Introduction to Electromagnetism
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Module - 05
Lecture - 46
Solving for Magnetic Field of a magnet in presence of Magnetic Materials

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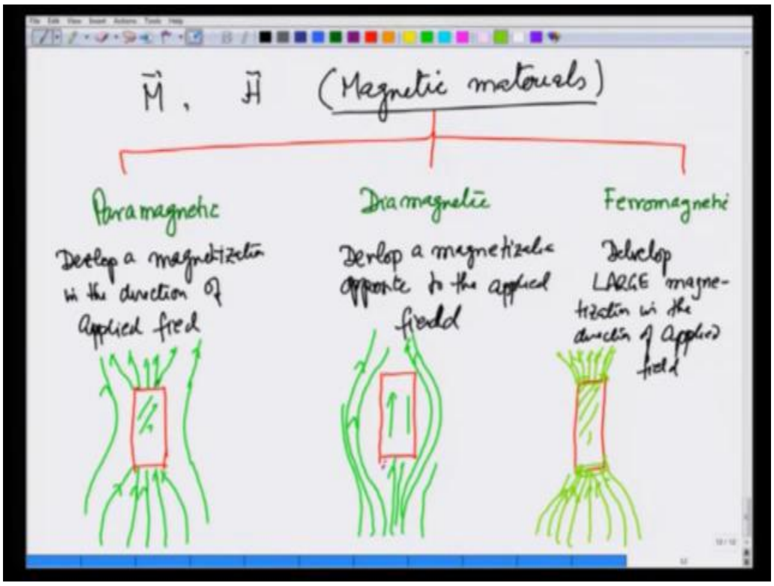

$\vec{M}$, $\vec{H}$ (Magnetic materials)

Paramagnetic

Develop a magnetization in the direction of Applied field

Diamagnetic

Develop a magnetization opposite to the applied field

Ferromagnetic

Develop LARGE magnetization in the direction of Applied field

Having set up our machinery for calculating magnetic field with magnetic moment m, having introduced an auxiliary field h we are now ready to deal with magnetic materials. So, we are going to deal with magnetic materials which can be broadly classified into three kinds. These are paramagnetic, diamagnetic and ferromagnetic. Paramagnetic materials develop a magnetization in the direction of applied field. So, if I take this sample apply a magnetic field it will develop a magnetization in that direction. Diamagnetic materials develop a magnetization opposite to the applied field and Ferromagnetic materials develop a large magnetization in the direction of applied field.

So, let us make this pictorially. If I look at paramagnetic material, suppose I take a sample of paramagnetic material and I put it in a uniform field this will develop a magnetization in the direction of the applied field. And therefore, it will develop a magnetization like this and that will give rise to additional magnetic field, both inside and outside. We just saw that inside if it is a long one it is going to be a uniform additional magnetic field. As a consequence what you notice outside here the field is opposite to the applied field. So, field will decrease a bit, but near the ends its strong therefore, the final lines that we are going to see in this case. Let me now erase this I will show you the final lines are going to look like it will become stronger near the end. So, these lines that we applied are going to turn in become weaker here. So, they will go like this strong again here. So, more lines come in and weaker here this is how a paramagnetic material the lines around it would look when it is put in a uniform magnetic field.

On the other hand, if I look at diamagnetic material and put it in the similar uniform field it will develop a magnetizationally opposite direction. If it develops magnetization in the opposite direction what does it mean? That means, the new field lines that will create are going to be like this opposite to the applied field inside in the direction of applied field outside. And therefore, the way field is going to look is going to be something like this it will become weaker inside. So, this again takes this field it will become weaker inside. So, that means, lines will get repelled and stronger here near on the sides and you see lines have gone away and it will be weaker inside compared to outside, this is a diamagnetic material.

In a ferromagnetic material strong magnetization gets developed and therefore, most of the lines are going to go in it is like paramagnetic material, but takes in many more lines and very weak field outside. You can see this if you take these materials near iron filings in a ferromagnetic material lot of iron filings gets stuck near the ends. Because of this behavior what will happen is that a paramagnetic material will go towards stronger field its energy gets lowered a diamagnetic material will go away from strong field.

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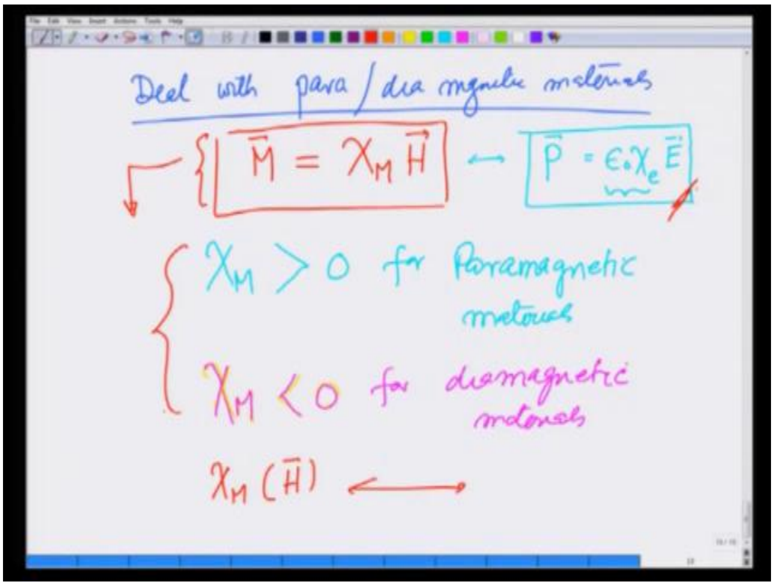

Deal with para / dia magnetic materials

$$\boxed{\vec{M} = \chi_M \vec{H}} \longleftrightarrow \boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$$

$\chi_M > 0$ for Paramagnetic materials

$\chi_M < 0$ for diamagnetic materials

$\chi_M(\vec{H}) \longleftrightarrow$

It will move towards weaker field and a ferromagnetic material off course will go very strongly towards a stronger field what we will be doing in this is deal with para slash dia magnetic materials. Let me just again define what they are these are the ones that develop a magnetic moment proportional to the applied field or to define it very precisely this becomes equal to a magnetic susceptibility chi m times h.

Again similarity with electrostatics is there because in electrostatics what we had was if there was an electric field somewhere it had developed epsilon naught chi electrical e polarization at that point slight difference in the way we have defined susceptibility for electrical case and magnetic case. In the electrical case I had susceptibility defines such that epsilon naught chi times e gave me polarization there e is the field. In the case of magnetic materials m is defined as proportional to h the auxiliary field not the field which applies the force times chi m there is no mu 0 here right. So, this is chi m is going to be greater than 0 for paramagnetic materials and chi m is less than 0 this color is not coming well. So, I will change the color chi m is less than 0 for diamagnetic materials. What this boxed relationship here in red is giving you is the constitutive relationship that we had in electrostatics now it is for magnetostatics.

For ferromagnetic materials chi m depends on the applied field itself. So, it becomes a non-linear behavior we are not going to deal with this and this leads to hysteresis and things like those we are going to deal with all this these materials. Techniques are going to be similar to what we did earlier in the case of electrostatics.

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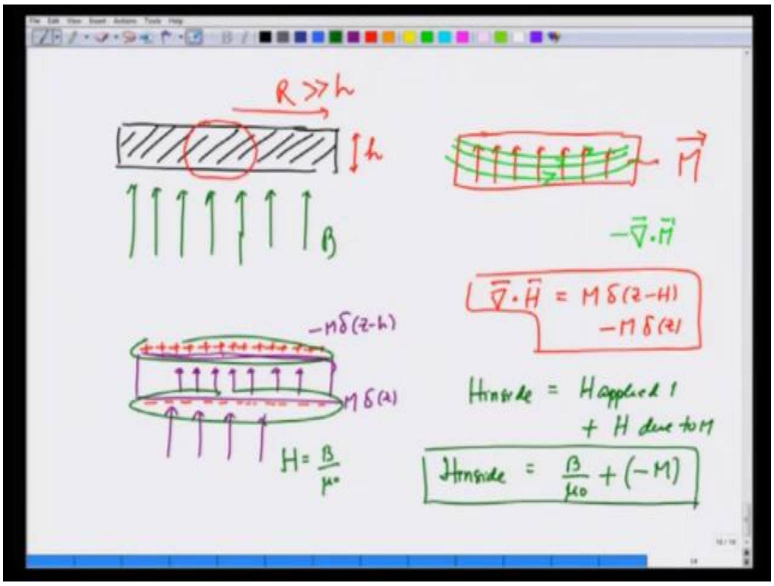

So, I would request you to go and look at that lecture again because now, I am going to solve examples. As the first example let us take the case where I take a sample of magnetic material paramagnetic or diamagnetic and put it in a uniform field b. The sample is large enough such that its radius r is much greater than its height let us say h. So, that I can ignore the fringe effects or in other words I am looking in this region in between somewhere here where the effect of the edges is really negligible.

Now, what will happen when I apply b. this b will give rise to a magnetic moment. So, let us say this fellow develops a magnetic moment m. This magnetic moment in turn can be thought of as the bound currents like this it is a uniform magnetic moment. Or, I can think of minus divergence of m as that magnetic charge that gives rise to h let us solve the problem in both ways. So, if I look at this material I had applied b and if I take this uniform magnetic field inside then the divergence of m at the upper surface is minus m delta z minus h if I take the upper surface at h and at the lower side it is m delta z. So, it is like a surface charge if this is divergence the del dot h is going to be minus del dot m. So, this is going to be m delta z minus h minus m delta z. So, h is going to be like we are giving rise to an h which is with positive charge here and negative charge here. So, Del dot h satisfies this equation. So, h outside is nothing, but b over mu 0 h inside is going to be the applied h plus h due to m. Divergence of h applied is 0 because it is a uniform h. So, h inside is going to be h applied which is b over mu 0 plus h due to m and as I said this comes because this is positive charge on the top and negative charge on the bottom? So, this is going to be minus m there is no epsilon 0 nothing here. So, h inside is b over mu 0 minus m.

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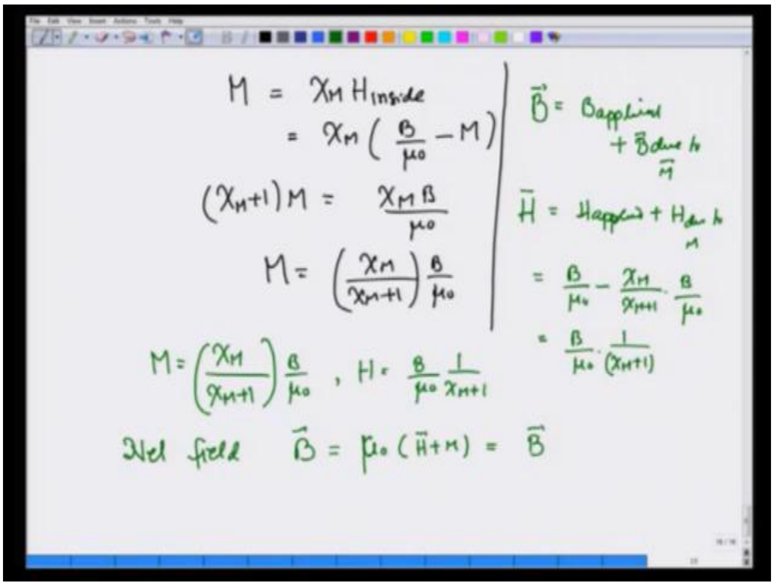

$$M = \chi_M H_{inside}$$
$$= \chi_M \left(\frac{B}{\mu_0} - M \right)$$
$$(\chi_M + 1) M = \frac{\chi_M B}{\mu_0}$$
$$M = \left(\frac{\chi_M}{\chi_M + 1} \right) \frac{B}{\mu_0}$$

$$\vec{B} = B_{applied} + \vec{B}_{due\ to\ \vec{M}}$$
$$\vec{H} = H_{applied} + H_{due\ to\ M}$$
$$= \frac{B}{\mu_0} - \frac{\chi_M}{\chi_M + 1} \cdot \frac{B}{\mu_0}$$
$$= \frac{B}{\mu_0} \cdot \frac{1}{(\chi_M + 1)}$$

$$M = \left(\frac{\chi_M}{\chi_M + 1} \right) \frac{B}{\mu_0}, \quad H = \frac{B}{\mu_0} \frac{1}{\chi_M + 1}$$

Net field $\vec{B} = \mu_0 (\vec{H} + M) = \vec{B}$

Now, we will go through a loop of self consistency because I know this m is nothing, but its same direction. So, I am not writing a vector sign on top its nothing, but chi m times h at that point. So, h inside which is nothing, but chi m b over mu 0 minus m and this gives me chi m plus 1 m equals chi m b over mu 0. Or m is equal to chi m over chi m plus 1 b over mu 0 this is a solution for m and therefore. B which is b applied plus b due to m or h which is h applied plus h due to m is going to be equal to h applied is b over mu 0 minus m which is nothing, but chi m over chi m plus 1 b over mu 0 which gives me b over mu 0 1 over chi m plus 1. So, h using all this we have found that m is chi m over chi m plus one b over mu 0. H is b over mu 0 one over chi m plus 1 and the net field b is going to be equal to mu 0 h plus m and that is comes out to be b. So, b inside remains the same it is not affected by chi m.

This is not surprising since in this material by gauss's law I have if I choose this box out here I have del dot b is equal to 0 which gives that b perpendicular should be continuous and that is precisely the result we are seeing.

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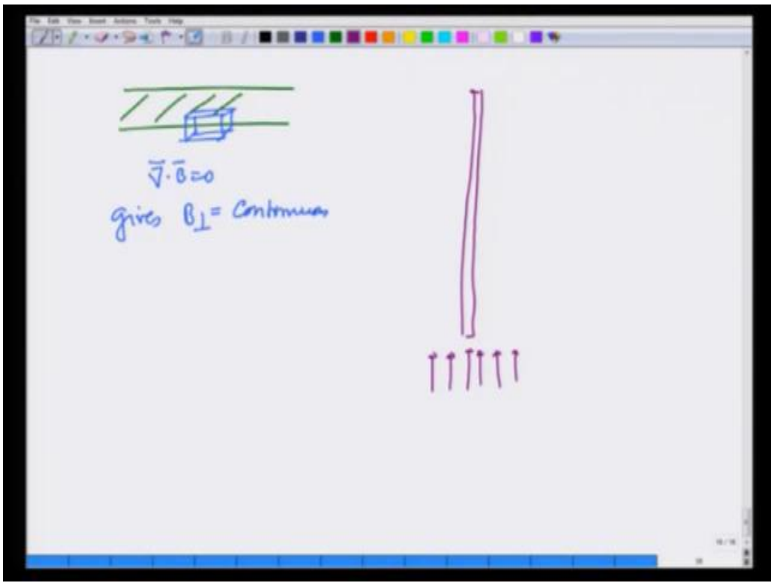

$\vec{\nabla}\cdot\vec{B}=0$

gives $B_\perp$ = Continuous

Another example you may want to try is the long solenoid which is put again in a uniform field this I will leave as an exercise.

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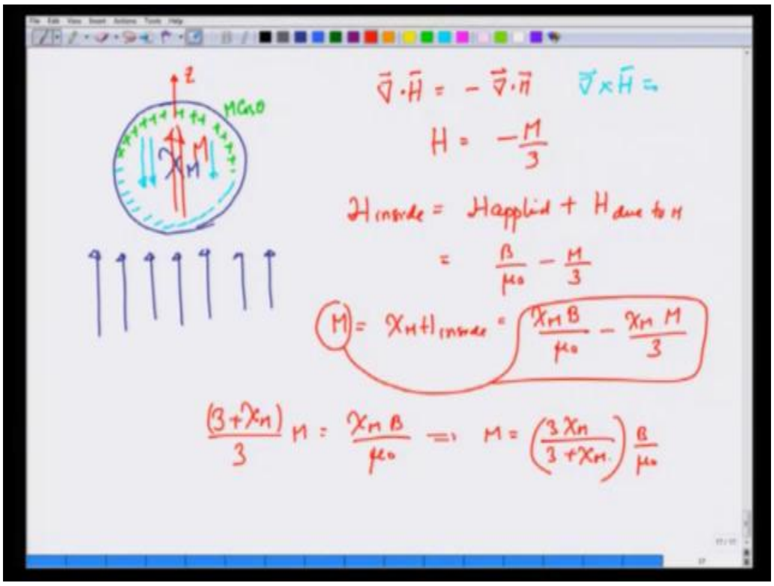

$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$ $\quad \vec{\nabla}\times\vec{H} =$

$H = -\frac{M}{3}$

$H_{inside} = H_{applied} + H_{due\ to\ M}$

$= \frac{B}{\mu_0} - \frac{M}{3}$

$M = \chi_M H_{inside} = \frac{\chi_M B}{\mu_0} - \frac{\chi_M M}{3}$

$\frac{(3+\chi_M)}{3} M = \frac{\chi_M B}{\mu_0} \Rightarrow M = \left(\frac{3\chi_M}{3+\chi_M}\right)\frac{B}{\mu_0}$

The third example I am going to do again. I will come back to over stand at problem of sphere made of a material of chi m magnetic moment put in a uniform field from outside. We will start by assumption that it develops a magnetic moment m in the same direction let us take this direction to be z. Now, if I want to solve through h method I get del dot h is equal to minus del dot m and we have already seen. This means I have a positive charge kind of thing out here which is m co sign of theta and a negative charge on the lower side which is gain m co sign theta and therefore, the h gives rise to is in this direction an h and del cross h is 0 because there is no free current. So, h it gives rise to is going to be minus m by three. So, h inside is going to be h applied plus h due to m which is going to be b over mu 0 minus m by 3, but I also know then m is going to be chi m h inside and this is therefore, this is going to be chi m b upon mu 0 minus chi m m over 3 m equals this and this gives 3 plus chi m over 3 m is equal to chi m b over mu 0 implies m is equal to 3 chi m over 3 plus chi m times b over mu 0.

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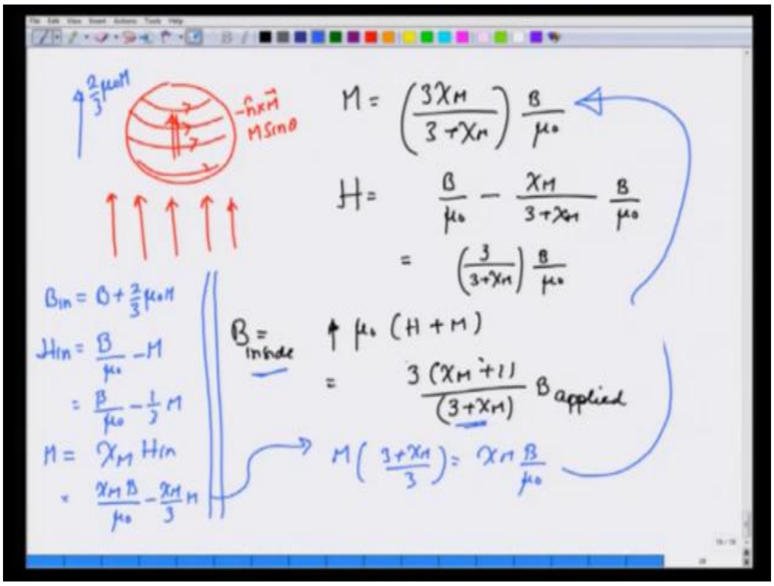

$$B_{in} = B + \frac{2}{3}\mu_0 M$$

$$H_{in} = \frac{B}{\mu_0} - M$$

$$= \frac{B}{\mu_0} - \frac{1}{3}M$$

$$M = \chi_M H_{in}$$

$$= \frac{\chi_M B}{\mu_0} - \frac{\chi_M}{3}M$$

$$M\left(\frac{3+\chi_M}{3}\right) = \chi_M \frac{B}{\mu_0}$$

$$M = \left(\frac{3\chi_M}{3+\chi_M}\right)\frac{B}{\mu_0}$$

$$H = \frac{B}{\mu_0} - \frac{\chi_M}{3+\chi_M}\frac{B}{\mu_0}$$

$$= \left(\frac{3}{3+\chi_M}\right)\frac{B}{\mu_0}$$

$$B_{\text{inside}} = \mu_0 (H + M)$$

$$= \frac{3(\chi_M + 1)}{(3+\chi_M)} B_{\text{applied}}$$

So, what we obtained in this if I have this sphere in a uniform field we obtain that m is 3 chi m over 3 plus chi m b over mu 0 and h therefore, is going to be h applied which is b over mu 0 minus m by 3 which in this case becomes chi m over 3 plus chi m b over mu 0 which is equal to 3 over 3 plus chi m b over mu 0. And therefore, b the net field inside is going to be equal to mu 0 h plus m which gives me 3 chi m plus one over 3 plus chi m b applied. So, you have seen the techniques used for electrostatics when we had a polarizable material in a electrostatic field. Similar techniques can be used here to calculate the field inside when a magnetic material is put in a field. Another way one could have done it is, when I assumed this m out here I could have thought of this m not as this through h method, but I could have thought of this m giving rise to these bound currents which are minus n cross m which actually gave me m sign theta.

And we have seen in the past few lectures ago that this actually gives rise to a field inside which is two thirds mu 0 m. So, to work this out then I know that b inside is going to be b applied plus two thirds mu 0 m and this gives h inside b in over mu 0 minus m which is equal to b m over mu 0 minus oh sorry b over mu 0 minus this is not b this is b over mu 0 minus one third m now m itself is equal to chi m h in which is going to be chi m b over mu 0 minus chi m over 3 m and this gives m 3 plus chi m over 3 equals chi m b over mu 0 giving m same as here on the top. So, you see I could have solved this problem using the current method also. The main point is I assume an m which is in the same direction as b which happens to be the true in these cases and then do a self consistent analysis to get the answer. Notice that finally, because of the paramagnetic nature if chi m is greater than one b inside is greater than b applied. However, this chi m is less than one then this is going to be different and going to be less than the applied field.