Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 45
Solving for Magnetic Field of a magnet I

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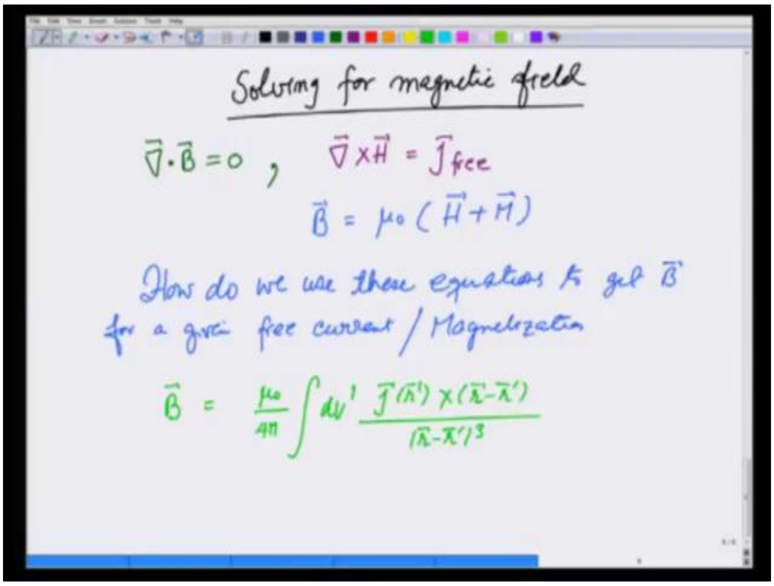

In this lecture, we are going to now focus on solving for magnetic field in different situations. The tools that we have on our hands are: one - divergence of B, which is the magnetic field is 0. We also have an auxiliary field H, which has curl equals j free. An additional information we have is that all the way we have defined H is that B is equal to mu 0 times H plus M. And the question is how do we use these equations to get B for a given a free current slash magnetization. So, depending on what situation we are at we will use the different techniques. One thing we know is that B is given as mu 0 over four pi integral d v prime j r prime cross r minus r prime over r minus r prime cubed. It can also be calculated from vector field.

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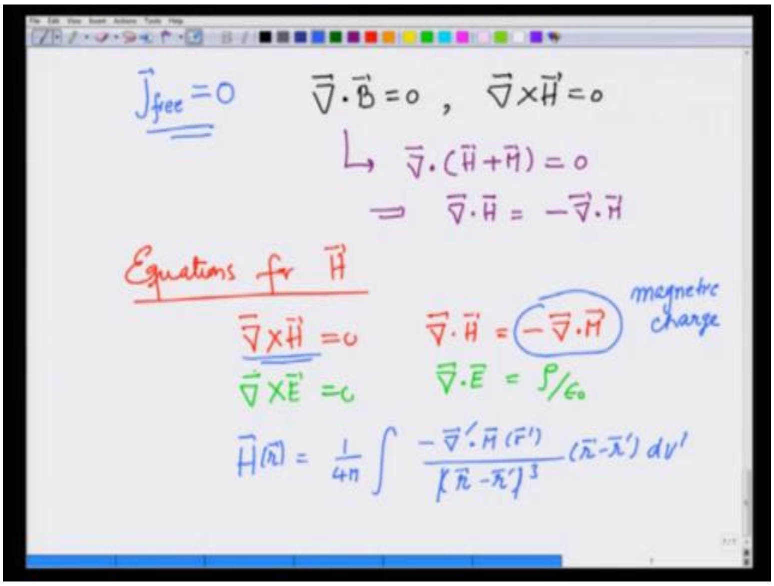

Now, let us look at different situations and see how we can exploit our previous knowledge to calculate magnetic field. Suppose we take a situation where j free is 0, situation in Electrostatics; where I had curl of E is 0 and divergence of E is equal to rho over epsilon 0.

So, in this case where there is no free current, I can treat this quantity here as some sort of a magnetic charge. And this quantity of course is curl of H is 0. So, I can do a calculation treating this minus del dot M as magnetic charge as if I am calculating the electric field due to discharge. Just to complete the analogy, in this case what I can do is to write H as there is no epsilon 0. One over four pi integral of minus del prime dot M r prime because that is a charge, divided by r minus r prime distance cubed r minus r prime vector d v prime. And this is H at r. Just like we write the electric field. And therefore, in situations where there are some sort of symmetry or I can calculate M and del dot M easily. It is very easy to calculate H.

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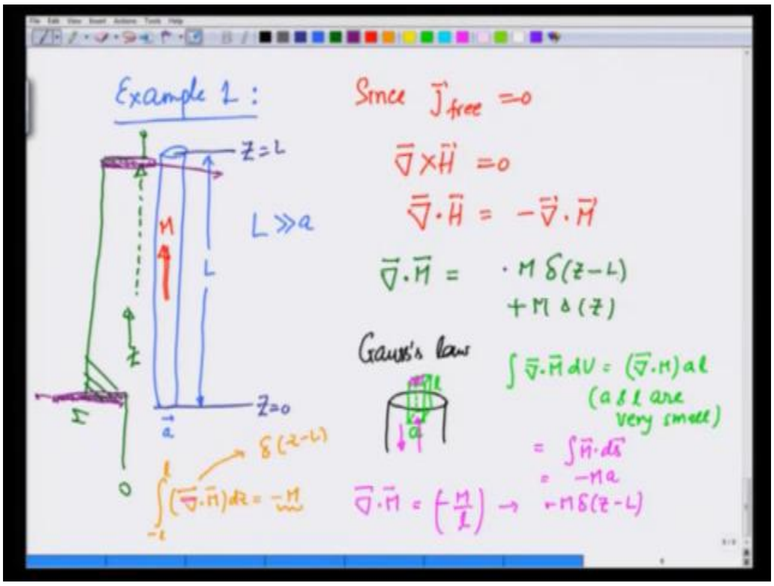

Let us take a few examples. Example one: suppose I have a long bar magnet, its length is L and radius a, such that L, or let me make L properly. The total length is L. L is much, much greater than a. So, I can ignore the fringe effects. And this carries a magnetization M along its length. Since j free is 0, I have curl of H is equal to 0 and divergence of H is equal to minus divergence of M. Let us see what divergence of M in this case is going to be.

The M, magnetization has certain value along the length and is 0 outside in this direction. So, this is finite value M and 0. If I take this direction to be z, the vertically up direction, you can see that curl of divergence of M is equal to M delta z at the upper point and similarly proportional to M delta z at the lower point.

Let us now fix certain values. Let us take the lower point to be z equal to 0; let us take the upper point to be z equals L. Then, I can write the divergence of M. M becomes smaller, it goes down. So, this is minus M delta z minus L plus, M increases at z equal to 0, so this is going to be a term M z.

If you want to be more convinced about it, I can use Gauss's law to show that this is so. For that, let me take this upper surface and make a small box here and calculate divergence of M d v in this box. If the lower surface area is a and the length of the box is l, this is going to be divergence of M a l; where a and l are very small. And by Gauss's theorem, this is going to be equal to M dot d s. Now, on the upper surface there is no M.

So, from the upper surface this gives you 0. On the lower surface d s is going out, so this is going to be minus M a. On the side surfaces, there is no contribution. So, M is parallel to the surface. And therefore, what I get is divergence of M is equal to minus M over l.

This l, you can take the limit going to 0. And in that case this goes to minus M delta z minus L. You can see it other way because minus divergence of M d z is equal to minus M. And no matter how small this d z is; from say minus L to L. No matter how small this d z is; I still get minus M. And therefore, this divergence of M has to be proportional to delta z minus L.

You can do the same thing on the lower side. So, what is happening is this divergence of M is like a spike at the lower; M equals L. I am showing it here with this purple block and minus like this on the upper side; goes all the way to infinity and goes all the way to infinity. So, let us make again in the next slide.

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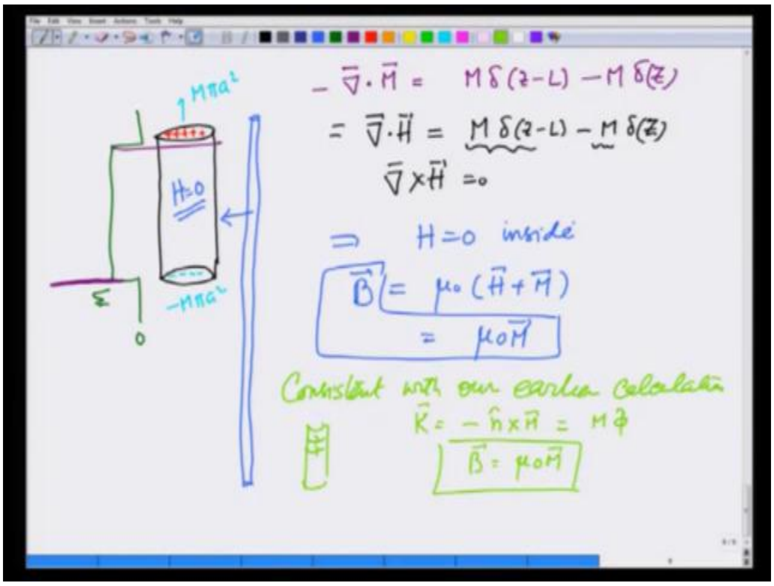

I have this long; and I am making a slightly different picture, so that you can see it clearly. A long bar magnet on which the magnetization goes down is M throughout the magnet and goes down again; 0, M. And its divergence has this delta function here and delta function here.

And therefore, minus divergence of M is going to be equal to M delta z minus L minus M delta z. And what is this equal to? This is equal to divergence of H, which is then is equal to M delta z minus L minus M delta z. And I know curl of H is 0. This is like; m is like in the surface charge, minus M is like surface charge. So, what I have here is as if the upper surface of this magnetic bar magnet has positive magnetic charge if you like and negative on the lower side. This amount of discharge is going to be M pi a square minus M pi a square.

And since the length of the rod is very, very large, so this is; if in reality it is more like a very thin rod like small charges up and down. And this is what is being represented here by this. So, H inside is going to be nearly 0. So, this implies H is 0 inside. And therefore, B is going to be equal to mu 0 H plus M, which is nothing but mu 0 M. Note that this is consistent with our earlier calculation, where we had treated this bar magnet as carrying surface current k; which was minus n cross M, which was M in phi direction. So, it became like a solenoid and that gave me a field, which was mu 0 M. This is the same, except this time we use a different technique. We use that auxiliary field H.

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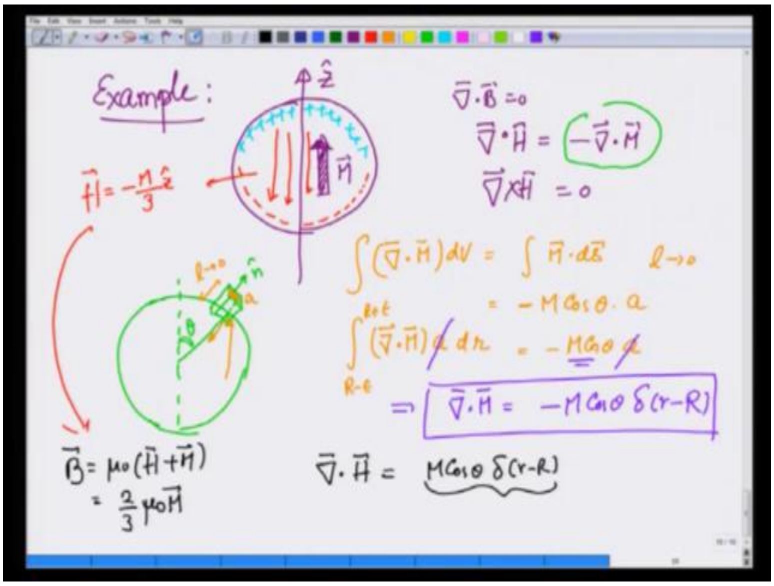

Let me do another example. A well-known example that we have been treating. And I said that this becomes nice example for magnetic field. In this example, I will take this magnetized sphere, which has magnetization; these are directions, this is magnetization M.

Again, since there is no free current, I have divergence of the 0; which gives me divergence of H is equal to minus divergence of M and curl of H is 0. So, again you see that this minus divergence of M becomes like the magnetic surface charge. How so? Let us see that.

If I look at the surface and I want to calculate divergence of M at an angle theta, then again by making a box here; which has n like this. Inside, the n is going to be in minus r direction and M is in this direction. I am going to take the length of the box; finally, going to 0. Therefore, when I calculate divergence of M d v, which is going to be equal to integral M dot d s by divergence theorem. And since this length l goes to 0, there is no contribution from the side surfaces.

In this case, although M has component along the perpendicular surface, but length is going to be 0. And therefore, the contribution from there is 0. The only contribution I get is from inner surface, where n is going in. So, this becomes M cosine of theta with the minus sign times this area a. If a is the area of this box and which is equal to nothing but divergence of M area times t r, if you like. From slightly inside, let us say r minus epsilon two r plus epsilon; which is equal to minus M cosine of theta a; a and a cancels. And no matter how small epsilon is; I get a finite value; minus M cosine theta. And this immediately tells me divergence of M is equal to minus M cosine of theta.

And therefore, I can write cosine of theta at the surface r minus R. And therefore, I can write that divergence of H is equal to minus del dot M, which is M cos theta delta r minus R. And what does this becomes? This becomes like the surface charge on the surface at small r equal to R; positive on the top and negative at the bottom because at the top, cosine theta is going to be positive. And therefore, this becomes like positive charge on the top with the cosine theta dependence and negative charge at the bottom with the cosine theta dependence. And I am calculating H. This becomes exactly like the previous problem where we had a polarization in the z a direction, which gives me sigma cosine theta as a surface charge. And therefore, H is going to be in this direction; in the opposite to z direction. And what is this value? H is going to be; I am writing on the left; minus M by three. There is no epsilon 0. There is nothing like that in this z direction; negative z direction.

So, what we have found is H is minus M by 3. And therefore, B is going to be mu 0 H plus M; which becomes two-thirds mu 0 M. This is the answer we have obtained earlier also; by treating this as a sphere carrying surface current with sin theta dependence. So, what you see in this lecture is that in the case where there is no free current and only magnetization, I can write the equation for H as if it is an electric field being given rise to by bound magnetic charge; del dot M.

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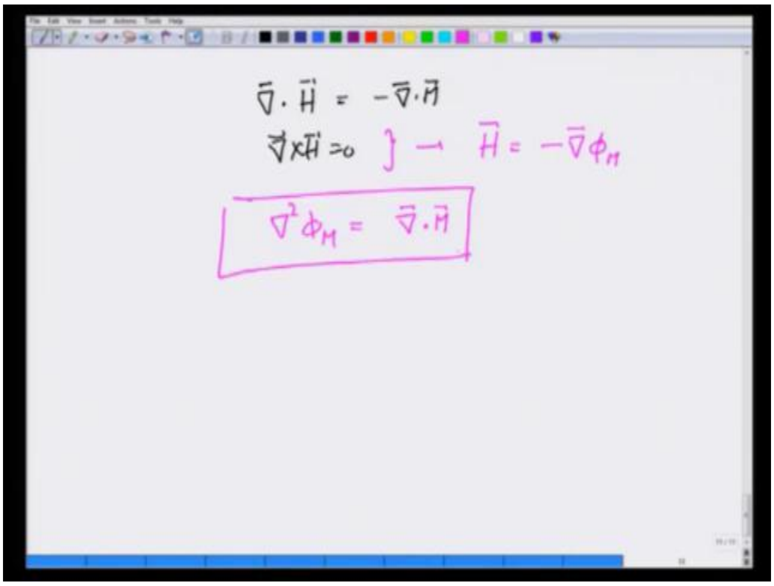

I can take the technique further and write for such cases; where del dot H is equal to minus del dot M. And del cross H is 0. If del cross H is 0, it immediately tells me that I can define a magnetic potential phi M. And therefore, write H as minus grad of phi M; which then gives me del square phi M is equal to del M, which is either Poisson's equation of phi magnetic. I can solve the boundary condition. From that I can get H. And from H, I can get B.