Introduction to Electromagnetism
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Lecture - 53
Quasistatic approximation

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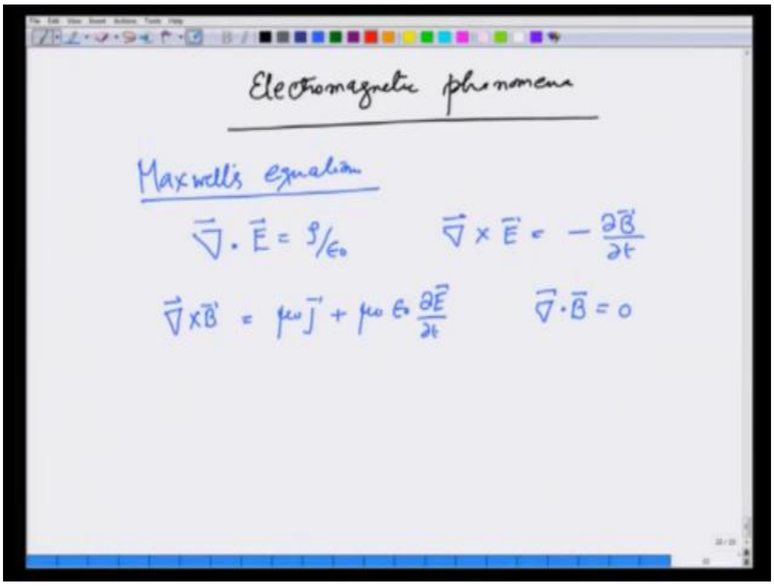

By now we have learnt that electromagnetic phenomena. Electromagnetic phenomena is described completely by Maxwell's equations, so let us write them. They are the Gauss's law that tells you divergence of E is equal to rho over Epsilon 0 curl of E which is Faraday’s law is equal to minus d B d t. Curl of B depends both on the regular current J plus displacement current which is arising out of the change in electric field with respect to times Epsilon 0 d E d t and the fourth equation is divergence of B is 0. Let us now see explore these equations a better in this lecture and see, what they predict.

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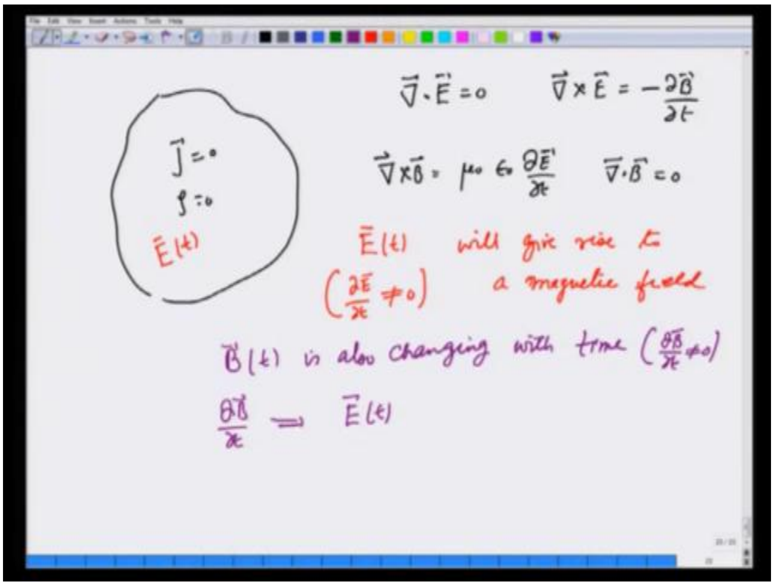

For example, if I want to take a situation in free space, now let us see I mean some space where all these fields are coming and there is no current here, no charge here, only fields are coming from outside. So, then I would have if I would describe E and B, divergence of E is 0, because I am taking no current inside, so this is inside this volume. Curl of E would be equal to minus d B d t, curl of B will be equal to mu 0 Epsilon 0 d E d t and divergence of B would be 0.

So, let us see when these fields come in what happens. So, let there be an electric field E inside which is changing with time. So, this E which is changing with time, so let us write that d E d t is not equal to 0 will give rise to a magnetic field. It gives rise to a magnetic field which will also change with time, because coming out of something which is changing arbitrarily in time. So, this magnetic field B t is also changing with time and this changing magnetic field so; that means, d B d t is not equal to 0.

This change in magnetic field implies E t again, it gives rise to another field and that E t gives B again and so we keep on going on, where do we stop. For example, when I consider the problem of displacement current in a capacitor in the previous lecture, we said this change in electric field between the plates is giving rise to a magnetic field and we stop. We did not even consider that, that magnetic field which is changing gives rise to another field again.

Similarly, in a problem where we were calculating the induced electric field applying for, by applying for a Faraday's law. For example, in a solenoid where the current is changing, we stop by calculating the electric field and we did not bother that electric field is changing and that will give rise to another magnetic field. Where do we stop? How do we decide that?

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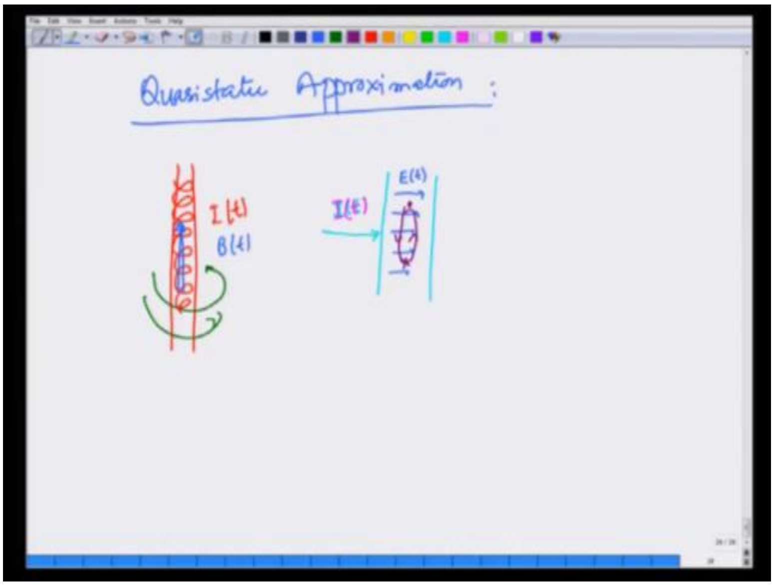

So, we were using in all these, something called the Quasistatic approximation, where we stopped after calculating the magnetic field or electric field and did not go beyond to the next step. So, I want to spend some time in understanding this Quasistatic approximation. So, let us see what we did. For example, we did something in which is solenoid, whose carrying a current which was time dependent I t and therefore, the magnetic field inside this B was changing and that we said by Faraday's law gave rise to an electric field going around direction.

I am just making arbitrarily, it could be the other way. Similarly, in the other example what I did, I took a parallel plate capacitor and I said there is a current which is coming in, which is I t. This current is time dependent, and therefore electric field in between… The electric field in between E changes with time which gives rise to a magnetic field, circular magnetic field which is coming up, but we stopped at that magnetic field.

We did not go beyond that. That may also give rise to further an additional electrical field. Why did we stop there? What gives us the right to do so? We will analyze that in this lecture under Quasistatic approximation. These two calculations have been done under that approximation and we want to understand, when we can apply this. So, to do this let us again give rise to look at the equations, time dependent equations that tell us that this fields give rise to each other.

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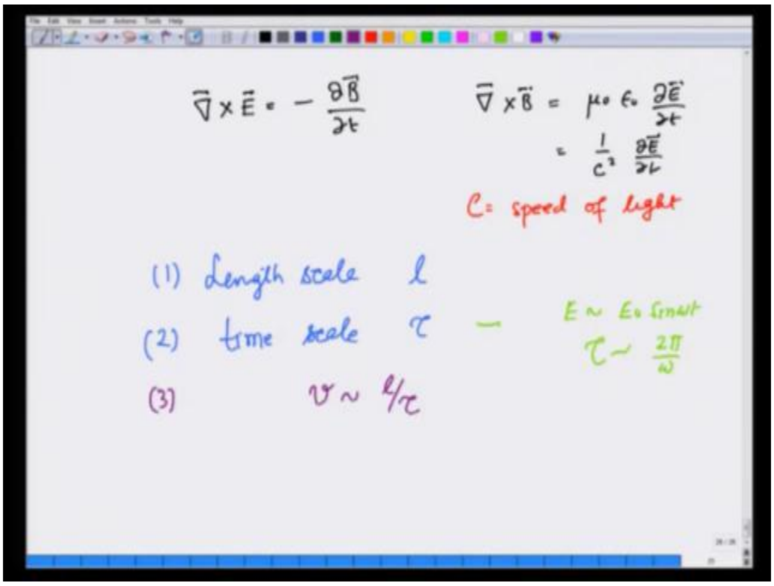

So, we have curl of E which is minus d B d t, curl of B which is equal to mu 0 Epsilon 0 d E d t. And let me write mu 0 times Epsilon 0 as 1 over C square d E d t, where C is the speed of light, because light is electromagnetic wave and seen naturally into all these. So, we are going to write it as 1 over C square. Now, let me assume that the length scale in the problem that we are solving in this is l. What it means is, if I am looking at the capacitor that the problem I saw in the previous lecture.

The capacitor may be of the size of, you know a few centimeters, 10 centimetres or something. I may be observing fields at a distance of few meters, so length scale would be a few meters. 2, let us assume that the time scale is of the order of tau. Again what we mean by that is, let us suppose I am applying a signal or an electric field which is changing in time, the time period is omega. Then, for that signal let me write this E which is E 0 sin omega t, then the time scale tau would be of the order of 2 pi over omega that is the scale we are talking about.

And third related quantity would be that maybe the system is moving with some velocity and that velocity is of the order of a l over tau. So, these three things are there and let us see under what circumstances, how should l and tau and all these things, should be related, so that we can apply this Quasistatic approximation.

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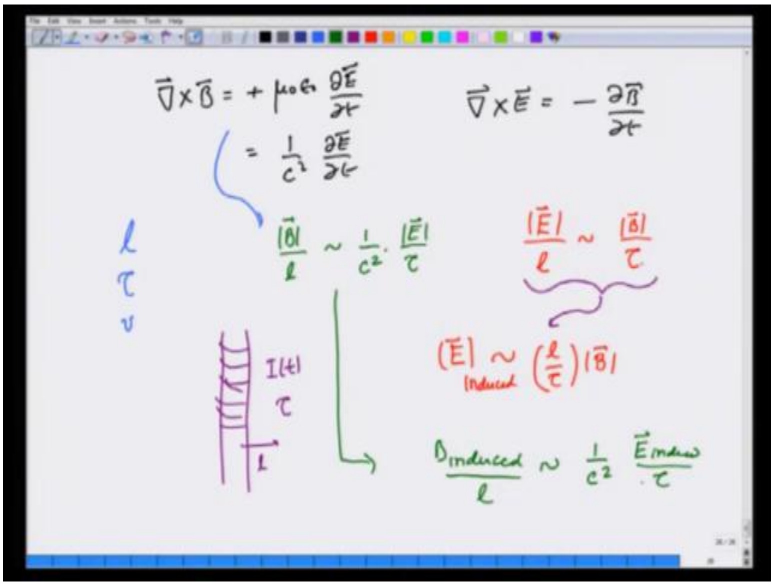

I will write these equations again, curl of B is plus mu 0 Epsilon 0 d E d t which is 1 over C square d E d t and I have curl of E which is equal to minus d B d t. So, with the time length scale l, time scale tau and velocity or speed v, I can write this equation roughly as curl of B is like B divided by l, magnitude of b divided by l that is the magnitude curl of B. And this is roughly equal to 1 over C square d E d t, so it will be magnitude of E divided by tau.

Similarly, on the other equation I have magnitude of E divided by l of the order of magnitude of B divided by tau and let us see, how we play out of this. Let us take that example of a solenoid. In the solenoid, there is current I t changing in with time scale tau and therefore, this change in magnetic field gives rise to the electric field. So, we use this equation and I am observing it at some distance l. So, we are going to say that E is going to be of the order of l over tau times p, whatever the magnetic field is. This will be the induced electric field, so let me write this induced.

This induced electric field is also changing in time and therefore, this gives rise to this new magnetic field. So, let us write it B induced that is the further induced due to this change in electric field. B induced divided by l is going to be of the order of 1 over C square that E induced divided by tau.

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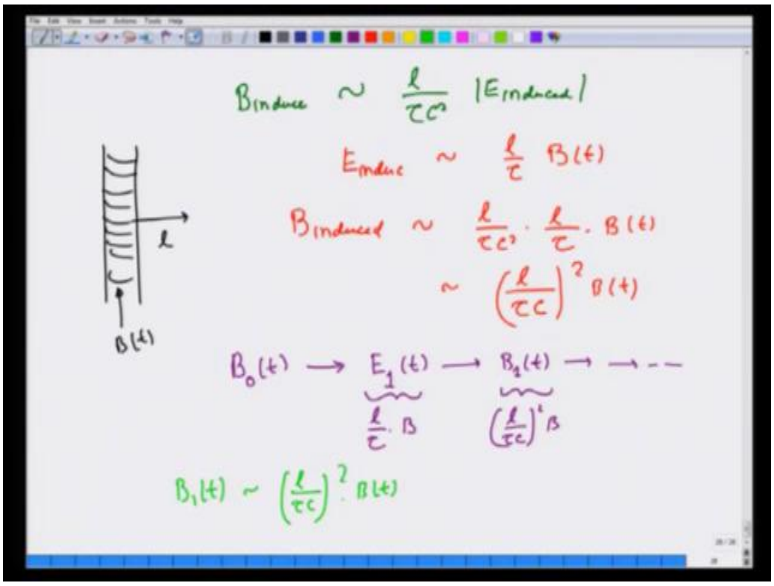

So, B induced is of the order of l over tau C square E induced. Let me remind you again, since this is a new slide. I am in this region where there is a solenoid, I am looking at some distances l, the induced electric field and then consequent induced magnetic field due to this B inside which is changing in time. So, we had all ready found that E induced is of the order of l over tau times is B t that is changing originally in the solenoid and therefore, B induced is going to be of the order of l over tau C square times l over tau times B t which is l over tau C square B t.

So, let us see what is happening. I have an original B t which I will call B 0 t, this gives rise to an induced field, let me call it E 1 t which gives rise to a field B 1 t, and then further like this. This is related to be as l over tau B and this is related to the original B as l over tau C square B. So, B 1 t is l over tau C square B t.

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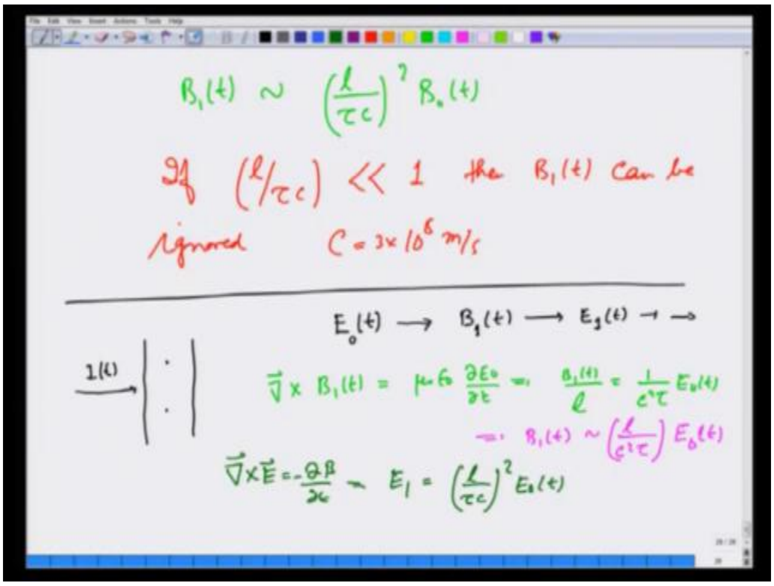

So, if B 1 t is of the order of l over tau C square B 0 t. If l over tau C is much, much, much less than 1, then B 1 t can be ignored and that is why we kept ignoring it. Because, you are talking about distances which work much smaller than tau C. C is very large, C is of the order of 10 raise to 8 meters per second, it is actually 3 times 10 raise to 8 meters per second. So, unless you are really talking large distances or very short time periods, so that C tau is very small, then the B 1 is going to be almost negligible.

I can apply similar argument, the other way. The other way was we took this capacitor in which there was a current I, t was coming in and there was this magnetic field being produced. So, originally I had E t, let us call it E 0 t which is giving rise to B 1 t which in turn again will give rise to another E 1 t and so on. Again, I can now argue that B 1 t, because I have dell cross B is equal to mu 0 Epsilon 0 d E 0 d t.

I can argue from this that B1 t, I am going to have B 1 t over l is equal to 1 over C square tau E 0 t, which gives me B 1 t of the order of l over C square tau E 0 t. And again using curl of E equals minus d v d t, I get E 1 which is equal to l over tau C square E 0 t. So, again we see that if l is much, much, much smaller than tau C, I can stop at this order and see what the effect is.

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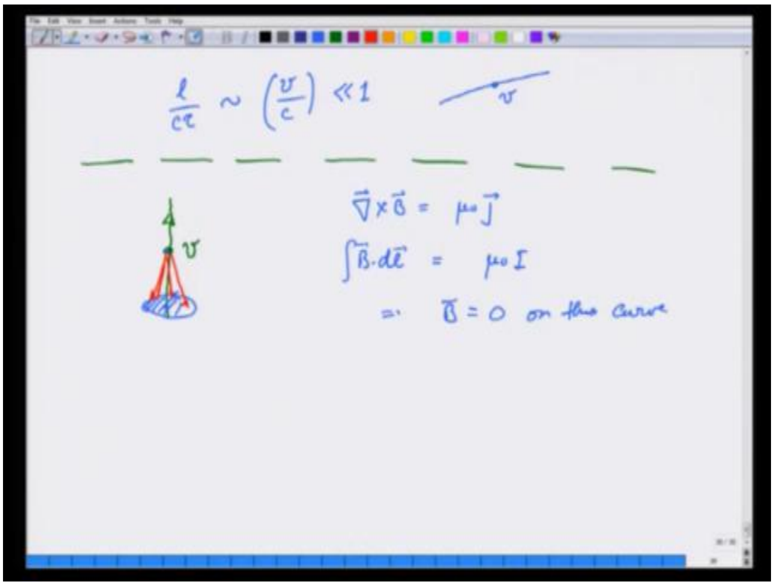

Finally, if l over C tau is very small that means now let me write l over tau as some speed divided by C. Then, the v over C is much, much, much less than 1, then also I can use this kind of approximation. For example, it could be the case of one charge particle moving with speed v. In that case, I will replace l by tau which is equal to this speed. Now, another manifestation of this displacement current is going to be precisely this example.

If there is a charged particle which is let us say moving with respect to, moving the speed v, then if I had this equation curl of B equals mu 0 J only, the free current. And if I want to take Stokes theorem around a region where the charge particle has passed from, the charged particles appear. Then, I would have B dot d l by Stokes theorem is equal to mu 0 I, but I is 0 through this area, because charge particle is not there, it is moving somewhere else.

And therefore, this would imply that B is 0 on this curl whereas, in reality it is not that arises, because the displacement current. Because of this charge, there is this electric field and since charge is moving, electric field also changes with time and that produces that magnetic field. I will give that as an assignment problem.