Introduction to Electromagnetism
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Lecture - 43
Magnetic Materials – II: Bound Current Densities

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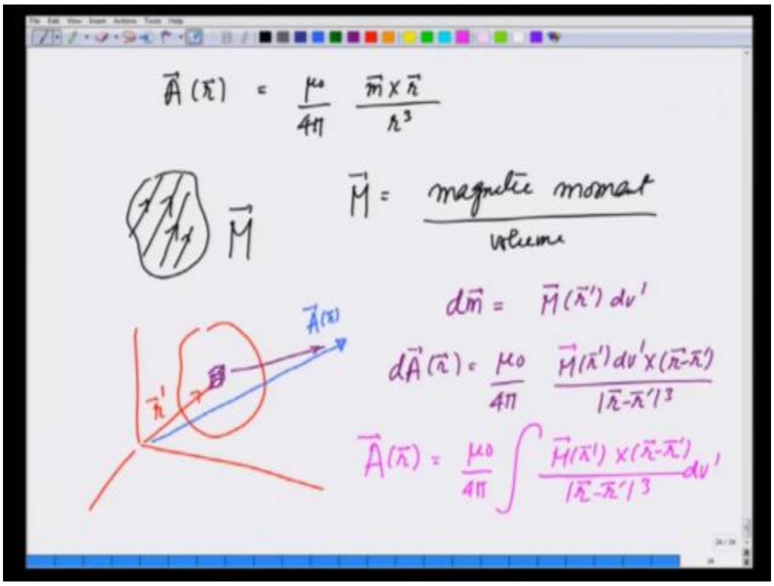

We have seen in the previous lecture that a point dipole gives me a vector potential, which is mu 0 over 4 pi m cross r over r cubed. Using this now we are going to show, if I have a material, magnetic material which has a permanent magnetization capital M, something similar to the permanent polarization in the electrostatic case. So, that M actually represents magnetic moment per unit volume.

Then this is equivalent to bound currents and surface bound currents. Again, similar to where a polarization in electrostatic case was equivalent to bound charges both the bulk bound charge and surface bound charge. Again to do this, we take help of vector potential, so when I have this system, let us make this is at r prime and suppose I am calculating vector potential at r. Then I will take a small volume here near r prime and this will have a magnetic moment d m, this is a small volume, which is equal to M at r prime d v prime.

And therefore, the vector potential d A differential vector potential at r, due to this is going to be mu 0 over 4 pi M r prime d v prime cross r minus r prime, where r minus r prime is this vector from that point to the point of interest, divided by r minus r prime cubed. And therefore, the vector potential A at r is nothing but mu 0 over 4 pi integral M r prime, this is a vector cross r minus r prime divided by r minus r prime cubed d v prime.

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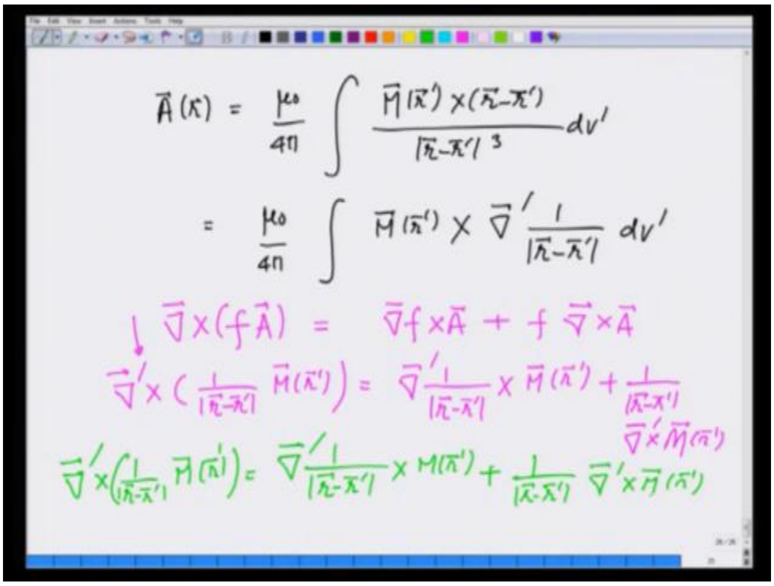

Now, we are going to do some vector manipulations here and show that this is equivalent M r prime cross gradient with respect to prime of 1 over r minus r prime d v prime. Now, I am going to use a vector identity, which is curl of f, where f is a scalar function times A is equal to gradient of f cross A plus f curl of A. Therefore, curl of 1 over r minus r prime which is like f M r prime and I should be taking curl with respect to the prime variable.

So, notice prime here should be equal to gradient of with respect to prime variable 1 over r minus r prime cross M r prime plus 1 over r minus r prime, curl of with respect to prime variable r prime. Let me write it again in different color, so that you see it clearly. Curl with respect to prime variable of 1 over r minus r prime M r prime is equal to gradient with respect to prime variable of r minus r prime cross M r prime plus 1 over r minus r prime curl of M r prime.

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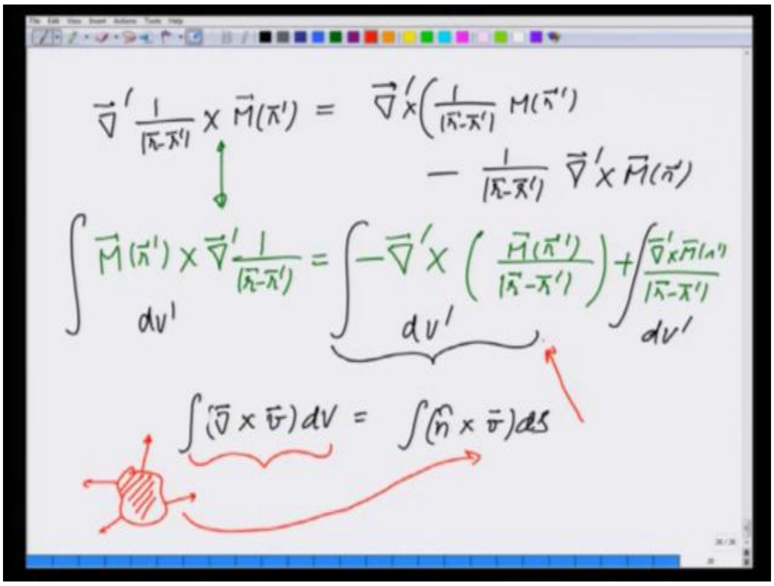

Let us invert this sense put 1 over r minus r prime with respect to prime cross M r prime is equal to curl of 1 over r minus r prime M r prime minus 1 over r minus r prime curl prime M r prime. And therefore, M r prime cross del prime of 1 over r minus r prime, I have just switched m n gradient. So, it will become a minus sign becomes minus curl of M r prime over r minus r prime plus curl of M r prime over r minus r prime.

And therefore, the integral of this over d v prime and this integral d v prime, integral d v prime, this again I am going to use an identity here, which says that integral of curl of a vector quantity over a volume is equal to n cross that vector quantity over the surface. So, what this means is that, if I have a closed surface enclosing a volume, this volume integral is equal to n cross v, where n is coming out of the surface over this surface, we can use this now here in the first integral.

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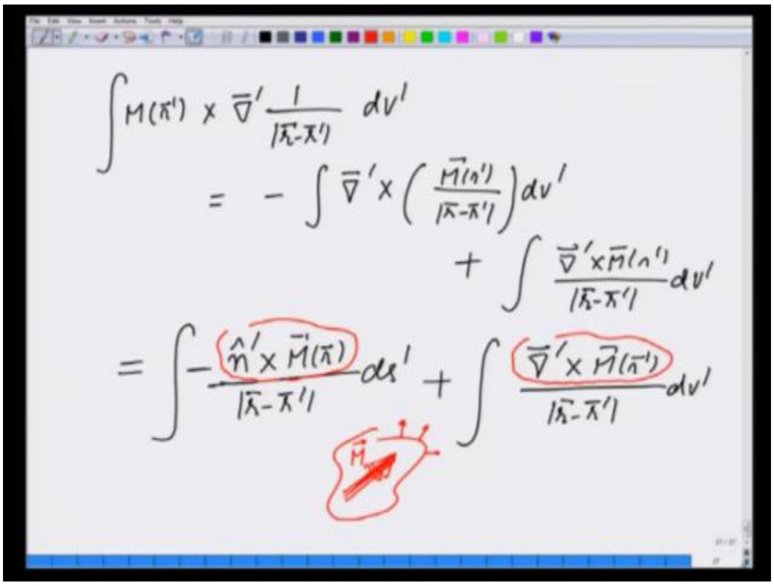

And therefore, I write M r prime cross gradient prime of 1 over r minus r prime integral d v prime is equal to which is minus gradient prime cross M r prime over r minus r prime d v prime plus integral curl of M r prime over r minus r prime d v prime. As equal to minus n prime cross M r prime over r minus r prime d s prime integral plus integral prime over M r prime over r minus r prime d v prime. So, what it means is, if I have a volume, where is there some magnetic moment M, then the surface n cross M with the minus sign gives me the surface current this term. And this is the like the bulk current, why, because I also know that A can be written A s.

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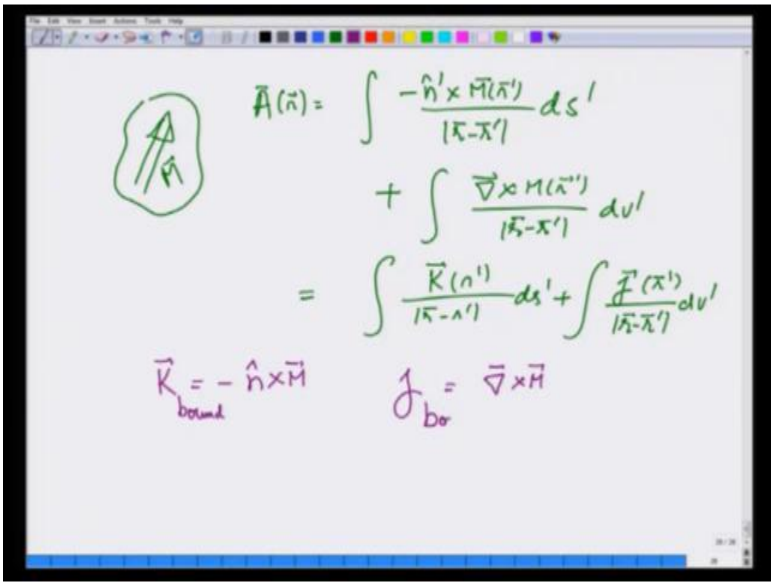

So, let us make it again here is a volume on which I have some M, then I can write A r as equal to integral minus n prime cross M r prime over r minus r prime d s prime plus integral curl of M r prime over r minus r prime d v prime. And this should be equivalent to a surface current K at r prime over r minus r prime d s prime plus a bulk current J r prime over r minus r prime d v prime.

So, we get from this magnetic moment that it is equivalent to a surface current K, which is equal to minus n cross M and a bulk current J, which is curl of M. And since, these are not free currents, we call them bound currents, just like we had bound charges earlier.

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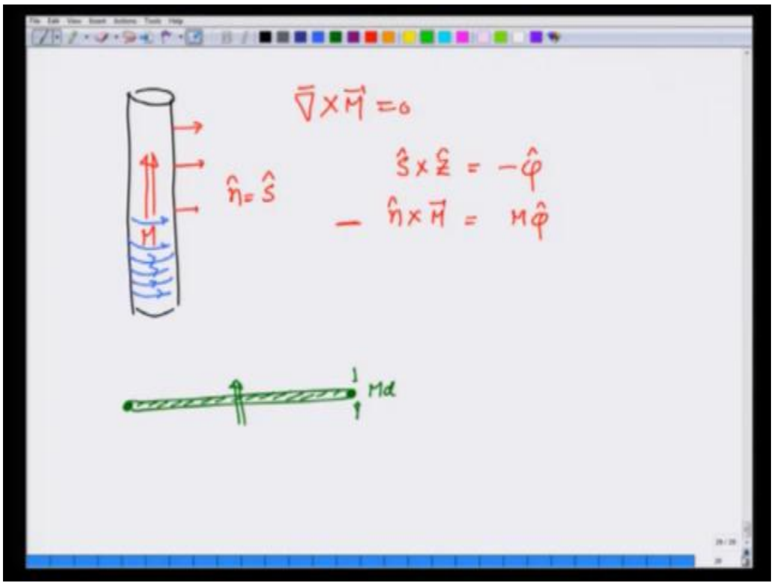

Let us take a couple of examples, if I have a long cylinder with magnetic moment in z direction, uniform magnetic moment in z direction, then curl of M is 0, n is coming out of the surface. So, n is in the same direction as S in the cylindrical coordinates and S cross z gives you minus phi, and therefore n cross M with a minus sign is nothing but M phi.

So, this long cylindrical magnet having uniform magnetization is equivalent to having surface current, which is equal to M flowing in phi direction and no bulk current. So, this becomes like a solenoid, the field will be constant inside. So, it will be like a solenoid field. On the other hand, if I have a very flat thin wafer like or thin disc like thing with M in z direction, this will be like a ring of current, where the total current will be given by M times this width d, that will be the current going through.

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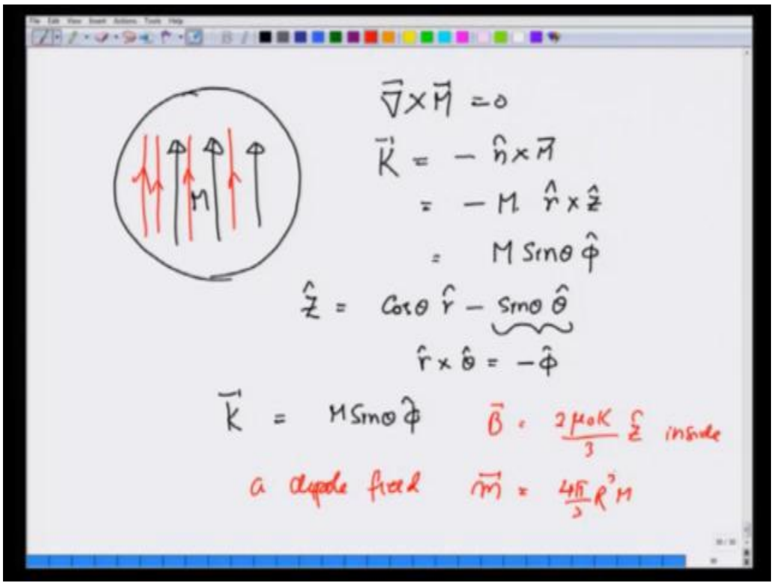

This third example, I can take is that of a sphere with constant magnetization, then again curl of M will be 0 and the surface current K will be minus n cross M, which in this case will be minus M r cross z unit vector. And you can check this is going to give me M sine theta phi; that is very easy to see, because z is nothing but cosine of theta r minus sin of theta r cross theta gives me minus phi.

And therefore, K becomes M sine theta phi, but this we have already solved, this we have solved in a previous lecture. And therefore, this is going to give me a constant magnetic field B, which is 2 mu 0 K over 3 z inside and a dipole field corresponding to m equals 4 pi by 3 r cubed M outside.