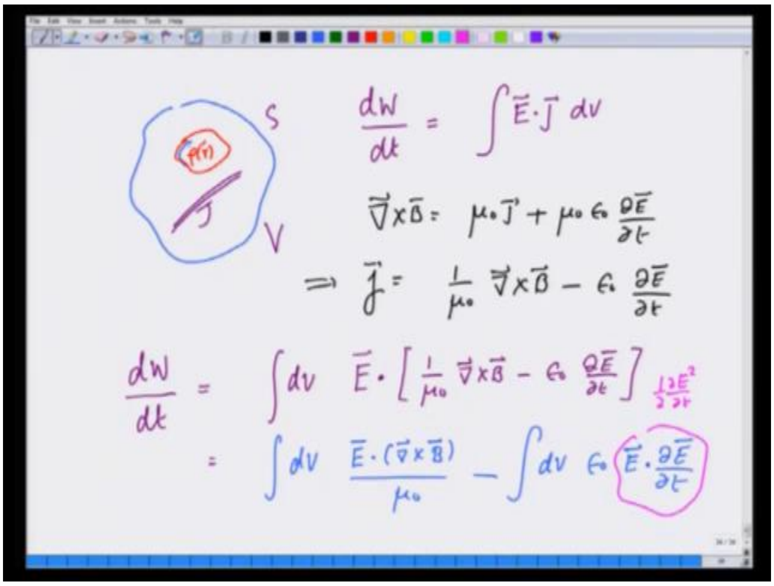

And therefore, if I see – if I go back to the earlier volume in which there is this charge density rho r and current density j; actually, I am writing them separately, but actually rho r flowing is j; then, the rate of change in the energy of this system inside this volume is nothing but d by dt of the energy content. And, I should remove this d by dt; this is nothing but E dot j integrated over the volume of this region. Let us now use the Maxwell's equation, which says that curl of B is mu naught j plus mu naught epsilon naught dE dt. This gives j is equal to 1 over mu 0 – curl of B minus epsilon 0 dE dt. And therefore, I can write dw dt is equal to integral over this volume dv E dotted with 1 over mu 0 curl of B minus epsilon 0 dE dt; which can be written as dv E dot curl of B over mu 0 minus dv integral over the volume – epsilon 0 E dotted with dE dt. This term together is nothing but one-half E square d by dt.

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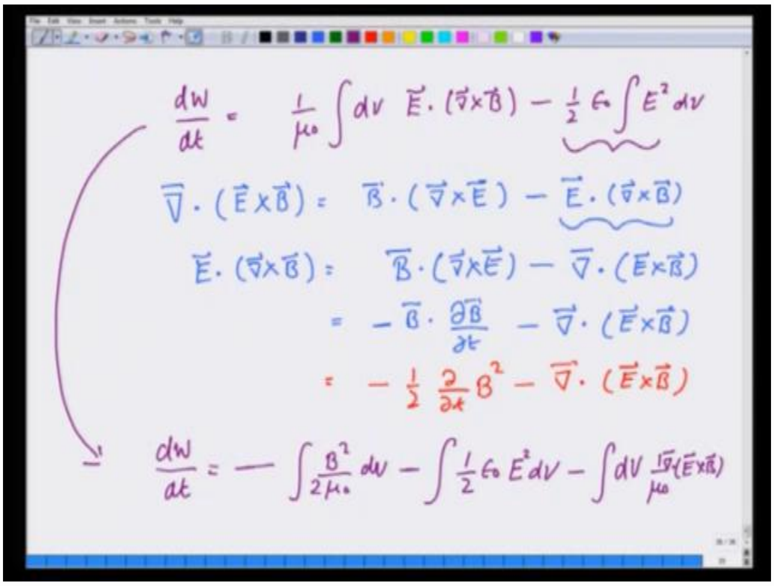

And therefore, I can write dw dt is equal to 1 over mu 0 integral over the volume – E dot curl of B minus one-half epsilon 0 E square dv. Recall that this is the energy stored in the electric field in this volume. Now, I am going to use a vector identity, which says that, divergence of E cross B is nothing but B dot curl of E minus E dot curl of B. Therefore, I can write E dot curl of B as B dotted with curl of E minus divergence of E cross B. Recall that, curl of E is minus dB dt. And therefore, I can further write this as B dotted with dB dt with a minus sign – minus divergence of E cross B; which can be further written as minus one-half d by dt of B square minus divergence of E cross B. And therefore, I come back to this equation and write this as dw dt is equal to minus one-half integral B square over mu 0 to make it look better. Let me remove this half and put two mu 0 inside this minus sign dv minus – again one-half epsilon 0 E square dv. So, I have taken care of those two terms that gave me magnetic and electrostatic energy. And, then I have finally, minus integral dv 1 over mu 0 E cross B divergence of. So, I have gotten these three terms. Let us simplify them.

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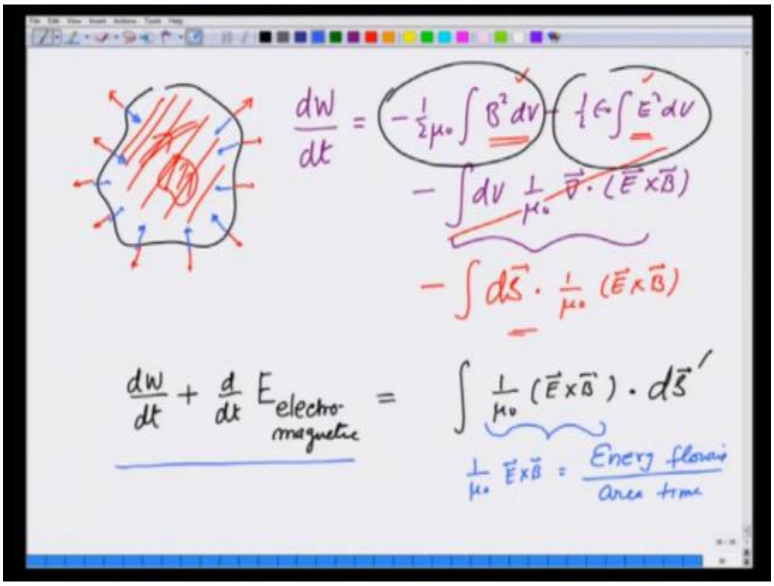

So, again I am talking about this volume inside which there are these currents and charge densities; and, what we've gotten is that, the change of the mechanical energy inside is equal to minus one-half mu 0 integration B square dv minus one-half epsilon 0 E square dv minus integral volume integral of 1 over mu 0 divergence of E cross B. I can now use divergence theorem and write this term as... I will remove this and write this as minus integral – the surface integral of 1 over mu 0 E cross B; where, the vector sign for this surface integral – the n is coming out. This is the surface element pointing out this way. Inside this volume... Inside this volume, whatever change is taking place is taking place due to these terms: one is the change in the energy in the magnetic field and electric field inside and this second term. To interpret this, let us write it slightly differently.

I will write dw dt, that is, the power going – power change inside the system plus d by dt of total energy E electro-magnetic. And, what we mean by this is the sum of the energy, which is here due to magnetic field and due to electric field; and I brought it to the left-hand side; therefore, it is plus – is equal to integral of 1 over mu 0 E cross B dot – let me write this as ds prime. So, that ds prime is nothing but the – this opposite of ds; that means, this ds prime is going into the volume. What does this mean? This means that, the change of the energy inside this volume, which is equal to the change in the mechanical energy plus the electromagnetic energy is equal to something going into the volume. And therefore, I will call this 1 over mu 0 E cross B as the energy flowing per unit area per unit time. Notice that, if E cross B is pointing into the volume, energy will be going in;

the left-hand side should be positive. On the other hand, if E cross B is pointing out – away from the volume. This E cross B dot ds prime will be negative; energy will be going out. And therefore, the energy – the left hand side term will be equal to something negative; the energy inside will be going down.

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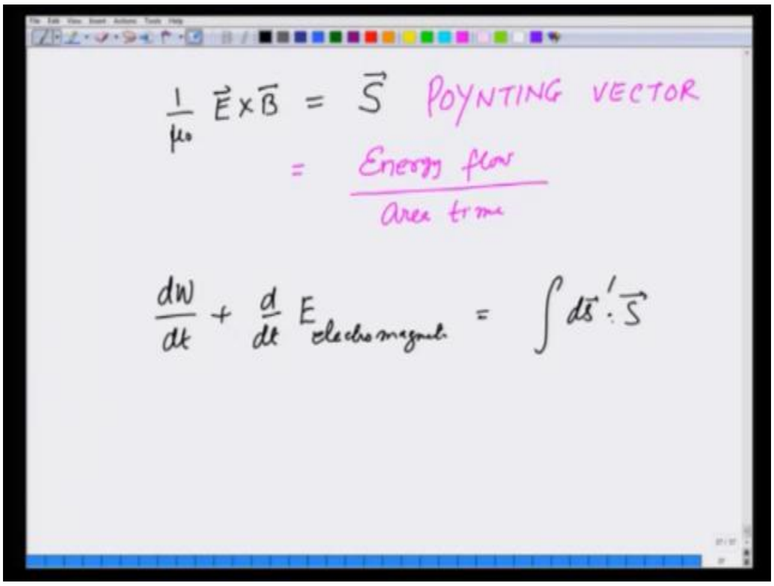

So, this is the interpretation of this term 1 over mu 0 E cross B. That is usually written as S and known as Poynting vector. And, this is equal to energy flow per unit area per unit time. Notice that, for S to be nonzero, both E and B have to be nonzero. So, it is only a combination of electromagnetic field; that means, if there is an electric field as well as the magnetic field; then only, this energy flows; otherwise, it is not there.

Next, we will do some examples to illustrate that, E cross B over mu 0 is really the energy flow and it satisfies the equation dw over dt plus d by dt of E electromagnetic equals integral ds prime dot s – the Poynting vector.