Introduction to Electromagnetism
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Lecture - 05
Electric Field due to a Charged Distribution

In the previous lectures, we have discussed the Electric Field due to Charges and Charge Distributions. In fact, while calculating forces on a given charge q, we had also calculated electric field due to a charge distribution, because if I take small q to be 1, it becomes the electric field. Let us do some more examples of calculating electric field in this lecture, I will do two examples here and both are important and will be used in the future.

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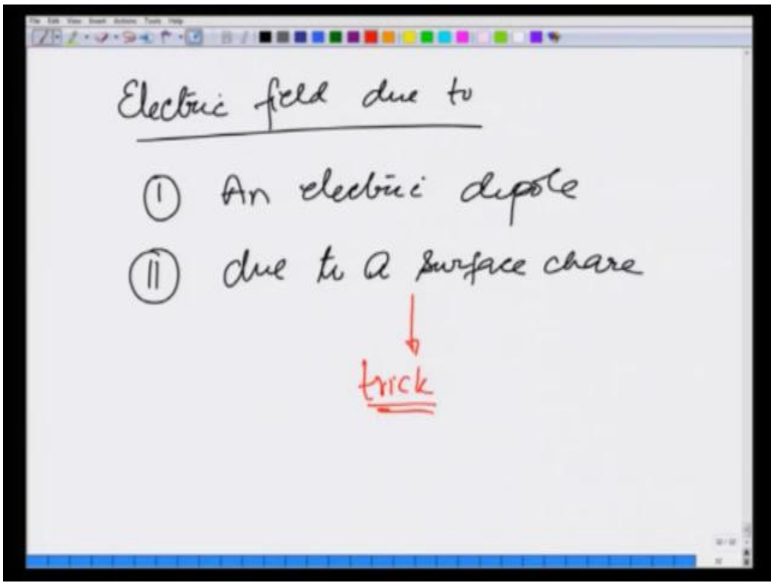

I am going to calculate electric field due to example 1 a, an electric dipole and 2 due to a surface charge. In example 2, you will also learn a trick to deal with such cases. In both the cases, we will be doing some limiting processes. So, this is quite an instructive and important lecture.

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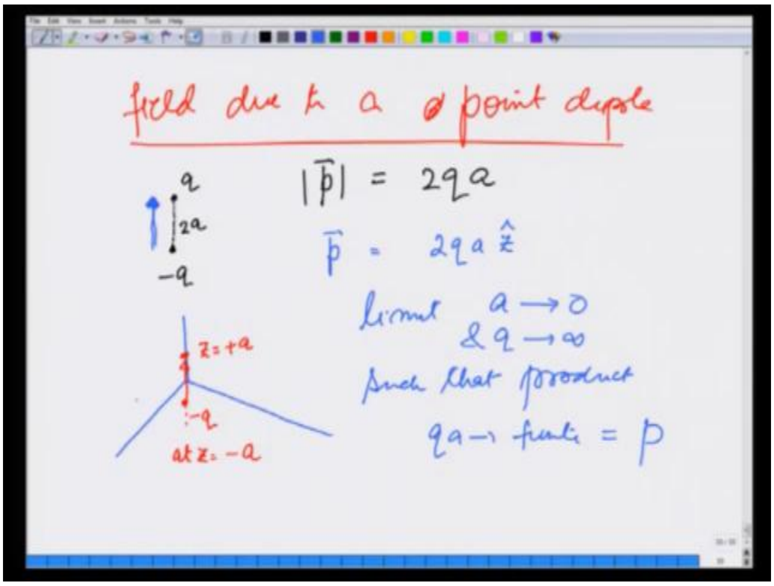

To calculate the field due to a dipole, I am going to be more specific a point dipole what we are going to do is take the dipole to be made up of 2 charges minus q and plus q separated by distance 2 a. So, that the magnitude of dipole moment is given by 2 q a, and the direction of dipole moment is from negative to positive charge.

So, p is going to have a direction from negative to positive charge, if I take the dipole to be sitting at the origin and in the z direction, then I will take the charges minus q at minus a at z equals minus a and plus q to be sitting at z equals plus a and I can write the dipole moment p of this to be equal to 2 q a in the z direction. To make it a point dipole finally, what we are going to do is take limit a going to 0 and q going to infinity, such that product q a remains finite and equal to some p value that makes it a point dipole. So, let us calculate the field due to this point dipole.

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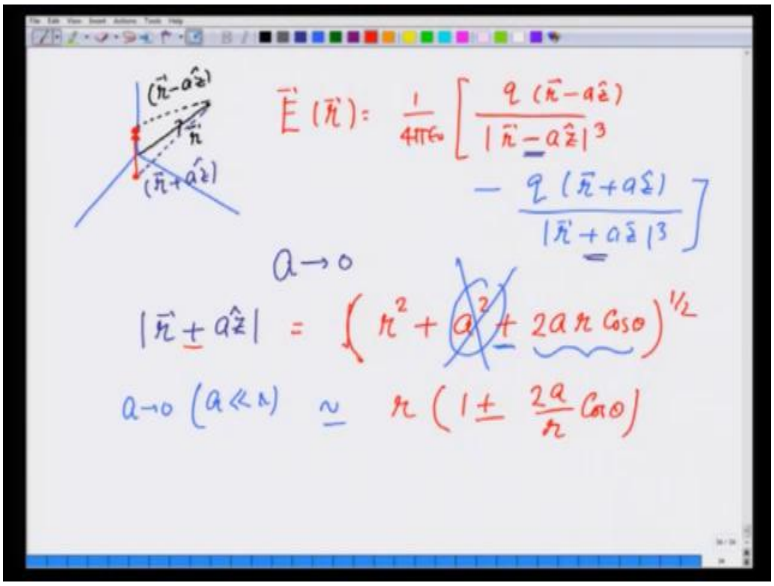

I am going to make it again, this is my point dipole plus q sitting here minus q sitting here. So, the field at some vector r is going to be 1 over 4 pi Epsilon 0 q the positive charge gives me the field q r minus a in the z direction. I will write a cube here and a vector r minus a z on the top, there is a field due to the positive charge and that due to negative charge is going to be minus q r plus a z over r plus a z cubed.

To make it more explicit, let me draw this point r, this is vector r minus a z and the one from the bottom here is r plus a z and we want to calculate this field and finally, take the limit q times a going to a finite number. Since, we are already assuming that we are going to take the limit a going to 0, I can expand these two quantities in the denominator. So, let us write modulus of r plus a z, I am going to include minus also right here which is going to be square root of r square plus a square plus 2 a r dot product of r unit vector with z unit vector which is going to be cosine theta raise to 1 half and for the minus sign it will be minus here.

Since, I am going to take a going to 0, so a is much, much, much less than r we are going to neglect a square term and keep only this term. So, by Binomial theorem I can approximate this as r 1 plus or minus 2 a by r cosine theta raise to 1 half which is...

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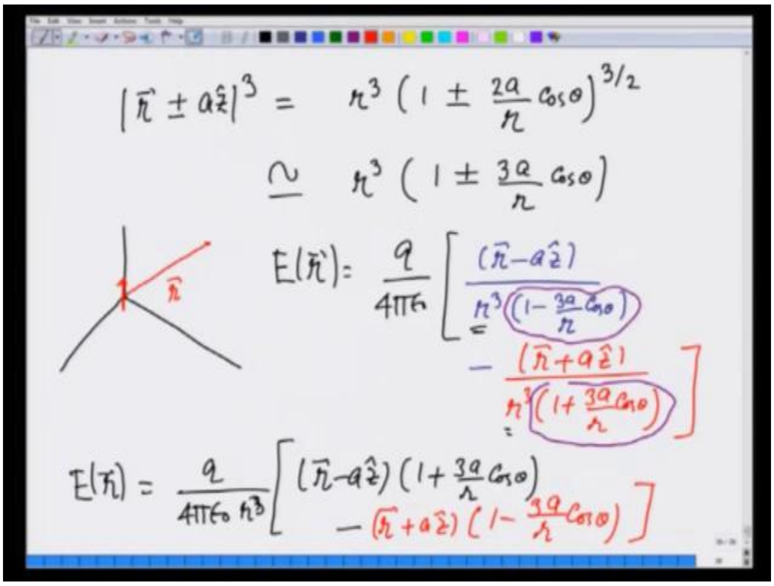

Thus we can write r plus or minus a z cubed as r cubed 1 plus or minus 2 a over r cosine theta raise to 3 by 2 which by Binomial theorem is r cubed 1 plus or minus 3 a over r cos theta. And therefore, the field due to this dipole sitting at the origin at a distance r is going to be given as E r equals q over 4 pi Epsilon 0. Inside I have vector r minus a z divided by r cubed 1 minus 3 a over r cos theta minus let me use a different colour r plus a z over r cubed 1 plus 3 a over r cosine of theta bracket closed.

Again I am going to use Binomial theorem and take the factors encircled in purple to the numerator and write this whole thing as E r equals q over 4 pi Epsilon 0 r cubed I am going to take out r cubed and inside, I am left with r minus a z times 1 plus 3 a over r cosine of theta minus r plus a z times 1 minus 3 a over r cosine of theta.

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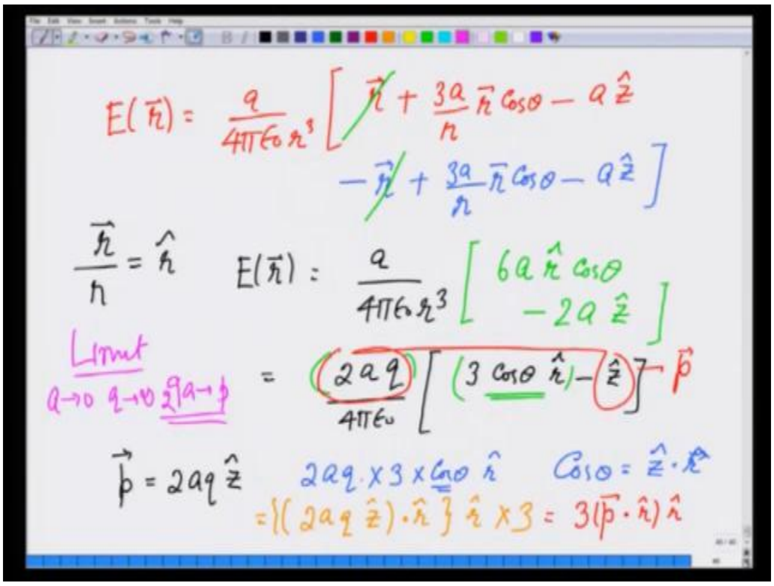

I will again expand this keeping terms only up to first order in a and what I get is E r is equal to q over 4 pi Epsilon 0 r cubed r plus 3 a over r, r vector cosine of theta minus a z minus r plus 3 a over r, r vector cosine of theta, this blue one is the second term minus a z let me check if I am doing it correctly, ((Refer Time: 09:24)) yes this is indeed correct. And now, if I use the fact that r vector divide by r is unit vector r, then I can write E r as q over 4 pi Epsilon 0 r cubed and by the way here this term cancels, inside I get 6 a r unit vector cosine of theta minus 2 a z which I can write as take 2 a out 2 a q over 4 pi Epsilon 0 3 cosine of theta r unit vector minus z.

I now want to be more general and realize that p vector is equal to 2 a q z and therefore, the term 2 a q combined with the first term here is 2 a q times 3 cosine of theta r, cosine theta is nothing but z dot r r unit vector. So, I can write this as equal to 2 a q z dot r times unit vector r multiplied by 3 which is nothing but 3 p dot r r. Similarly, 2 a q z this whole term combined with z is nothing but p and remember I am taking the limit that I am working under is that a goes to 0 and q goes to infinity, such that 2 q a goes to p.

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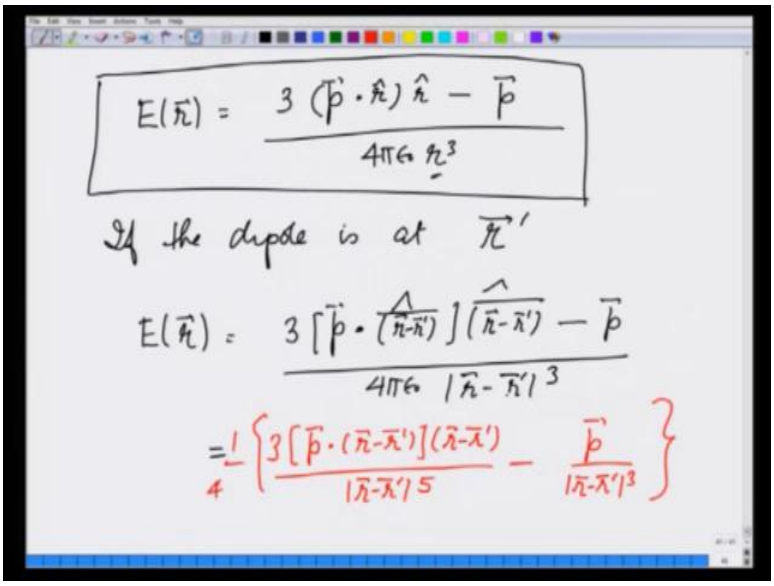

So, combining all these I can write E r as p dot r r with the 3 here minus p over 4 pi Epsilon 0 r cubed, this is the expression for a dipole sitting with any orientation now at the origin. Because, now I am taking the dot product, so it does not really matter dot product gives me that cosine theta already. If the dipole is at let us say r prime some point r prime, then this r would represent the distance from that point and I am going to have E at r is going to be 3 p dot unit vector in the direction of r minus r prime unit vector in that direction.

Unit vector r minus r prime minus p divided by 4 pi Epsilon 0 r minus r prime cubed, which explicitly I can also write as 3 p dot vector r minus r prime multiplied by vector r minus r prime. Since, I have multiplied by the vectors I will divide by r minus r prime raise to 5 minus p over r minus r prime cubed of course, there is a 1 over 4 pi Epsilon 0 outside.

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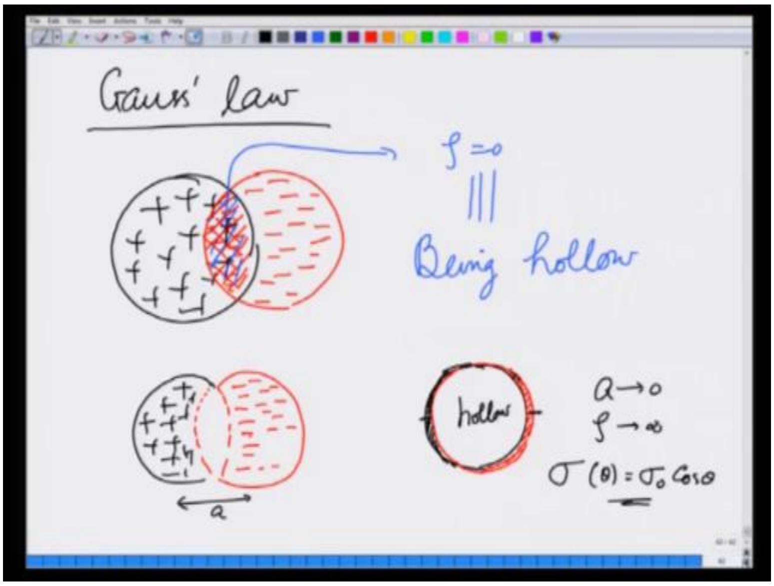

Next example I want to take we will use Gauss's law and in this I want to take a positively charge sphere superimpose it on a negatively charge sphere and calculate the field due to this super position of these two in this region, where they overlap. In a way you can think electrically this overlap region also to have no charge density, rho is 0. Because, positive and negative charges cancel, equivalently you can also think of this as being hollow, because electrically it does not matter.

So, in a way we are also calculating the electric field in a hollow region, this is hollow region when charge outside on one side is positive and charge on the other side is negative, these two centres are displaced by some distance a. I am finally, going to take the limit in this case, where these two charges with overlap for most of the regions. So, that this negative side will give me a very thin slice of negative charge, positive side will give me a very thin slice of positive charge.

And you can see in the middle the charges are going to have largest magnitude and it diminishes as you go to the side and this region is hollow by taking appropriate limits, limit a going to 0 and this charge density rho going to infinity. So, that the charge on the surface remains finite I will get the electric field due to a surface charge distribution rho theta or sigma theta which is some constant times cosine of theta and you will see that. So, again I am going to use a limiting process.

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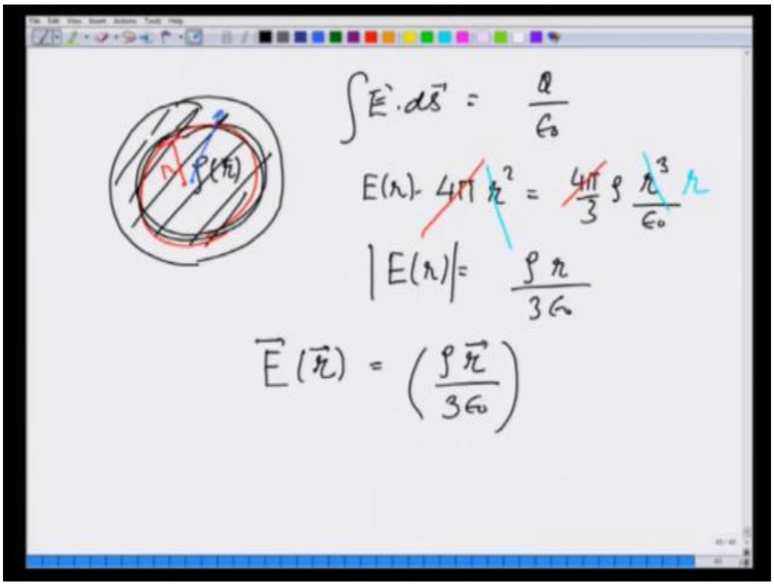

So, let us take a sphere with uniform charge density rho r and this problem you have solved in your 12th grade, if you apply Gauss's law and calculate the field inside at a distance r this field comes out to be rho r by 3 Epsilon naught. But, let us do it explicitly anyway E dot d s gives me charge enclosed divided by Epsilon 0. So, if I take the inner surface E dot d s is going to be E at r times 4 pi r square and charge enclosed is 4 pi by 3 rho r cubed divided by Epsilon 0 here 4 pi cancels, this r square cancels and gives me a single r.

And therefore, field at r comes out to be rho magnitude of field at r comes out to be rho r over 3 Epsilon 0, if I write the vector field v E at r it is direction is radially out and using this spherical polar coordinates I can write this as rho vector r divided by 3 Epsilon 0 this is the field.

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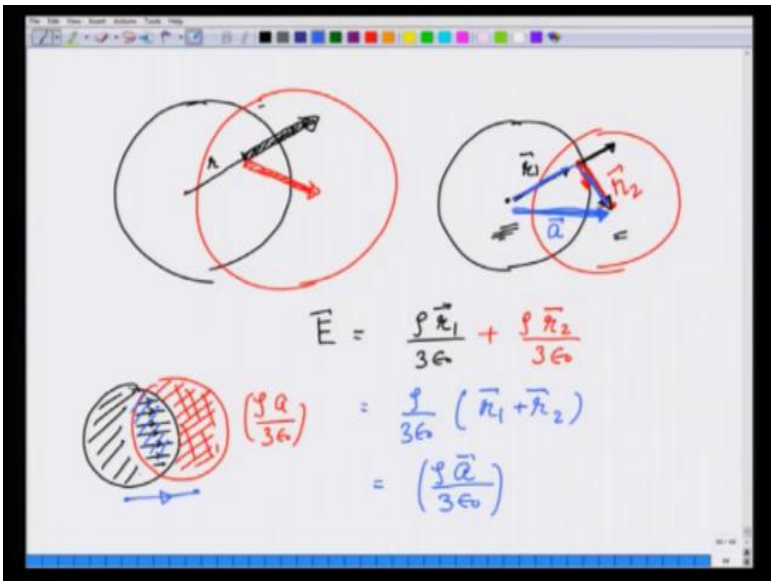

Now, let us superimpose these two charges that we were talking about, here is the positive charge, here is the negative charge, positive charge at this distance r gives me a field which is going like this radially out. If I look at the negative charge it gives me a field like this towards the centre, because the negative charge at any point. So, let me make this picture again, let me take any point, let us say point out here, here is electric field due to positive charge and here is electric field due to negative charge.

If I call this vector r 1 and call this vector from the point where I am calculating the electric field towards the centre r 2, then the electric field at this point is equal to some due to the field due to the positive charge plus that due to the negative charge. So, I am going to write this at rho r 1 divided by 3 Epsilon 0. And since r 2 is already in the direction where the in the direction of the electric field I can write this as plus rho r 2 over 3 Epsilon 0 combine them together I can take rho over 3 Epsilon 0 out I have r 1 plus r 2, this is r 1 vector, this is r 2 vector.

So, their sum is this vector whose magnitude that distance between the centres of theses spheres and vector is from positive charge towards the negative charge. So, I can write this as rho vector a over 3 Epsilon 0, this is very interesting result. Because, now you look at this contribution of the two fields, if I take this negative charge here and the positive charge here in this hollow region field is the same everywhere it is pointing from negative to positive side it is magnitude is rho a divide by 3 Epsilon 0 provided these two densities are the same that is how everything worked out.

And it is direction is from the centre of the positive charge towards the centre of the negative, this is very interesting result no matter what you do field inside is constant in this hollow region. Now, we are going to use this fact to derive field due to a surface charge distribution.

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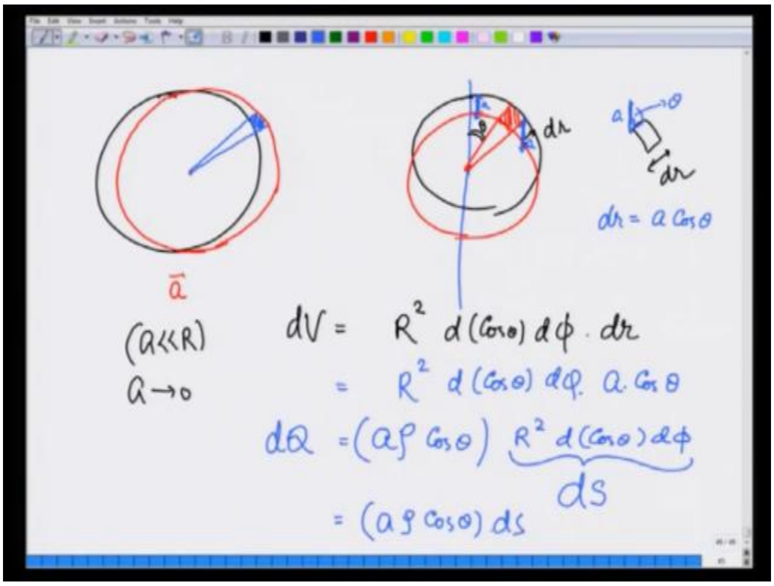

Let me now take these spheres to be overlapping over most of the regions and they are displaced by small vector a, these are the centres. Let us now take how much is the charge in this region, now since I am going to use this spherical polar coordinates it will be useful if I make this arrangement in the vertical direction, this is the centre of the two spheres. Because, the distance between them is very small I can take almost a common centre and I want to calculate the charge in this volume.

Remember, how much is the volume element, this small volume element d v is going to be again a is very, very small a is much, much, much less than the radius of these spheres a in fact, will tend to 0. So, I can write this as R square d cosine of theta, because this angle is theta d phi times d r, where d r is this distance along the radial direction. So, again I am taking this particular volume element, this distance is d r how much is d r? Remember what we said, we said these are displaced by distance a.

So, this distance here is a, this distance here is a and this is d r, this angle is theta. So, you can see that d r is a cosine of theta, so I can write this small volume element as R square d cosine of theta d phi times a cosine of theta. And therefore, the charge in this region d Q is going to be rho the density cosine of theta, let me bring a also here times R square d cosine theta d phi, recall from our one of our previous lectures this is nothing but the area element in the radial direction for this sphere. So, this is nothing but a rho cosine of theta d s.

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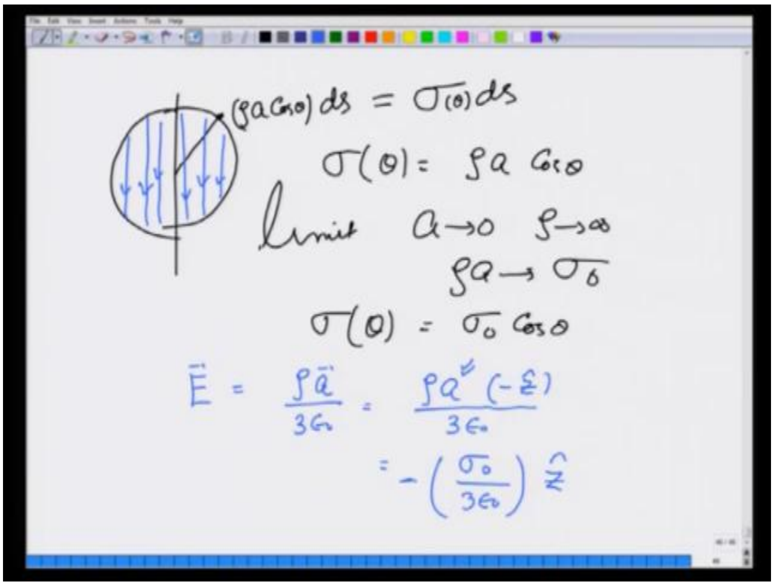

So, what I have the situation now if I think of this as single sphere, I have positive charge here which is rho a cosine of theta d s which I can write as a surface charge density sigma times d s, where sigma depends on theta and sigma theta is equal to some rho a cosine of theta. In the limit of a going to 0 and rho going to infinity, such that rho a goes to some sigma 0 I can write sigma theta as sigma 0 cosine of theta. But, these are two displaces spheres. So, what will the field look like, field will be in the direction of displacement from positive to negative side.

And therefore, the electric field which remember earlier we derived to be rho a over 3 Epsilon 0 is going to be rho a it is in the minus z direction divided by 3 Epsilon 0. But, rho a we have said is nothing but sigma naught. So, this is sigma naught over 3 Epsilon 0 z in the minus z direction.

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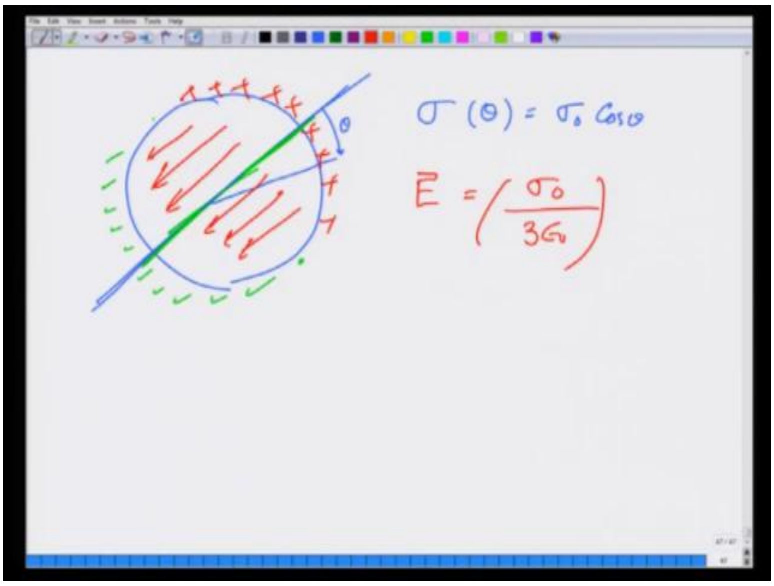

What do we conclude? We conclude that if I have a sphere over which sigma depends on the angle from some axis, some axis if you go away by an angle theta, if sigma theta is sigma 0 cosine of theta. So, that it is positive all over here and negative on the other side maximum being along this axis and then it diminishes and becomes 0 here. Then, E field is constant inside and as is given by sigma naught over 3 Epsilon 0 pointing from positive to negative side. That is a very important result, you also learnt in this how to use Gauss theorem with two charge distributions and it gives you a result that for a particular surface charge density you get a constant field inside this is sphere.