Introduction to Electromagnetism
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Module - 06
Lecture - 52
Displacement Current

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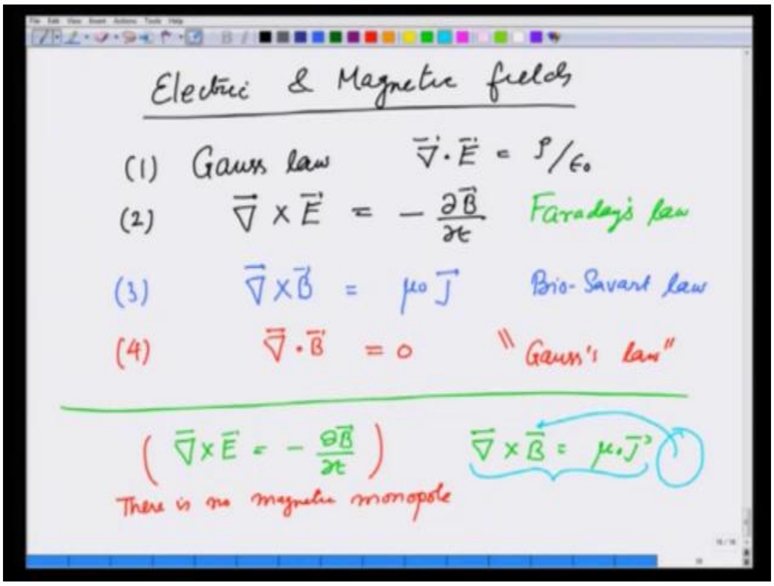

Let us now summarize what we have learnt so, far. What we have learnt so, far about electric and magnetic fields is that they satisfy one the electric field satisfies Gauss's law which is that divergence of electric field is given as rho over epsilon 0 where rho is the electric charge density. Then we have learnt about curl of electric field arising due to changing magnetic field which is the Faraday's law, keep in mind that curl of e is 0 arising from a charge distribution. Third we have learnt that curl of b and this I am talking in free space there's no medium in vacuum is equal to mu 0 j which you can say is Bio-Savart law. And finally, we have learnt that divergence of b is 0 which is related to that there are no magnetic monopoles magnetic field is always coming out of a current loop with which acts like a dipole. In a way you can call this like I will just put it in quotes Gauss's law for magnetic field we have learnt these four things.

Let us compare these equations you see if you look at equation two and three. So, let me write equation two and three side by side you have curl of e is equal to minus d b d t and curl of b which is mu naught j look at the two equations in the curl of e in the first equation we have electric field arising out of a changing magnetic field. But there is no current term which is basically related to there is no magnetic current, no magnetic monopole or free magnetic charge right so, there is no current. So, there is nothing coming out of current.

On the other hand, in the curl of b formula I have the magnetic field coming out of a electric current; however, I do have an electric field which changes with time. Is it possible that there could be another term here which gives rise to magnetic field that is the a symmetry here. See I could explain the absence of any current in this equation curl of e equation because there are no magnetic poles. But there is no reason why a d e d t term cannot be here. We will now show that it should indeed exist and it is known as displacement current and it was introduced by Maxwell and that will complete the entire set of equations describing electromagnetic field.

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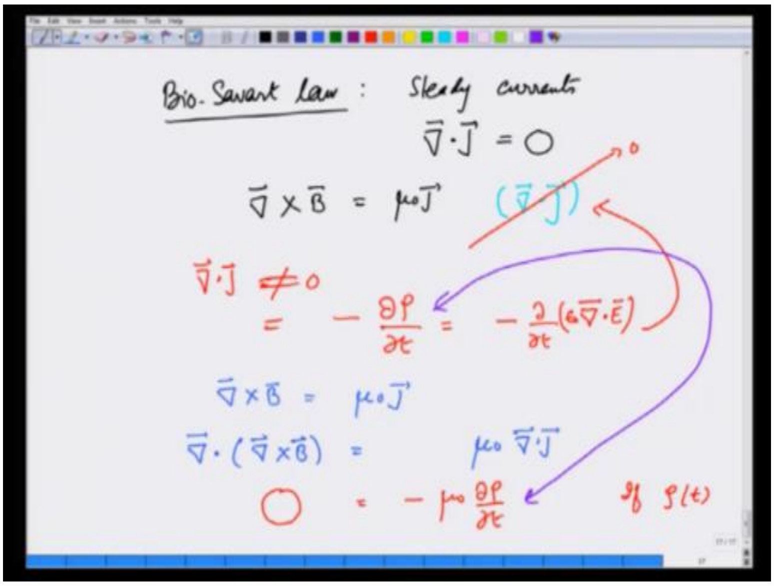

So, let me start with Bio-Savart law which you recall for steady currents and what you mean by steady currents these were not changing with time. That means, the divergence of the current density was 0 we use that somewhere right. And the formula gave me that curl of b was equal to mu 0 j. If you go back to that lecture you will notice that in deriving this formula along the way there was a term which was proportional to del dot j which we said was 0 because of this steady current what if that term is not 0 right. So, what if del dot j is not equal to 0 then it will be equal to minus d rho d t where rho is the charge density. And this term by using the gauss's law gives me minus d by d t of divergence of e times epsilon 0. So, there could be a term like this on the right hand side if I take curl of b, but do not take the current to be steady. Let us now see another way how it arises and this will be purely mathematical.

If I look this equation curl of b is equal to mu naught j and take the divergence on both sides divergence of both sides divergence of curl of b is equal to divergence of mu 0 a mu 0 divergence of j. Now divergence of curl is always 0 mathematically so, this term right left hand side is always 0 mathematics tells you that. On the other hand, if rho is a function or charge density is a function time right hand side has to be minus mu 0 d rho d t which we saw earlier here also. The right hand side is not 0 and therefore, there is an inconsistency in this equation, how do we remove that inconsistency.

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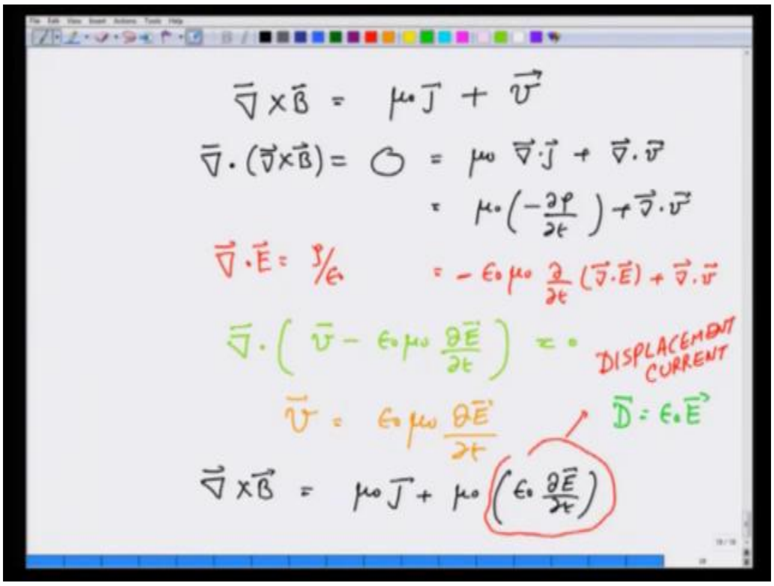

So, let us write del cross b is equal to mu 0 j and add another vector v. So, that when I take divergence from both the sides the answer comes out to be 0 on both the sides. So, if I take divergence of curl of b which mathematically is always 0 it is going to be equal to mu 0 divergence of j plus divergence of v which is nothing, but mu 0 times minus d rho d t plus divergence of v. By using the equation that divergence of e is equal to rho over epsilon 0 we can write this equation as epsilon 0 mu 0 d by d t of divergence of e with a minus sign plus divergence of v. So, what we have from these equations is divergence of v minus epsilon 0 mu 0 d e d t is equal to 0. So, we take this v to be equal to epsilon 0 mu 0 d e d t. So, the correct equation for the curl of b is therefore, curl of b is equal to mu 0 j plus mu 0 and this extra term epsilon 0 d e d t. This is like also like a current term right this is also like a current term, but it is in addition and this makes this mathematically consistent. This term that we have just added this extra term epsilon 0 d e d t is known as the displacement current because it is related to this quantity called displacement d which is epsilon 0 e in free space. So, let us write all these equations again.

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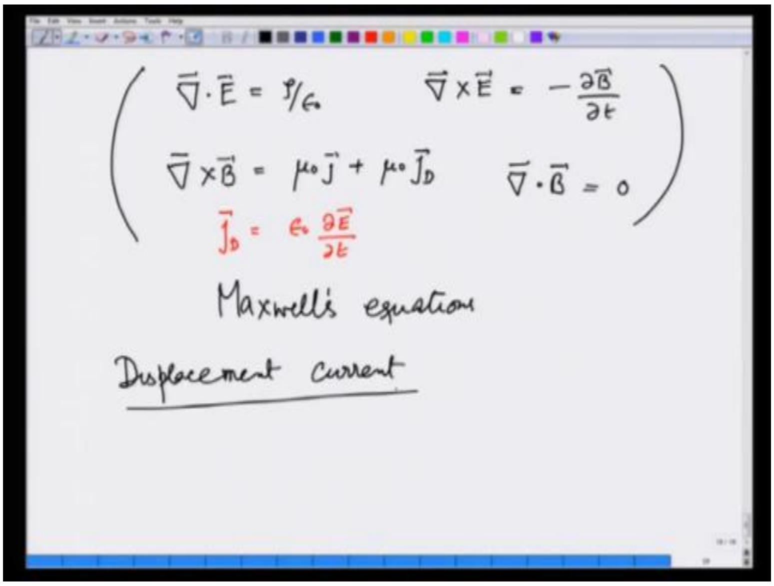

So, we have Gauss's law for electric field divergence of e is equal to rho over epsilon 0 then we have which is minus d b d t. Again I want to remind you this minus sign the curl of b equation whether they are changing with the time not changing with time electrostatic, magnetostatic everything is described by these four equations. So, entire electrodynamics classical electrodynamics is based on these four equations. In this lecture; however, our focus is going to be the displacement current because we want to understand this and understand displacement current and its effects.

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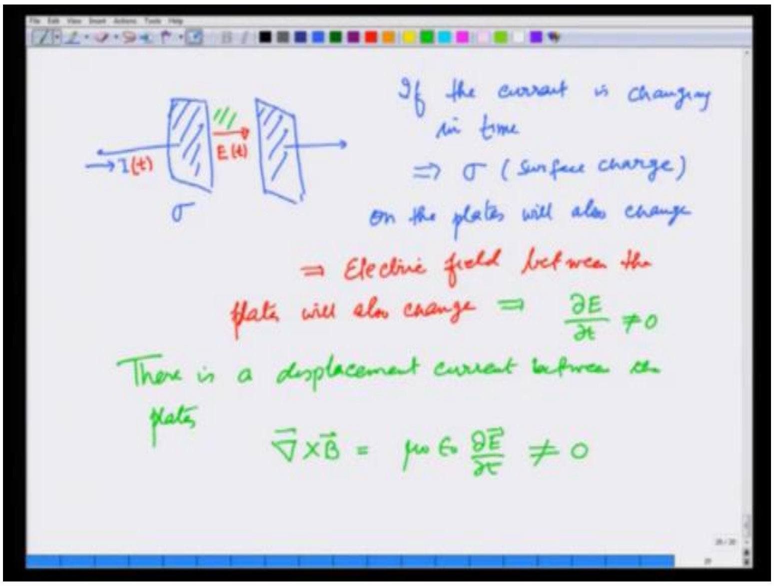

So, the often done example of this to show displacement current and its effect is taking a parallel plate capacitor connected to a wire that is bringing in current I as its going out the other way. If the current is changing in time if a current is changing in time then the charge that is brought in the sigma on the surface right will also change. So, then sigma or the surface charge on the plates will also change if that happens; that means, the electric field between the plates also change right.

So, what happens if this I is a function of time then this electric field here e becomes a function of time right. And this means this immediately implies that partial derivative of e with respect to time is not 0. There a displacement current displacement current between the plates and that should have some effect because now I know although there is no physical current curl of b is equal to mu 0 epsilon 0 d e d t and curl of b between the plates is not going to be 0. If curl of b is not 0 that means, there will be some magnetic field even between the plates although normally you would not expect it, let us calculate these quantities.

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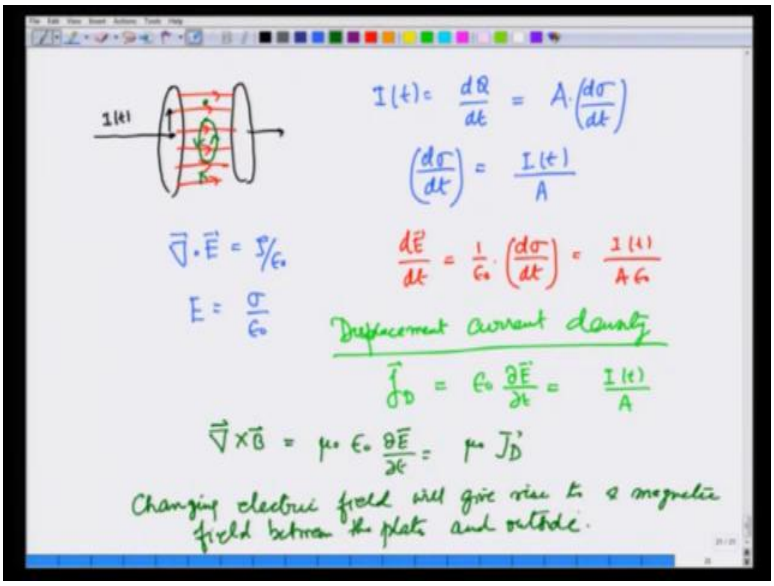

So, let us take this for simplicity let us take this plate to be circular. So, here is the circular plate here is the other circular plate, current I is coming in I t and its going out. For simplicity we will ignore we the any the current flowing time here so, that we will assume as soon as the current the charge comes in its splits evenly throughout the plate. So, I t is d q d t is going to be the area of the plate a times d sigma d t. So, d sigma d t is equal to I t divided by a I also know from Gauss's law which is true even in this dynamic situation rho over epsilon 0 that e is going to be equal to sigma over epsilon 0 between the plates and therefore, d e d t is uniform. So, I am not even worrying about writing a partial derivative e is uniform here d e d t which is going to be one over epsilon 0 d sigma over d t is going to be I t over a epsilon 0. And therefore, the displacement current displacement current density j, displacement current density j d it is a vector, but its vector in the same direction as the electric field is going to be equal to epsilon 0 partial e partial t is going to be equal to I t over a. That is a displacement current.

So, it is as if this current which is coming in has spread over this area a and then it is going through so, the current per unit area becomes I t over a. Because of this displacement current there's going to be field. So, magnetic field equation is curl of b is equal to mu 0 epsilon 0 d e d t or mu 0 j d. This is precisely the equation that is due to a regular current also and therefore, out here the b field is going to be circular like this coming out of the paper on top going into the paper at the bottom and this b field is going to be produced. So, changing electric field will give rise to a magnetic field between the plates and also outside, that is the effect of displacement current.