Introduction to Electromagnetism
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Lecture - 48
Induced Electric Field due to Changing Magnetic Field

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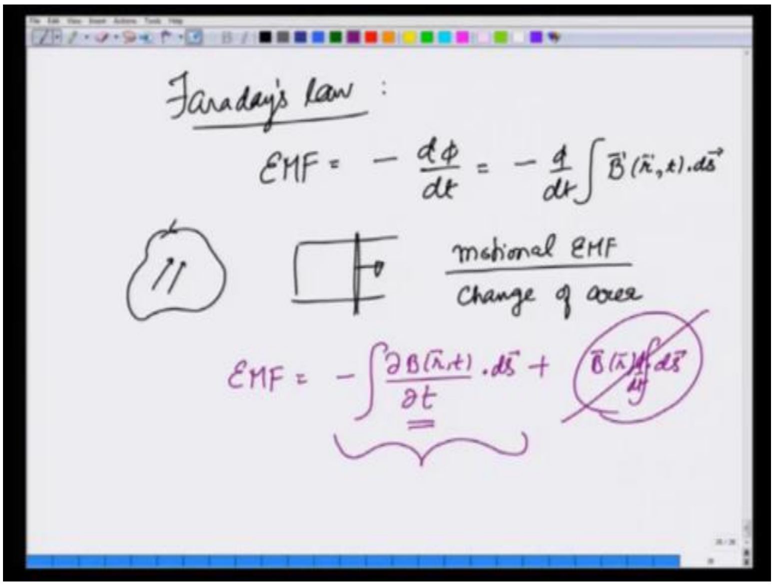

We have been discussing Faraday's law. And it says that EMF produced in a circuit is equal to minus d phi by dt, which is minus d by dt of B – add a point r; B may depend on t dot ds. And what it meant is that, if I take a wire and through it, there is some magnetic field; and, if I change the shape of the wire or change the magnetic field, there is going to be a current in this. In the previous lecture, we saw through an example, where we moved a rod out that, there was something called the motional EMF, which actually comes from change of area. However, the demonstrations I showed you were those, where no change of area took place. The only thing that was changing was B. So, it is also change in B that could give rise to an EMF.

So, let us see that. So, EMF could have two terms: minus – if B is changing with time – d by dt; I am putting a partial derivative here, because at a given r, only t is changing dot ds plus – I could have B fix; no dependence on t – dot ds; and, this could be changing with time. You have seen this effect; now, we want to focus on this. We want to focus on this, because this gives you a relationship between a field, which is generated due to change in magnetic field; a field which is generated is the electric field due to change in magnetic field. So, let us see that.

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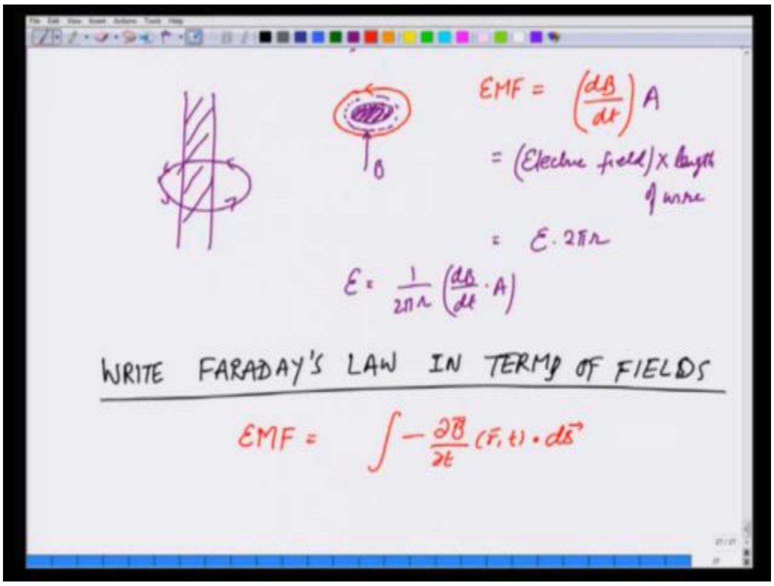

In the example where we saw in the demonstration that, there was a ring, which jump out; that means there was some EMF, which was generated to generate the current that made the ring jump. So, one can say… Suppose I have an area through which B is changing and I put a wire around it; then there is an EMF generated, which is equal to dB dt times the area of that shaded purple thing. So, I will just put that area in purple. This I could also write as electric field on that wire times length of the wire, which is E times 2 pi r.

So, E comes out to be 1 over 2 pi r dB dt times area. Does it mean that, electric field is only in that wire or does it exist everywhere? If I put the wire in the inner loop; then also, the current flows. So, that means, electric field generate there also. So, what we are going to now turn these equations into is – write them in terms of fields – write Faraday's law in terms of fields. Then it becomes independent of whether I am putting a wire there or not. If there is a change in magnetic field, it produces a field – electric field; something exists. With the change of this magnetic field, something is given rise to and we can see that, that is the electric field, because that derives the current; and, we want to develop the machinery for it. So, let us now write this EMF, which is integration minus db by dt of r, t dot ds; where, ds is the area.

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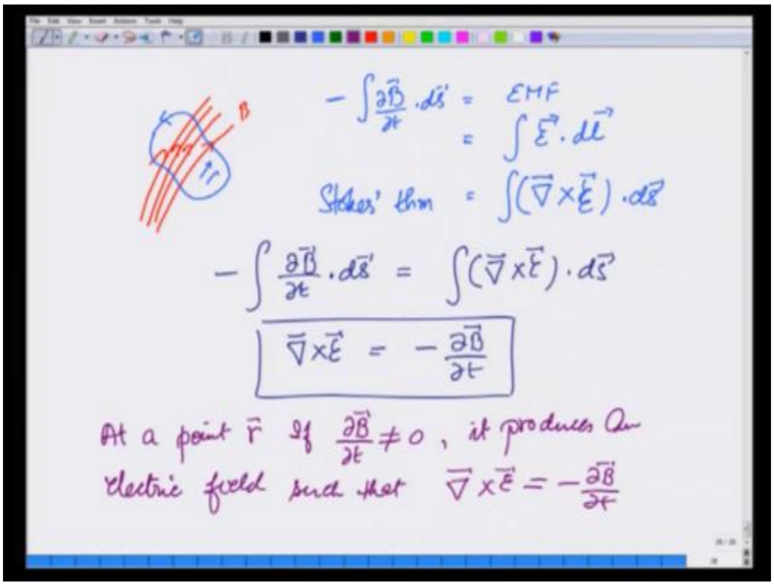

If I now take this region, through which B is changing; and, take a loop around it, through which I have taken this area ds; then I have integration dB by dt dot ds, which EMF with a minus sign in front. And EMF through this loop is going to be nothing but electric field E produced here dot dl, which then I can write as integration by Stokes' theorem curl of E dot ds. So, through Faraday's law, I get this relationship at minus integration dB by dt dot ds is equal to integration curl of E dot ds for any loop, because I can take this wire of any size in this; there will be a current flowing. Therefore, I can easily conclude, because ds is arbitrary – that loop size arbitrary that, curl of E is equal to minus dB by dt. What does it mean? It means that, at a point r, if dB by dt is not equal to 0; that means, the magnetic field is changing; it produces an electric field such that curl of that electric field is equal to minus dB by dt.

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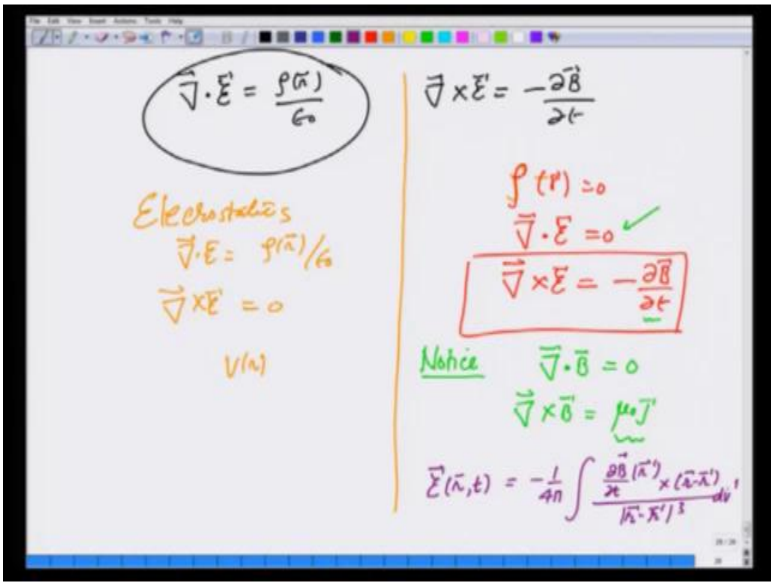

So, now, we have another source of producing electric field. So, we have two sources: one which gives me divergence of this electric field equals rho r over epsilon 0; and, another one which gives me curl of electric field as equal to minus dB by dt. This we have solved quite a bit. In this case, if there was no change in the magnetic field, only electrostatics was there; in that case, what we had; that divergence of E was rho r over epsilon 0. And curl of E was 0. And this give rise to a potential v r and we developed the entire machine to deal with it.

Let us look at this case – the dynamic case; where, suppose the charge is 0. Suppose I take rho r to be 0; then divergence of E is 0; but curl of E is equal to minus dB by dt. So, in this case, the source of electric field that is produced is the changing magnetic field. And the curl E is 0. Notice the similarity; notice the similarity of these equations with the equations for B, which give me the divergence of B – 0. And curl of B is equal to mu naught j. Source of B was J – the current density here; the source of E becomes dB by dt and divergence of both is 0. The similarity then tells you that, the solution for electric field produced by changing magnetic field should be equal to minus 1 over pi integration dB by dt at r prime across r minus r prime over r minus r prime cubed integrated over dB prime, because similar equations, similar boundary conditions should have the same answer.

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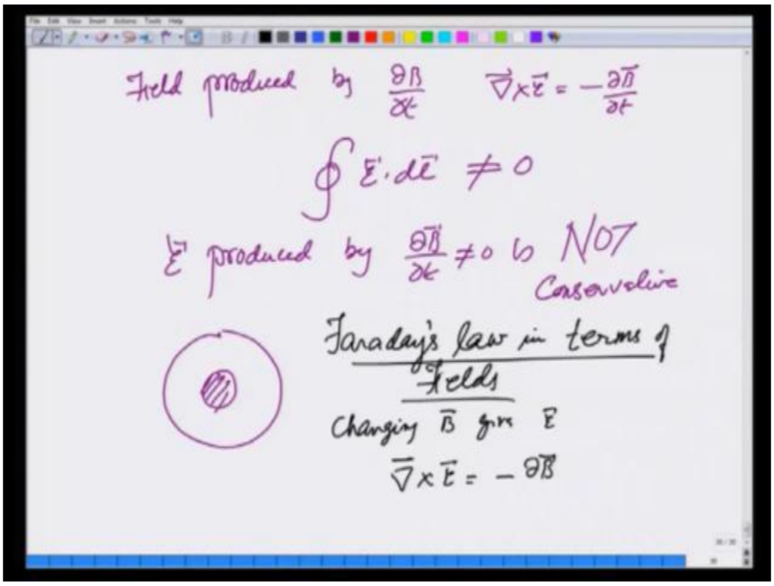

Notice that, because this field produced by dB by dt has curl nonzero; that means integration of E dot dl by Stokes' theorem is not 0. But, that we already know, because E dot dl is nothing but EMF. So, this field is not conservative. E produced by change in magnetic field is not conservative; that means if I take a path – circular path around a changing magnetic field, more I go around, more energy I gain. That is the meaning of consideration; otherwise, if it was conservative, every time I went around, the network done will be 0. But, here I go around; more is the energy that I gain. So, this is Faraday's law of fields, but it tells you is that, changing B gives E; and, the equation is curl of E is equal to minus dB by dt. This minus sign again comes because of the Lenz's law. It gives the proper directions when I saw the equation.

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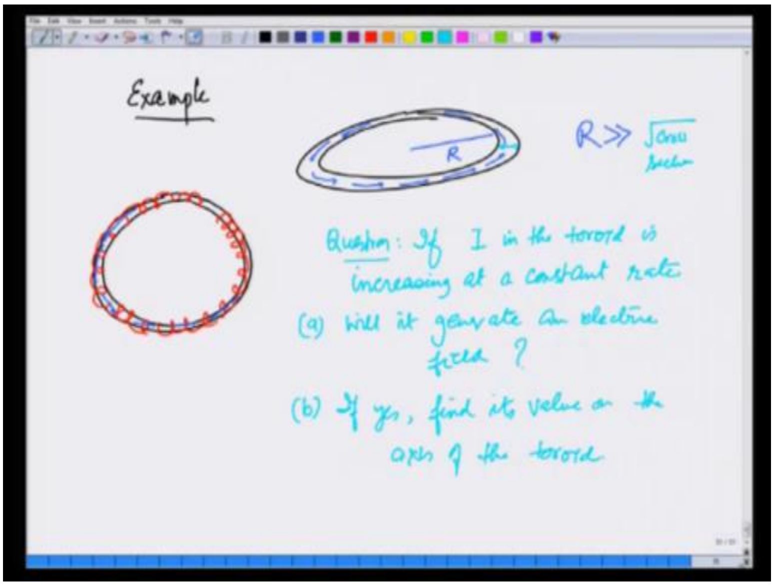

Let us now solve an example using this. The example I take is that of a toroid; that is, it is a thin tube, which is running around on which we have wrapped a current carrying wire. If you like, this is a long thin solenoid, which has been turned into around ring, so that the field inside this is nonzero depending on the direction of the current; outside field is 0. So, what I have is this ring in which there is field inside shown by blue. Let us take the radius of this to be much much much greater than the thickness or the cross section. It is a much greater than square root of cross section, so that, it is like a thin tube making a ring. If I change the current; if I change the current in this, the question being asked is if I in the toroid is increasing at a constant rate; a – will it generate an electric field? and, b – if yes, find its value on the axis of the toroid. So, let us look at this. What is happening now?

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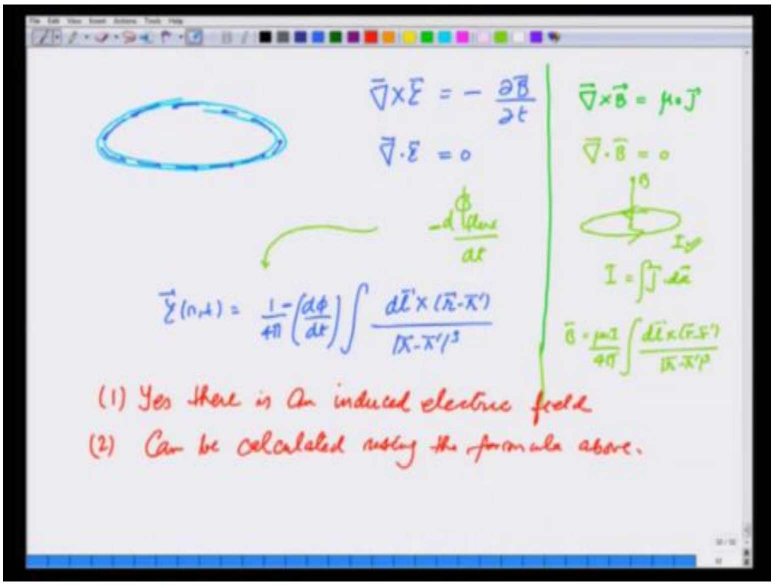

What is happening is that, I have thin toroid; and, as the current is increased, this field inside it is increasing. If the field changes; that means it produces an electric field; and, that is governed by curl of E is equal to minus dB by dt. I will keep writing on the right-hand side. The similarity with the magnetic field – curl of B is equal to mu 0 J. Divergence of E because there is no charge is equal to 0. I have divergence of B is equal to 0. So, if I were to make a similar thing for field and magnetic field, it will be similar to a current carrying ring and I am trying to find the B field on the axis of this ring. What is the current in this case? Current is J dot da; where, da is the cross section of the ring. Here therefore, similar to I, what I would have is flux by dt with the minus sign; minus d by dt flux will play the role of I.

How do I calculate the magnetic field here? I calculate the magnetic field by dl cross r minus r prime over r minus r prime cubed integration mu 0 I over 4 pi. So, I will calculate the electric field here on the axis or anywhere – E r, t is going to be… There is no mu 0; 1 over 4 pi; I is replaced by d phi by dt with the minus sign – integration dl across r minus r prime over r minus r prime cubed. And this integral can be easily calculated as we have done in an assignment in the past. So, to answer the question that I raised in the previous slide is – yes, there is an induced electric field; and, can be calculated using the formula above. So, I finish this lecture by saying that, we have now looked at Faraday’s law in a slightly different way in terms of fields; and, said that – interpreted in a way that, any changing magnetic field gives rise to an electric field, which drives a current. And this is precisely what you saw in the demonstration, where there was no area changing, but only magnetic field was changing. But, it was lighting and LED; it was making the rings jump.