Introduction to Electromagnetism
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Lecture - 42
Magnetic Materials – 1

Having calculated vector potential and magnetic field in free space, we now want to move on to magnetic materials, and see how to calculate fields there.

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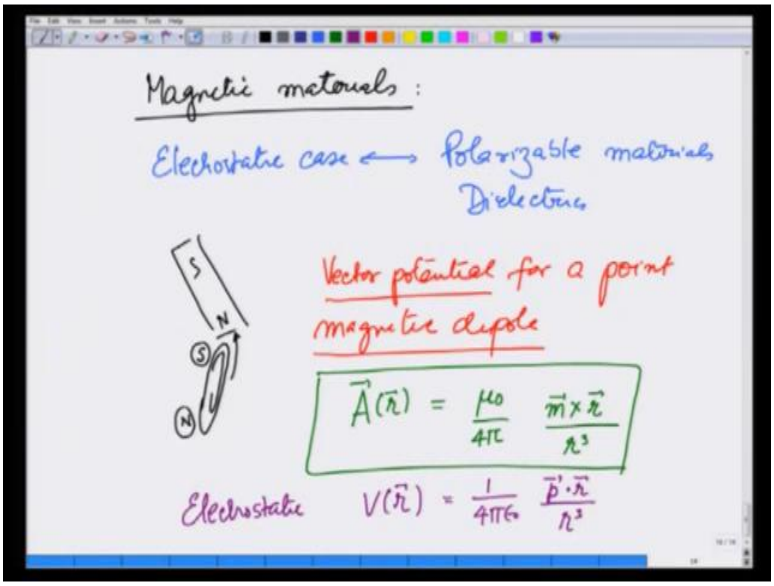

Recall that we had such a discussion for the electrostatic case, where we talked about polarizable materials and also dielectrics. Similarly, we have magnetic materials, where if they put them in the magnetic field, they get magnetized, they develop a dipole moment.

Now, the simplest example is, if I take a magnet and bring a pin close to it, it gets attracted. Why it gets attracted is, because this magnetic field develops a magnetic moment or a magnet in this pin itself and the pin gets attracted towards the original magnet. This obviously, develops in such a magnet that S and N ends are like this. So, we want to understand this or we able to calculate or relate the magnetic moment of these media or magnetic material to apply fields and so on.

So, to start with, what I will do is first to develop the subject, I will first calculate the vector potential for a point magnetic dipole. Because, finally when we want to interpret the magnetic moment, we will do it through bound currents and so on and for that, it is important that we know what the vector potential looks like for a small magnetic dipole. And my claim here is, let me give a trial here that A at point r, for a magnetic dipole point dipole sitting at the origin is equal to mu naught over 4 pi m cross r over r cubed.

This is similar to recall that for an electrostatics, the electrostatic potential at r due to a point dipole sitting at the origin was 1 over 4 pi Epsilon 0, P dot r over r cubed. So, what is happening when you go to magnetic field is this, this dot product is getting replaced by a cross product and instead of the magnetic dipole, electric dipole. Now, I have a magnetic dipole m, so let us verify this.

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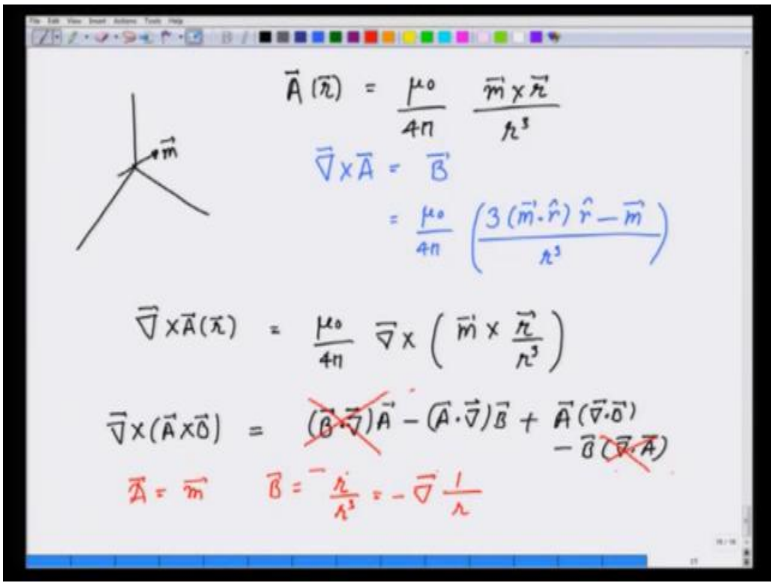

So, we are saying that, if I have a magnetic dipole m sitting at the origin, the vector potential due to this is mu 0 over 4 pi m cross r over r cubed. How do I verify this, this vector potential should be able to give me through curl the field due to a point dipole, which should be equal to mu 0 over 4 pi 3 m dot r unit vector, r unit vector minus m divided by R cubed, curl of A gives me this undone.

So, let us do that, when I take curl of A, it gives me mu 0 over 4 pi, curl of m cross product r over r cubed. Now, I am going to use this identity curl of A cross B is nothing but B dot del operator operating on A, minus A dot del operator operating on B plus A divergence of B minus B divergence of A. And take my A vector in this case to be m, which is a constant vector and B vector to be r over r cubed, which is also equal to 1 over r gradient of 1 over r.

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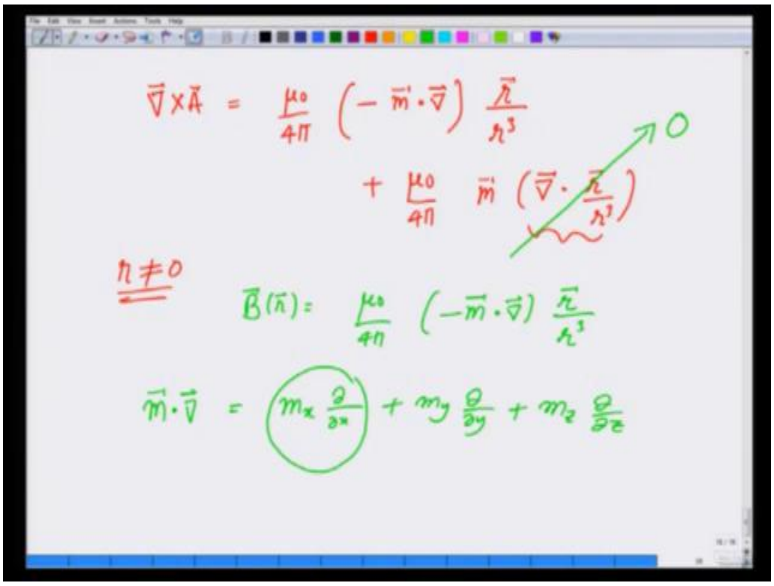

Now, since A is a constant vector, this part drops, this part drops. So, I am left with curl of A in our case to be equal to mu 0 over 4 pi. Let ((Refer Time: 05:56)) me just go back and check, minus m dot del operating on B which is r over r cubed plus mu 0 over 4 pi m divergence of r over r cubed. Now, for r not equal to 0; that means, away from the origin or away from the magnet divergence of r over r cubed is 0. So, this fellow goes out.

So, I am left with B r is equal to mu naught over 4 pi minus m dot del operating on r over r cubed, where m dot del operator is nothing but m x partial x plus m y partial y plus m z partial z. Let us calculate this, let us calculate one of the terms and that will give us an idea what the other terms will be like.

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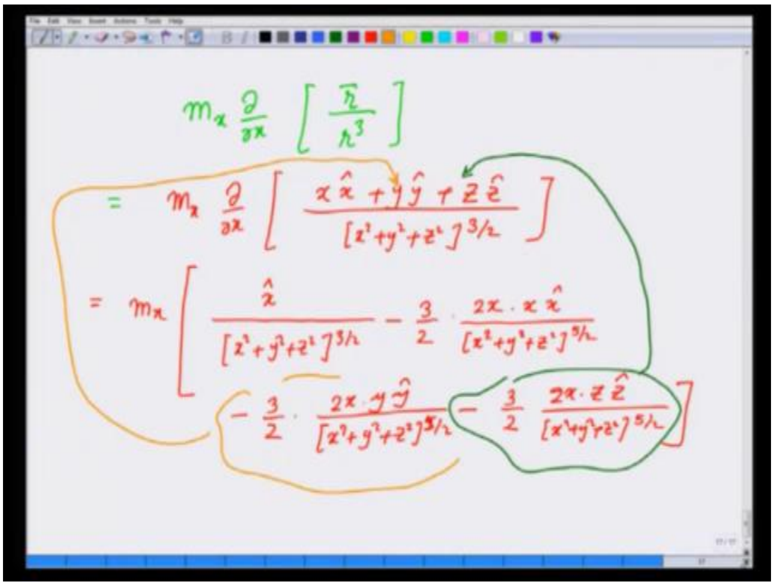

So, let us calculate m x partial x acting on r over r cubed, which is equal to m x partial x acting on x square plus y square plus z square raise to 3 by 2 x in unit x direction plus y in y direction plus z in z direction. This will give me m x, the first term gives me x unit vector divided by x square plus y square plus z square raise to 3 by 2 minus 3 by 2, 2 x times x in the x direction divided by x square plus y square plus z square raise to 5 by 2 minus 3 by 2, 2 x y in the y direction divided x square plus y square plus z square raise to 3 by 2 minus 3 by 2 x square plus y square plus z square raise to 5 by 2. This is 5 by 2, 2 x times z in the z direction.

Let me make a correspondence this term comes from the differentiation of z over x square plus y square plus z square term. The second term, this term here comes from the differentiation of this y term and a first two terms come from the differentiation of the first.

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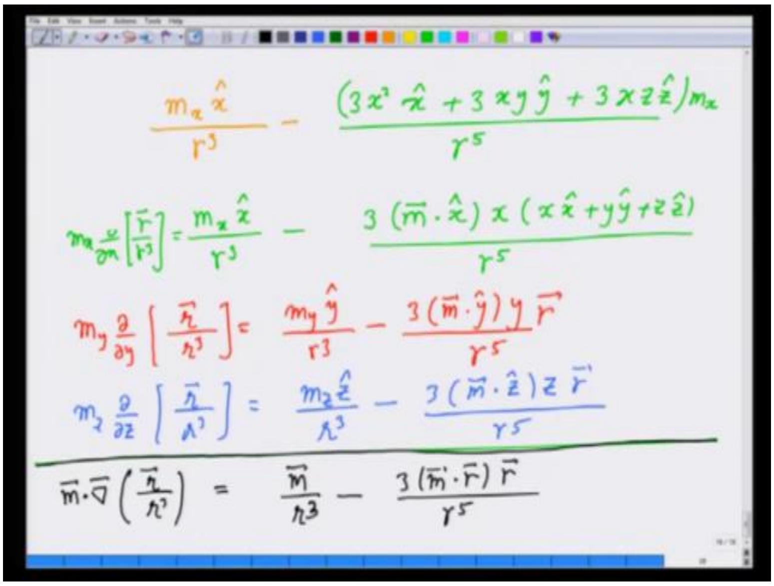

So, what are we left with if I just differentiate with respect to x, I am getting this to be equal to m x, x unit vector over r cubed minus let me go back and cancel certain terms this 2 cancels everywhere, this 2 cancels everywhere, this 2 cancels everywhere. I am getting 3 x square x unit vector plus 3 x y, y unit vector plus 3 x z, z unit vector over r raise to 5 times this whole thing is m x.

So, I can write this as m x, x unit vector divided by r cubed minus 3 m dot x unit vector that gives me m x, I can take x out, then I am left with x x plus y y plus z z vector over r raise to 5. Similarly, if I take m y partial y of r over r cubed, I am going to get m y y over r cubed minus 3 m dot y y. This x x plus y y plus z z, I am going to write as r vector divided by r raise to 5 and at the end, when I take m z d by d z of r unit vector over r cubed I am going to get m z z over r cubed minus 3 m dot z z r over r raise to 5.

Let me write the first term, what that is, the first term is nothing but m x d by d x of r over r cubed is equal to this. If I add all this, what do I get, I get m dot del acting on r over r cubed. As precisely what the left hand side is, is equal to m x x plus m y y plus m z z gives me m vector again over r cubed minus m dot x unit vector x y unit vector y z unit vector z gives me 3 m dot r vector times r is outside divided by raise to 5.

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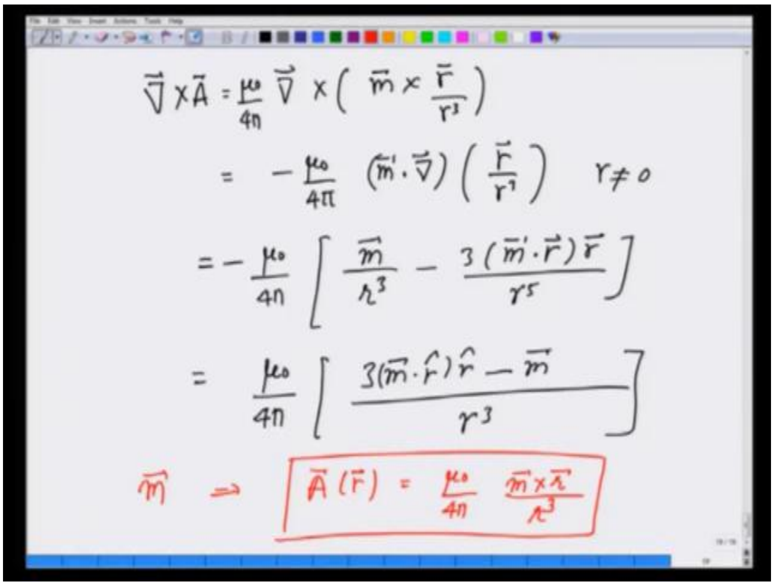

So, if I put it all together, we had started with del cross A, which is del cross m cross r over r cubed and we have gotten it to be minus, there is a mu naught over 4 pi, mu naught over 4 pi m dot del acting on r over R cubed r not equal to 0. And this is minus mu naught over 4 pi, inside I get m over r cubed minus 3 m dot r vector, r vector divide by r raise to 5, which I can then write as mu naught over 4 pi 3 m dot unit vector r, unit vector r again. So, r square is taken out.

So, I am left with r cubed in the denominator minus m over r cubed, which is a same expression as B for a point dipole sitting at the centre. So, what we have shown is, if there is a point dipole m sitting at the origin, this gives me a vector field r, A r which is mu naught over 4 pi m cross r over r cubed. And this is as important relationship as we had for electrostatic case, where we had the electrostatic potential of a electric dipole sitting at this origin was equal to P dot r over r cubed.

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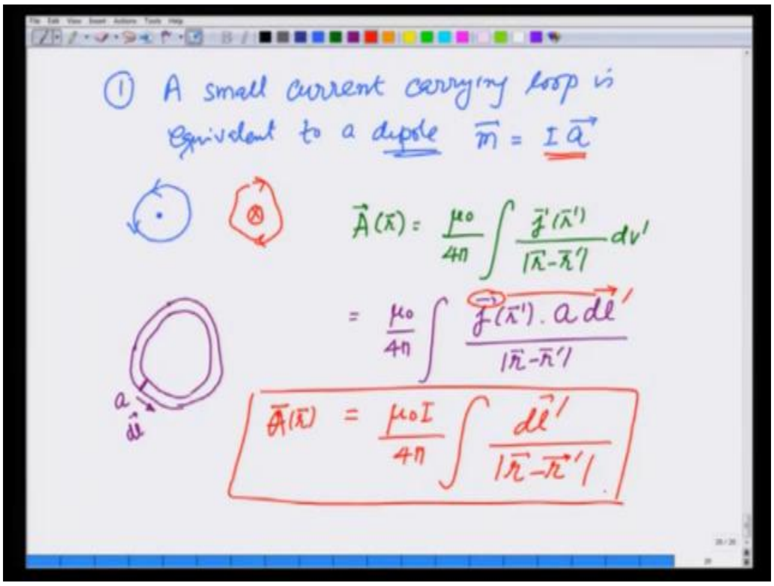

Now, using this we are first going to show that number 1, a small current carrying loop, I will specify what small is, is equivalent to a magnetic dipole m is equal to the current times. The area with the vector A of the loop, where A is again given by the current direction, if the current direction is counter clockwise, A would be coming out. If the current action is clockwise the area would be going in.

Again, we will calculate the vector potential due to this and show that vector potential goes to as if it is coming out of a magnetic dipole like this. So, for A current distribution A r is given as mu naught over 4 pi integration of j r prime over r minus r prime d v prime. Let us take a current carrying wire, let me make it little thick and let this cross section at end point A and this is the current direction is d l.

So, now, earlier you remember we came from I to j, now I am going to go back to I from this. So, I will write this as mu naught over 4 pi j vector r prime d v prime, I am going to write as that area A, which I have shown here cross sectional area of the wire times d l divided by r minus r prime. Now, I am going to take this vector sign from here, I will put it over d l and j a, I know because A is the cross sectional area perpendicular to the current carrying direction is I. So, I can write this as mu 0 I over 4 pi integration d l should put a prime here, d l prime vector over r minus r prime, this is my A r due to A wire that is carrying current I.

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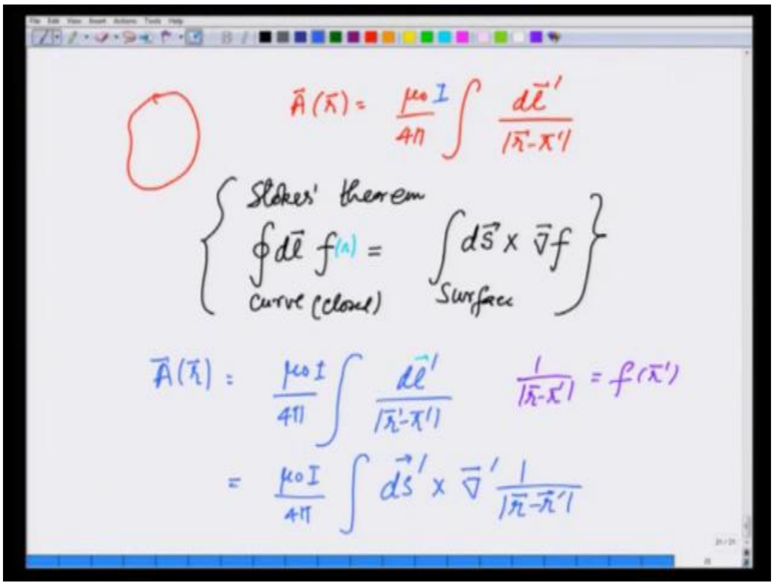

So, a current carrying wire A at r is given as mu 0 over 4 pi integration d l prime over r minus r prime. Now, using stokes theorem, one can actually show that d l times a function f over a close circuit is equal to d s cross gradient of that field f, integral over the surface. So, over a closed curve, if I take d l f, it is equal to d s cross gradient of f. So, what do I have now, I apply this to, this I will give as an assignment problem for you to solve, but not to submit.

I have A r is equal to mu 0 over 4 pi, integral d l prime over r minus r prime, I will take 1 over r minus r prime as my f of r prime, where the f is a function here, this is a vector. So, this is going to be equal to mu 0, there is an I also out here, which I have missed I, I over 4 pi integral of d s prime vector cross gradient with respect to prime variable 1 over r minus r prime, which is nothing but.

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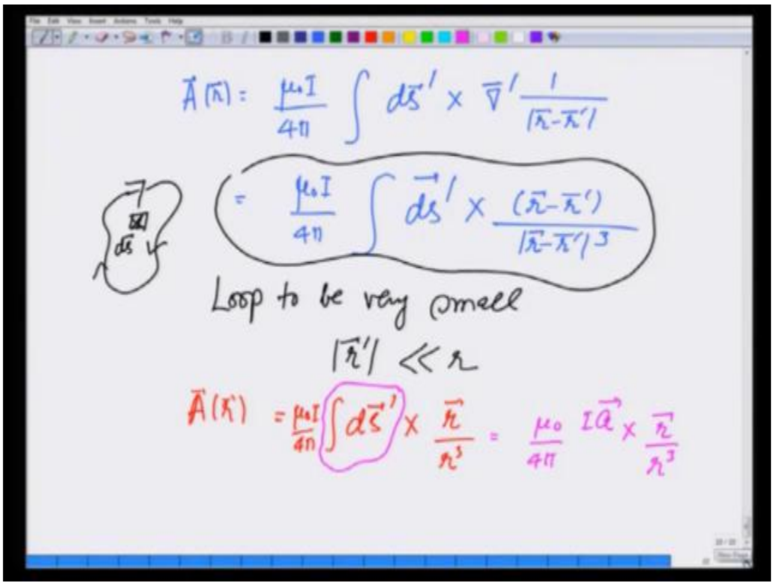

Let me write it again A r is equal to mu 0, I over 4 pi integral d s prime cross gradient with respect to primed variable over r minus r prime, which is nothing but mu 0 I over 4 pi integral d s prime vector cross r minus r prime over r minus r prime cubed. That is the gradient with respect to prime. Let me remind you again, I am taking this current loop and if the current is say going clockwise, then d s everywhere I am taking as going in. So, it is given by my right hand convention.

Now, let us take this loop to be very, very small and what do you mean by small; that means, magnitude of r prime is always much, much, much less than r. Then, as the first approximation, I can write this whole thing by ignoring r prime as A r is equal to integral mu 0 I over 4 pi d s vector prime cross r over r cubed, where d s prime now is our primed variable, and therefore does not depend on r gives me the total area of the loop. So, this becomes mu 0 I, area of the loop over 4 pi here cross r over r cubed.

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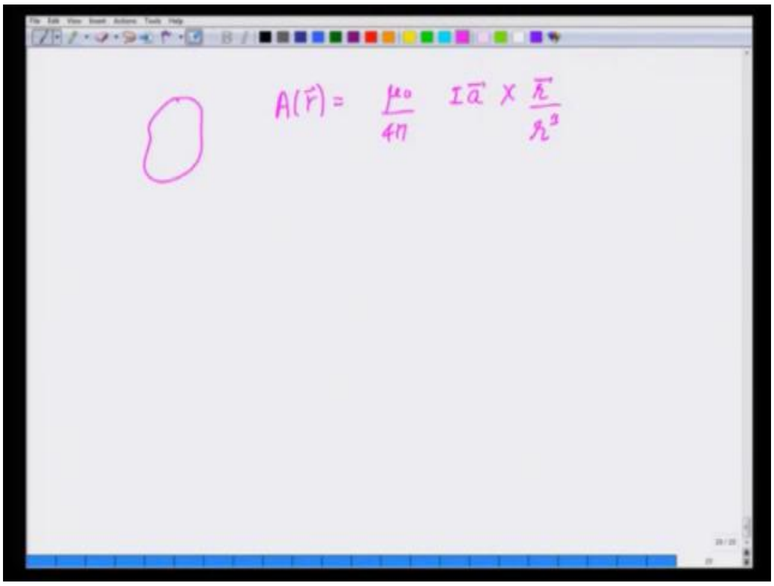

So, what I have shown is that, if I have using all these vector identities, if I have the current carrying loop, which is small, then A r is given as mu 0 over 4 pi, I a cross r over r cubed, where r is the distance from the loop.

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