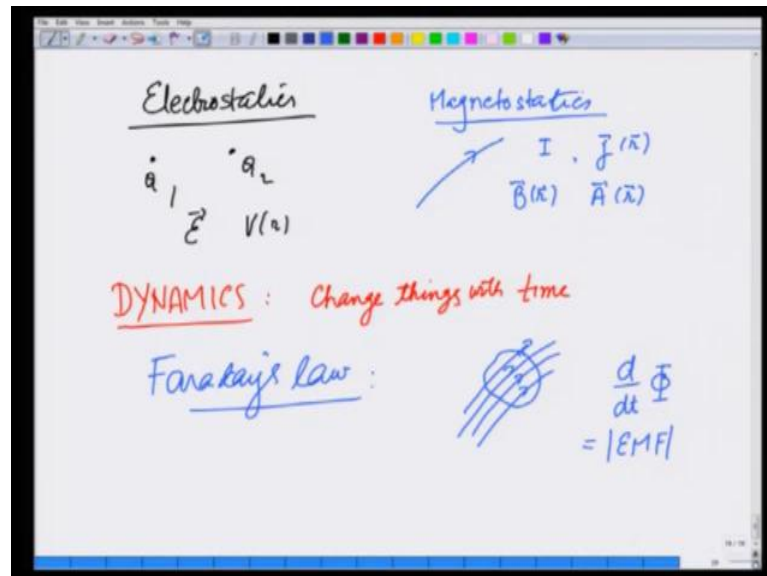


**Introduction to Electromagnetism**  
**Prof. Manoj K. Harbola**  
**Department of Physics**  
**Indian Institute of Technology, Kanpur**

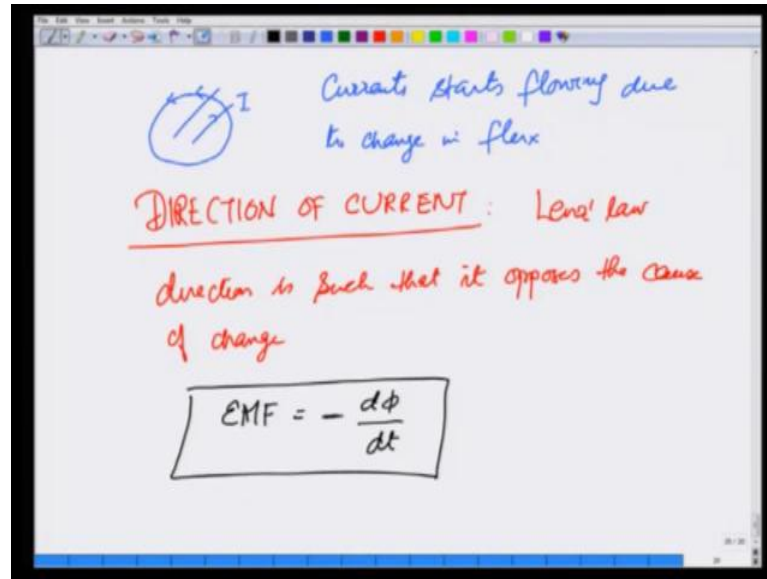
**Module - 05**  
**Lecture - 47**  
**Faraday's Law**

(Refer Slide Time: 00:18)



So, far in this course we have focused on static situations. So, we have dealt with electrostatics which concerned itself with fixed charges and corresponding electric field and the potential  $v$  or we dealt with magnetostatics, which dealt with steady currents that means, the current did not depend on time  $j$  or current density the corresponding magnetic field and the vector potentials and so on. Now, we are going to go into dynamics and what that means is we are going to change things with time and see what is the effect of this. So, first thing that comes in this is Faraday's law what faraday first thought should happen and then observed is that if I have a circuit and I change the flux of magnetic field passing through it. So, we change  $d$  by  $d$   $t$  of the flux passing through it then this induces an  $e$   $m$   $f$  and the units. So, you know as such that  $e$   $m$   $f$  is equal to  $d$  by  $d$   $t$  the flux the magnitude of  $e$   $m$  alright.

(Refer Slide Time: 02:11)



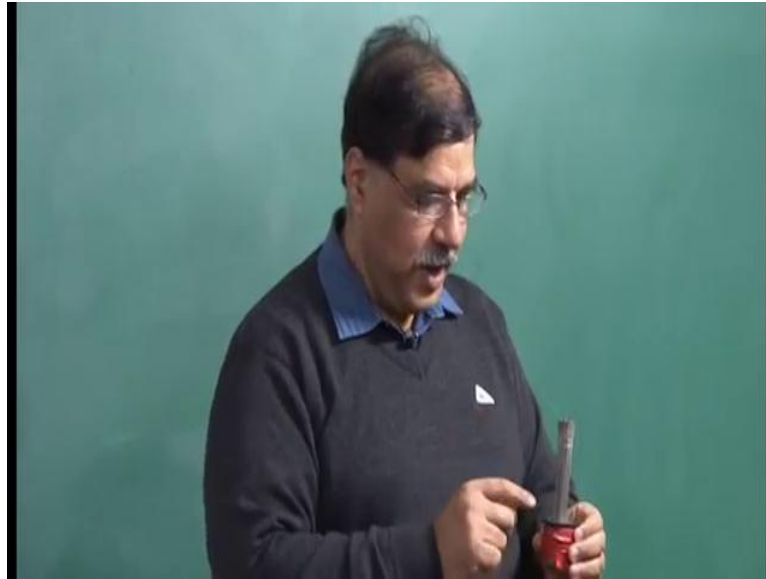
What it means is that if I have a circuit then because of this e m f the current will start flowing in this current starts flowing due to change in flux. What about the direction of current and that is given by Lenz's law which says that the direction is such that it opposes because of change and we will put that mathematically in a minute. Now, what is means is that if I take a circuit and let the area of the circuit change or the magnetic field through it change then there will be an e m f generated. Let me show this to you through a demonstration I want to acknowledge professor h c verma at I i t kanpur who has developed these demonstrations for school going kids. In the first demonstration I have this tube on which a lot of wire has been wound I put this magnet inside.

(Refer Slide Time: 03:39)



Now, if I shake the magnet as it comes near the wire the flux is going to change because as the magnet comes nearer the field becomes stronger and as it goes away field becomes weaker. Again the flux is going to change and that should produce an e m f to show that an e m f is produce that wire is connected to an l e d out here and this l e d should glow and let us see that. You see the red light coming. So, as the magnet is moving around its changing the magnetic field and the change in the magnetic field changes the flux and that generates an e m f. So, this is a first that you have seen that if I throw a magnet through a coil this is which is what faraday did that produces an e m f in that. The second thing is that the direction of e m f is opposite such that it opposes the change.

(Refer Slide Time: 04:43)



So, here we have a coil in which we have put this again developed in professor h c vermas lab put these iron spikes which make the magnetic field very strong here. As soon as you will see as you switch on the switch. So, that the current starts flowing the ring in this will try to go away because the current generated or e m f generated in the ring is going to be such that it is going to oppose the change. So, I will just switch it on and you will see that the ring jumps out and that is the Lenz's law let us now how do this mathematically. So, what we have seen is that we have e m f which is equal to  $-d\phi/dt$  and to show the Lenz's law I put a minus sign in front. You may ask how does this minus sign help and let us discuss that a bit.

(Refer Slide Time: 05:43)

$$\text{EMF} = - \frac{d\phi}{dt}$$
$$= - \frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

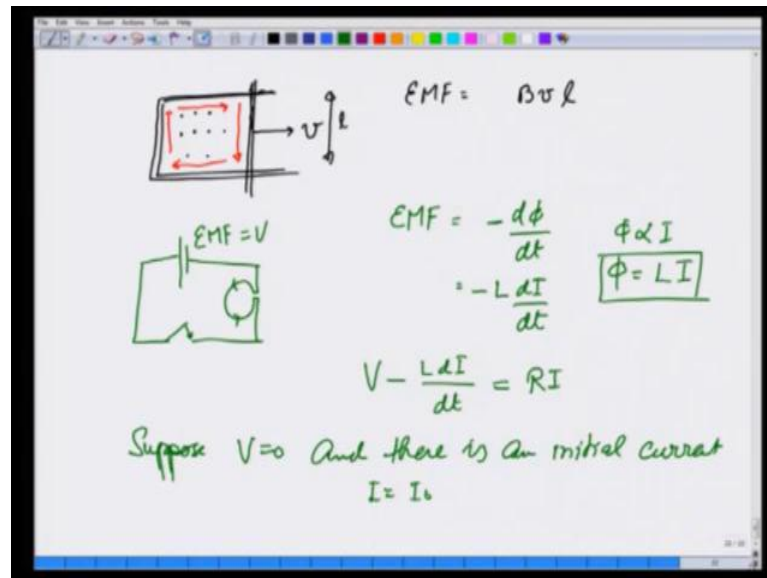
Diagram: A closed loop with magnetic field lines passing through it, labeled with  $\phi$ .

$$\int - \frac{\partial B(r,t)}{\partial t} \cdot d\vec{s}$$
$$\int \vec{B} \cdot d\vec{s}$$

Motional EMF

So, e m f is equal to minus d phi over d t which is equal to minus integration b dot d s d by d t where b is going through this area passing through this area and d s is the area element in this area. So, b dot d s integrated over gives me the flux there are two ways that this can change either b can change with time. So, I will put d by d t with the minus sign dot d s and the area remains fixed. Or I can have b constant in time, but this area changes that means, area goes in and out and therefore, the flux changes. This is due to movement of the boundaries whatever e m f comes is sometimes referred to as motional e m f and this is due to the change in magnetic field.

(Refer Slide Time: 07:09)



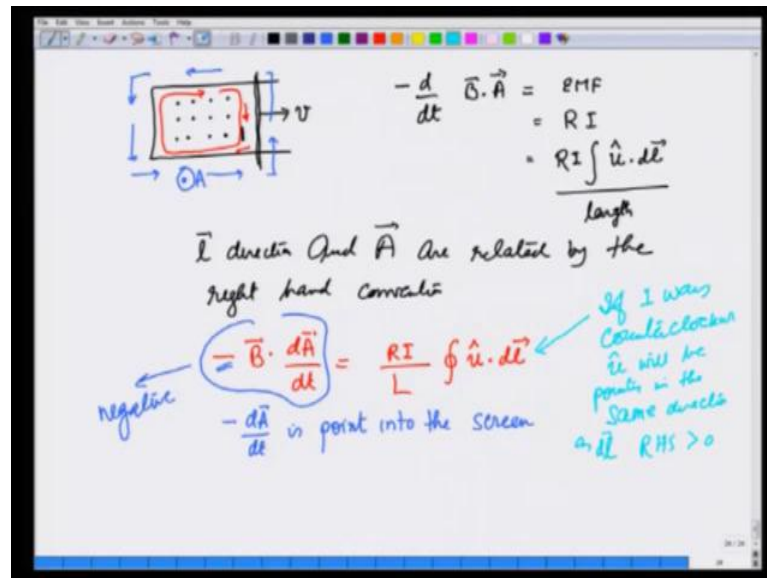
So, first let us look at an example of motional e m f you have very familiar with. If I take a wire and put a rod on this metallic rod in which the field is coming out and I pull this with velocity  $v$  then the area is changing and this change in area gives me an e m f which is equal to  $b v$  times  $l$  where  $l$  is the length of this rod  $v l$  gives you the rate of change of area. And the direction of current is going to be such that it changes it opposes the change. So, the current should be such that it opposes the rod and it opposes this being pulled out. So,  $I v$  is coming out  $I$  cross  $b$  it will give me a full force like this therefore,  $I$  should be going down in the rod and therefore, in the loop it should be going like this. Another example of this would be suppose I take a battery put a switch and put it through a coil. So, that when the current passes through the coil there is a flux through it and any change in current would change this flux. Now e m f is going to be proportional to or equal to  $d \phi$  by  $d t$  with the minus sign. Now I will make this minus sign mathematically meaningful  $\phi$  you know is going to be equal to in such a coil is going to be proportional to  $I$  and this is proportionality constant is usually called the not usually its always called the self inductance. So, I am going to put an  $l$  out here. So, this is minus  $l d I$  by  $d t$  now we will see the importance of this minus sign. So, as soon as I put on the switch the equation I am going to have is e m f which is e m f equals  $v$  the other e m f which is generated in the system is minus  $l d I$  by  $d t$  should be equal to  $r I$  if  $r$  is the resistance of the system. Suppose  $v$  was zero and there is an initial current  $I$  equals  $I_0$  in that case.

(Refer Slide Time: 09:57)

The image shows a whiteboard with handwritten mathematical equations. At the top, the equation  $L \frac{dI}{dt} = RI$  is written, with a circled minus sign to its left. Below it, the equation  $L \frac{dI}{dt} + \frac{R}{L} I = 0$  is written. A red arrow points from the circled minus sign to the plus sign in the second equation. Below that, the solution  $I = I_0 e^{-R/L t}$  is written and boxed in green. A red arrow points from the boxed solution to the equation  $L \frac{dI}{dt} = RI \Rightarrow I = I_0 e^{R/L t}$  written in red below it.

This equation becomes zero minus  $L \frac{dI}{dt}$  equals  $R I$  or  $L \frac{dI}{dt} + \frac{R}{L} I$  equal to zero. This would happen for example, if in the circuit there was a current flowing and I suddenly took switch off. Now this solution gives me  $I$  equals  $I_0 e^{-R/L t}$ . Now notice if instead of this minus sign here if I had a plus sign if I had a plus sign I will get  $L \frac{dI}{dt} = RI$  and this will give me an answer  $I$  equals  $I_0 e^{R/L t}$  in this case after I turned the switch off it is the current is increasing with time which is in violation of what we observe it is in violation of energy conservation in this case it is decreasing with time and eventually going to zero. So, you see the importance of that minus sign out here and that is the statement of Lenz's law.

(Refer Slide Time: 11:13)

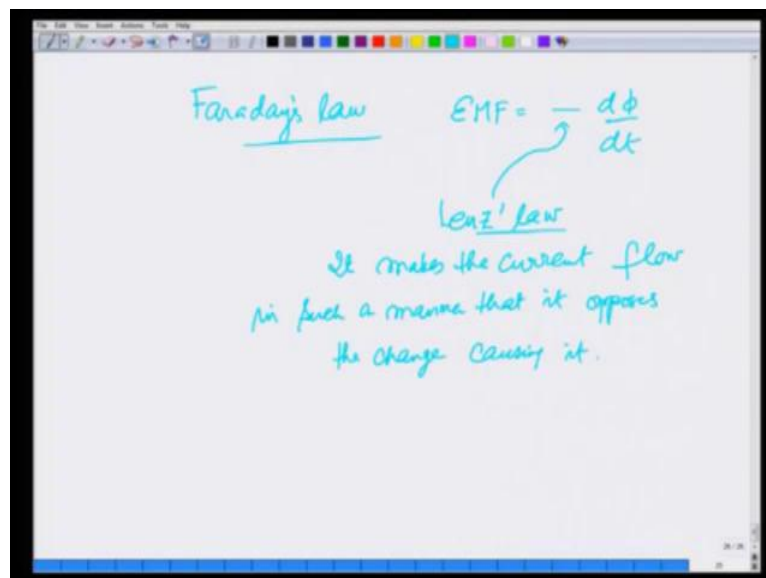


The second example through which I show the significance of this minus sign is what we have already solved is that this metallic circuit on which I have this rod which is being pulled to the right this is velocity  $v$ . And we have already taken that field was coming out of this screen and in that case we saw that the current was going clockwise that is what opposes. So, this is what we physically saw let us see it mathematically. So, I have minus  $d$  by  $t$  of  $b$  in this case it is a uniform field. So, I can write it like this  $b d a$  is equal to  $e m f$  which in this case I can write as  $r I$  and to understand the significance of this minus sign I am going to write  $I$  in a slightly different manner. I am going to write as  $r$  integration  $I$  is there  $u$  where  $u$  is the unit vector in the direction of the current. So, here it is going to be like  $\oint u \cdot d l$  where  $d l$  is the lined element along the path divided by the total length this keeps it  $r I$ . Now  $l$  direction and direction of  $A$  are related by the right hand convention. So, if I take my fingers around  $l$  clockwise or counterclockwise thumb gives me the direction of  $a$ . Now, in this case  $b$  is fixed  $a$  is changing. So, I am going to write this as  $-\vec{B} \cdot \frac{d\vec{A}}{dt} = \frac{RI}{L} \oint \hat{u} \cdot d\vec{l}$ . Let us assume I take  $a$  also to be coming out suppose I took area  $a$  to be coming out. If I took area  $a$  to be coming out  $u$  and  $l$  will be counterclockwise like this as  $v$  is being pulled out  $a$  area is increasing  $d a$  by  $d t$  is positive and  $-\frac{d a}{d t}$  is pointing into the screen and therefore, this entire product  $-\vec{B} \cdot \frac{d\vec{A}}{dt}$  is negative because of this minus sign here.



If the current direction was also counter clockwise just look at the right hand side here if I was counterclockwise then u would be pointing opposite to l. Oh sorry u would be pointing in the same direction as l u will be pointing in the same direction as d l and d l s will be positive. You see the inconsistency of the left hand side and the right hand side. On the other hand if u is pointing clockwise if the current is clockwise then u and d l are in the opposite direction and you get the right sign. So, you see the significance of this sign this actually relates to energy conservation. Again because if the current went clock or counterclockwise the rod will keep on moving faster, faster and faster because it will be pushed out. On the other hand when it goes clockwise it is stopped.

(Refer Slide Time: 15:50)



So, what we have covered in this lecture is that change in flux by Faraday's law gives e m f which is given as minus d phi by d t. And this minus sign is given there to show Lenz's law its mathematical effect is that it makes the current flow in such a manner that it opposes the change causing it. In the next lecture, we will be turning this whole Faraday's law into in terms of fields.