Introduction to Electromagnetism
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Module - 08
Lecture - 51
Energy stored in a magnetic field - II
Solved examples

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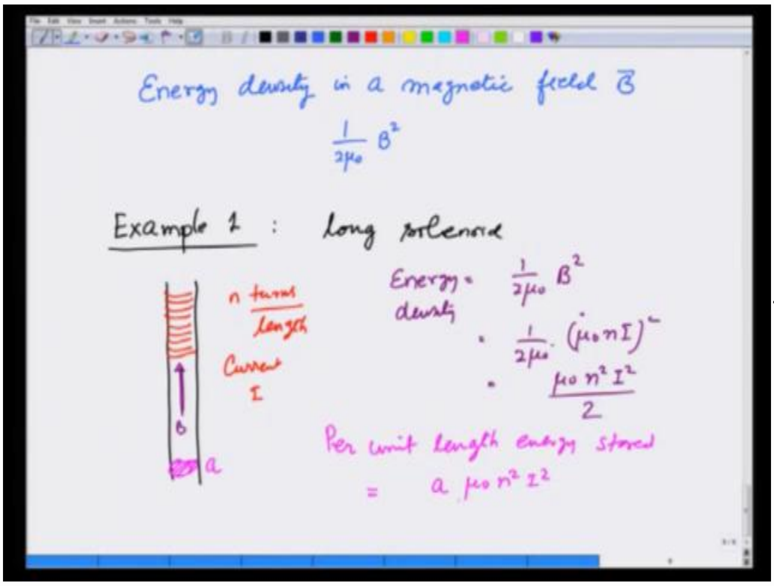

We have learnt that energy density in a magnetic field b is given as 1 over 2 mu 0 b square. Let us now use this to see some examples. So, example one I will take a long solenoid that has n turns per unit length. So, it is carrying current I and it has n turns per unit length and it is carrying current I so, that there is a magnetic field established here b. Then the energy is going to be 1 over 2 mu 0 b square this is the energy density b in a solenoid in a I know what it is this going to be 1 over 2 mu 0 b is mu 0 and I square therefore, the energy density is given as mu 0 n square I square divided by 2.

Let us if see this make sense per unit length energy stored therefore, is going to be the area of cross section let us call it a times unit length so, that is going to the volume. So, energy stored per unit length is going to be area of cross section a mu 0 n square I square divided by 2 and this should be equal to.

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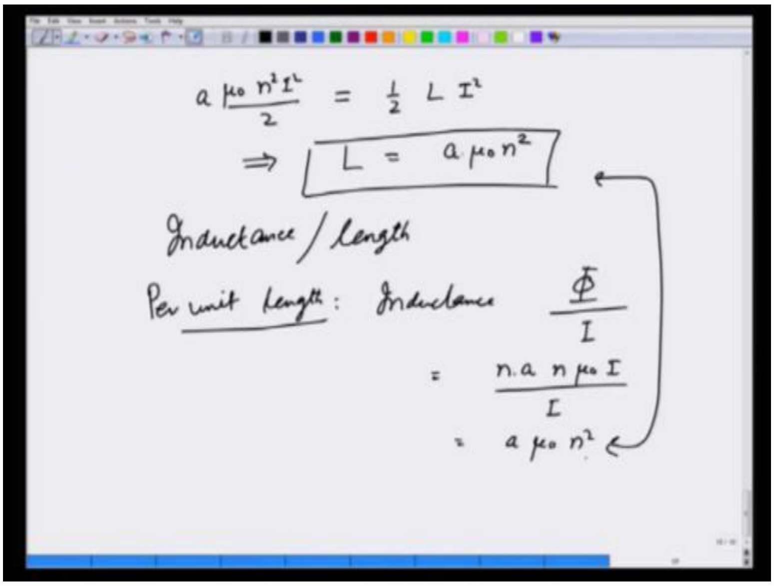

So, energy stored per unit length is mu 0 n square I square divide by 2 times a this should be equal to one half l I square and this gives l is equal to a mu 0 n square this is the inductance per unit length. This makes sense for a solenoid in per unit length the inductance will be phi total flux that is passing through all this n turns divided by I which is going to be n times the area for though these n turns the field is n mu 0 I divided by I which is a mu 0 n square so, these two match. So, that is one example where we have seen how the energy calculated by the energy density of b is same as it matches with the inductance through the inductance method. Should not surprise you because after all we derived that formula starting from an inductance formula

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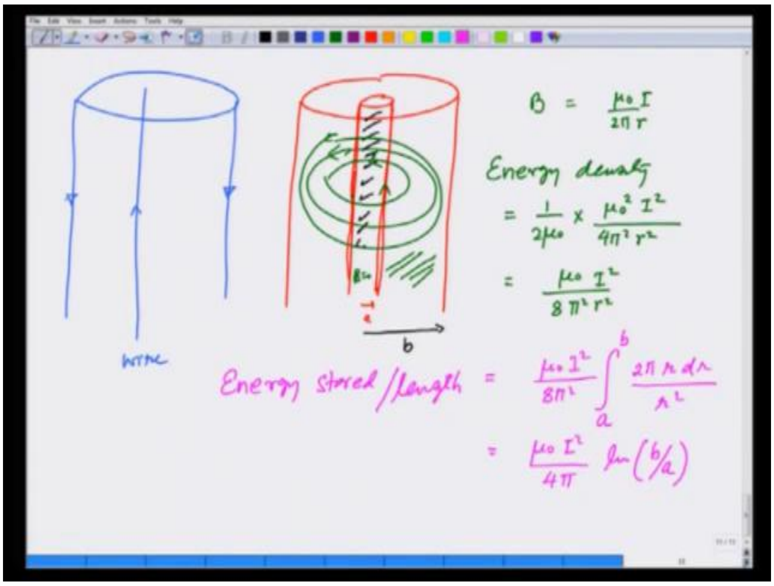

Next example, I will take a current carrying wire enclosed in a shell through which the current comes back. To save myself from mathematical difficulties right now I take this wire to be such that it has a radius a and is hollow so, that the current flows only on the surface of the wire. So, current is flowing on the surface inside there is nothing. So, this radius is a and let the outer radius be b by ampere's law b in this wire is going to be mu 0 over 2 pi r times I and this b if the current is going up if the is the current is going up b field is circular like this. Therefore, the energy stored since b 0 inside b 0 inside this hollow b is only in this region. So, the energy stored energy density is going to be 1 over 2 mu 0 times b squared mu 0 square I square over 4 pi square r square which is equal to mu 0 over 8 pi square r square I square.

If I want to calculate energy is stored per unit length, energy stored per unit length this will be equal to mu 0 there is a constant I squared over 8 pi square integral a to b 2 pi r d r divided by r square and this comes out to be mu 0 I square over 4 pi above b over a.

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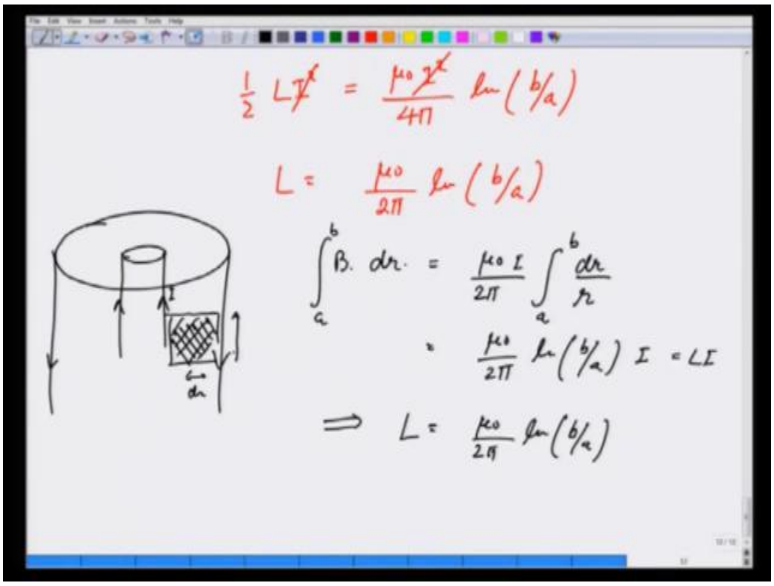

This by definition should be equal to 1 half l I square 1 half l I squared is equal to mu 0 I square over 4 pi log of b over a I square cancels and I get l equals mu over to pi log of b over d. Let us see if this makes sense again in this wire which is carrying current I and outside that that it is coming back the flux per unit length will be in this area which I am showing by this shaded region. And this is going to be times b r times per unit length d r is this unit length is this way this would be the flux r wearing from a to b which is equal to mu 0 over to pi I integral d r over r a to b will t gives me mu 0 over 2 pi log of b over a times I I should be equal to l I. And therefore, this gives me l equals mu 0 over 2 pi log of b over a, the two match. You may be wondering how I am calculating energy from that I am getting in inductance and then trying to match them. In next example we show the importance of that.

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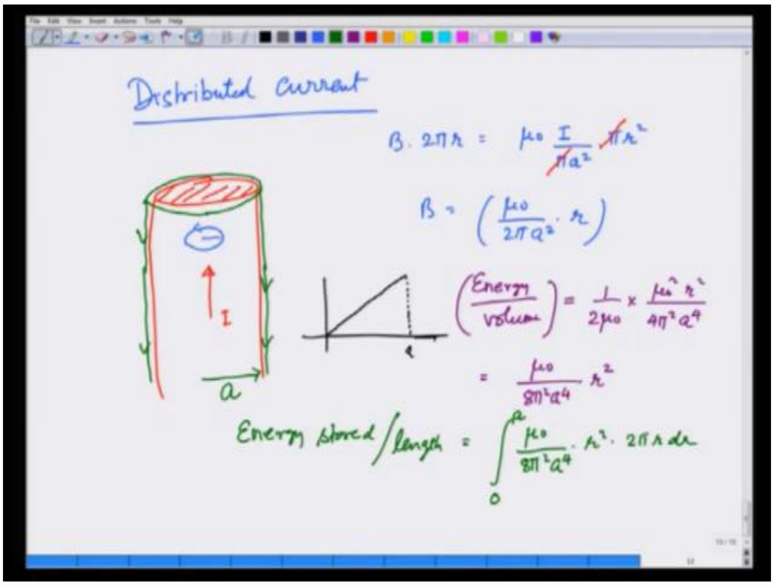

In the next example I take what is known as distributed current and what we will do in this will take a wire which is solid and it is carrying current I. And the current comes back through outer casing which is matching exactly the same radius as a wire so, through this outer casing that the current comes back. Let us calculate the magnetic field inside this wire and the energy stored in that assuming that the current is distributed over the entire wire evenly let this radius be a. Then, by ampere's law if I would calculate the magnetic field at distance r from the axis of the wire we are going to have b times 2 pi r is equal to mu 0 total current is I which is distributed over pi a square and therefore, current enclose would be I over pi square times pi r square.

Let us cancel this pi and therefore, I get b to be mu 0 over 2 pi a square times r. So, the field inside the wire is increasing linearly up to a and then it becomes 0 because for outside the net current encloses 0. And therefore, the energy stored energy per unit volume is going to be equal to 1 over 2 mu 0 times mu 0 square r square over 4 pi a square a raise to 4, which is equal to mu 0 over 8 pi square a raise to 4 r square. Let us now calculate the energy stored per unit length and that is going to be mu 0 over 8 pi square a raise to 4 r square times 2 pi r v r integrated over r going went from 0 to radius a

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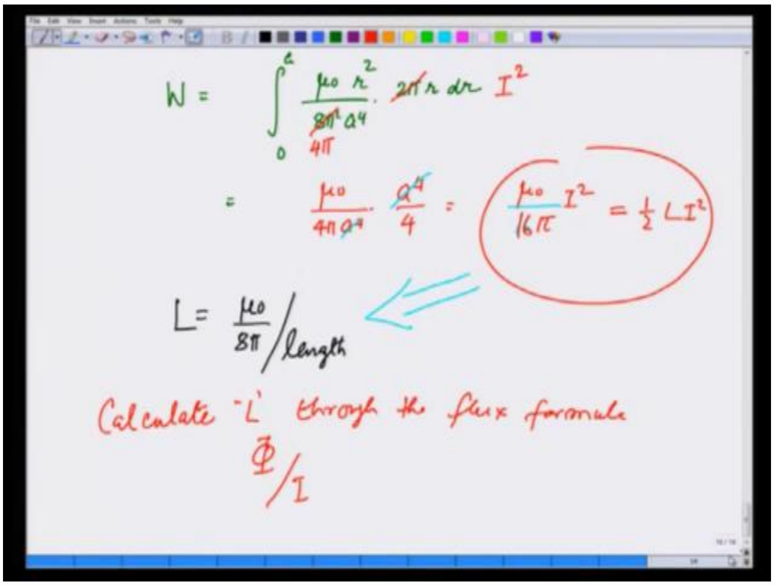

Let us calculate this and this comes out to be yes calculating energy which is equal to integration 0 2 a mu 0 over 8 pi square a raise 2 4 times 2 pi there is r 2 pi r r r square 2 pi r the r which is equal to this 2 pi cancels here n and gives me 4 pi. So, this comes out to be mu 0 over 4 pi a raise to four times a raise to 4 divided by 4 which is mu 0 a raise to 4 again cancels over 16 pi. I have forgotten one I square I square and this should be equal to 1 half l I square these together then give me that l would be equal to mu 0 over 8 by per unit length. What if I calculated the calculate l through the flux formula that is I calculate total flux divided by r.

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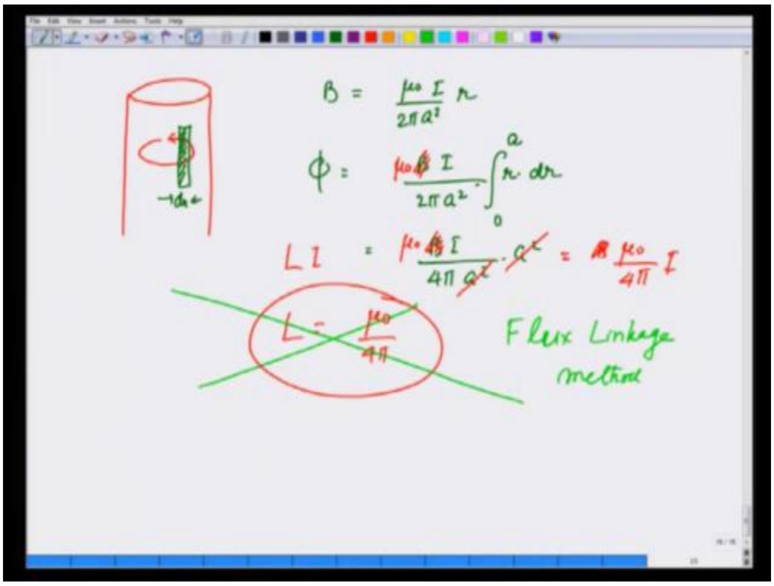

Let us see what answer we get then answer I am going to get in that case is let take this wire in which magnetic field lines are going around and b you have already calculated the I direction which is equal to mu 0 over 2 pi a square I times r. So, if I want to calculate flux per unit length I will take this area of thickness d r and calculates flux is going to be b I over 2 pi a square r times b r 0 to a which gives me b I over 4 pi a square time is square a square cancels and I get sorry this s not be is mu 0 it is mu 0.

So, this gives me mu 0 over 4 pi I and if I equate this to l I get l equals mu 0 over 4 pi which answer is correct earlier answer is mu 0 over 8 pi this time getting mu 0 over 4 pi. So, in the cases where there is a distribution of current you will get two different answers through the energy method or through flux by I this actually answer is not correct. Because in this case what is happening is that the current is distributed and therefore, the flux is due to many different currents and one has to use something called the flux linkage method. In which you take flux due to each small current multiply that by I submit over and then get the answer, but is safer to use than the energy method where b at any point is given to all the currents and the answer you get is correct.