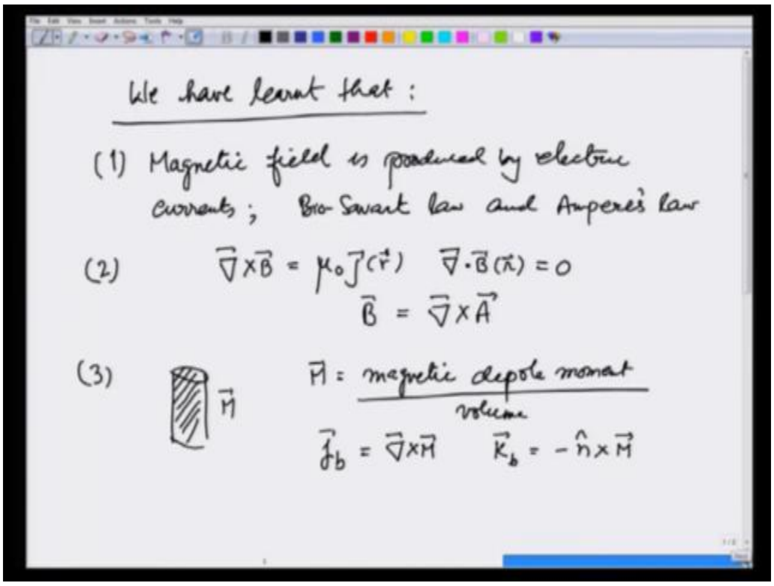
Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 44
The Auxiliary Field – H

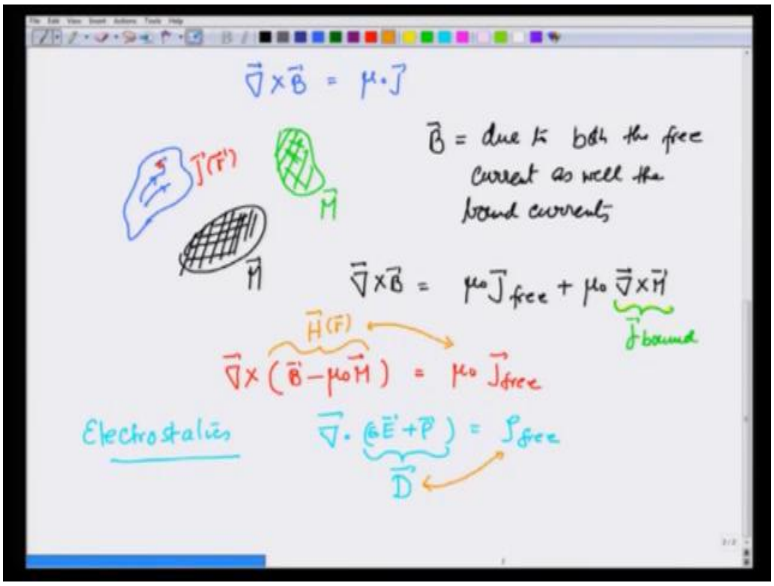
(Refer Slide Time: 00:32)



With the background given so far, we are now ready to look at how magnetic field behaves in presence of magnetic materials. Before we do that, let me first review just in a few lines or a couple of minutes what we have learnt so far. We have learnt that: one: magnetic field is produced by electric currents. The related laws that we learnt are Bio savart and Amperes law. Second, we learnt that curl of B is equal to mu zero j r in free space and divergence of B at any point is always zero. And therefore as a coronally, we have also learnt that B can be expressed as curl of a vector potential, which we kept calculating. Third we learnt, when I take a magnetic dipole or a magnetized medium with magnetization M; where M is the magnetic dipole moment per unit volume. Then, this is equivalent to a bound current j b, which is curl of M and a surface current k bound which is minus n cross M. And we have solved certain examples using these.

What we want to do now is develop a machinery to using these to deal with situations, where there are magnetic materials sitting that develop magnetic moment when a field is applied. So to do this, let us start with the equation curl of B is equal to mu naught j.

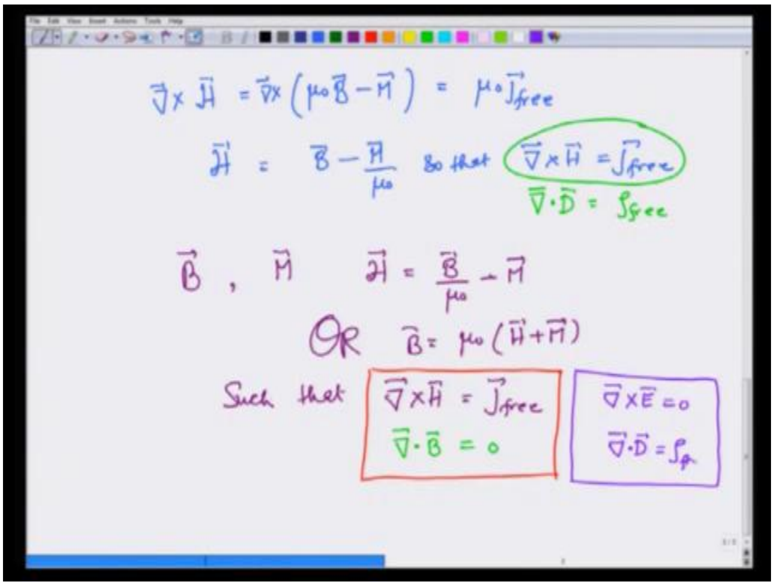
(Refer Slide Time: 02:46)



Now in presence of currents, if there is a current density j, now let me write with red; j r prime. And there are suppose these magnetic materials that get magnetized one here, one here and so on. That develop a magnetic moment M, when put in this field. What happens then? Now this B, due to both the free current as well as the bound currents. So, what I should be writing in calculating this magnetic field is curl of B is equal to mu naught j free, where free is the current which is done from outside, plus mu naught curl of M. Remember, what is curl of M? We have just discussed curl of M is j bound.

And therefore, I can write this equation as curl of B minus mu zero M is equal to mu zero j free. It is interesting that this is very similar to equation we wrote in Electrostatics in presence of polarizable medium. Recall what did we write there? We wrote curl of epsilon zero E plus P was equal to rho free. Just like we identified in Electrostatics, this vector epsilon zero E plus P. As it is placement vector we are going to identify here, this vector, here B minus mu naught M as an auxiliary vector H. Just like D was not electric field, but just an auxiliary vector; which was related to free charge. We are introducing a vector H, which is related to free current density. It is not the magnetic field. If a charge moves in this region, the force on it will be determined by B. H is an additional vector which we are introducing that is related only to free current, which we can control from outside.

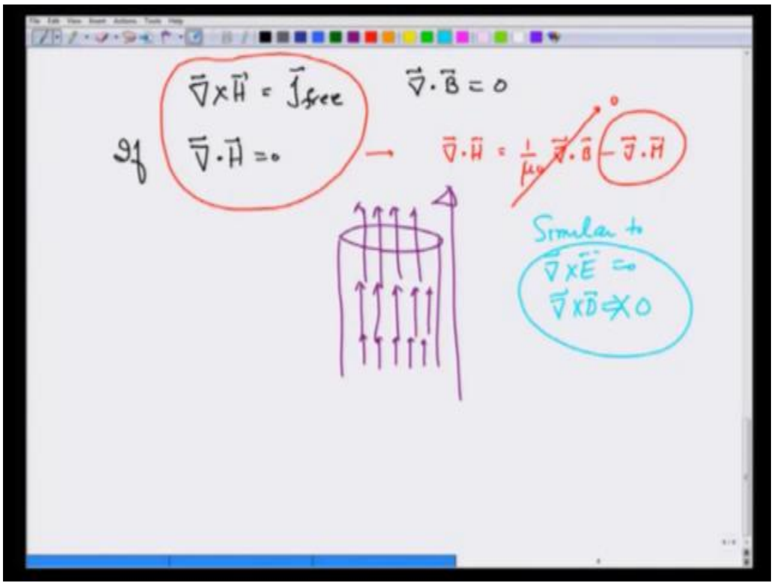
(Refer Slide Time: 05:55)



So what we have introduced is a field H, which is mu naught B minus M; whose curl is equal to mu naught j free. It will be even better if I divide by mu naught all throughout. And define H as B minus M over mu zero, so that curl of H is equal to j free. Just like, again recall that divergence of D was equal to rho free. This is similar to that. That is why I divided by H mu zero. So to again sum it up, magnetic field B and if there is a magnetization M, we have identified a vector H which is equal to B upon mu zero minus M or equivalently B equals mu zero times H plus M, such that curl of H is j free. That is the curl equation.

What is the other equation I have in the magnetic field? It is divergence of B, which is always zero. So, I have got these two equations in presence of magnetic material or in presence of magnetic moment and free currents, both. Again, these are very similar to the equations we had for electrostatic situation; where I had curl of E equals zero and divergence of D equals rho free.

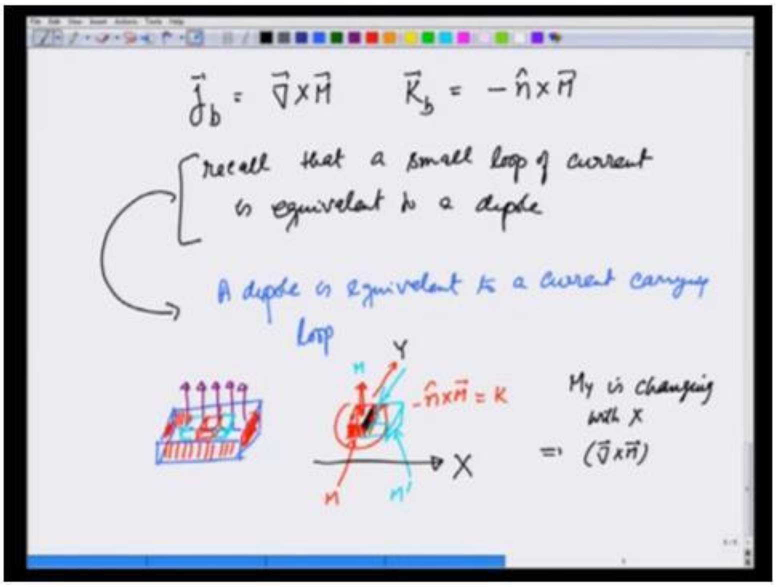
(Refer Slide Time: 08:03)



This is very similar. Starting from curl of H is equal to j free and curl of B, divergence of B is equal to zero. The question is how do we get the magnetic field? If divergence of H was zero, then it would have been very easy because these two equations together would have given me equations; which are there for B. Curl of B is equal to j and divergence of B is equal to zero. But, divergence of H need not be zero; as I will show here.

Divergence of H is equal to one over mu zero; divergence of B minus divergence of M. And this quantity need not be zero, although divergence of B is zero. Why? Suppose I have any material, take a simple material, and I have M varying. For example, it could be shown by the length of the arrow, this length, then length could be increasing. It could be even more out here. So, as I am moving in the direction this way, that component of M is increasing and divergence of M is not zero, so divergence of H need not be zero. Again, this is similar to curl of E being zero. But curl of D, remember, was not zero. And therefore, we need additional technique or some more information to solve for the magnetic field in presence of magnetic materials.

(Refer Slide Time: 10:11)



Before we end this lecture, I want to give you a feeling for how the bound currents arise out of magnetic moments or how we can interpret them. Recall that we had j bound, which is curl of magnetic moment and surface current, which is minus n cross M. Also recall that a small loop of current is equivalent to a dipole. I can turn this argument around and say that a dipole is equivalent to a current carrying loop.

Now, look at a situation. Suppose, there is a material that has some magnetic moment. let us say in the direction going up, a uniform magnetic moment, then if I look at a small portion of it, here, this small portion has a dipole moment in the direction going up and that would be equivalent to a current going counter clockwise along these surfaces; which are now going to darken. So on these surfaces, on the side surfaces there is a current going.

If this whole thing has uniform magnetic moment, then what is going to happen is all these currents, which are on the adjacent surfaces, notice if there is an adjacent block out here that I take on this, the green current is going to be going down; the red current is going to be going up. And since M is uniform, they are going to cancel. And therefore, the only places where the current is going to be left without being cancelled is going to be on the surfaces; outer surfaces of this block. And what is the amount of these currents? It is n cross M with the minus sign. So outside, I am going to have n cross M left and that is the surface current that we talk about which is k.

On the other hand, if these M at this point at the red point, let me show it here, red point. And the green point here was slightly different. Then as I move to the right, as I move to the right, the vertical component, so let us say this is X direction; the Y component of M is changing. And therefore, the currents at the interface are not going to change. If the currents do not change, that means if M varies, then the currents and these surfaces do not change and they contribute to the bulk current. Here, M y is changing with X. And therefore, this is related to curl of M. And that is what gives you a bulk current. So, this is a physical interpretation of bound bulk current and bound surface current. Similar to this similar interpretation we had for the bound charges and bound bulk charges, bound surface charges, coming out of a polarization.