Introduction to Electromagnetism
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Lecture - 41
Vector potential from Current Densities – 11

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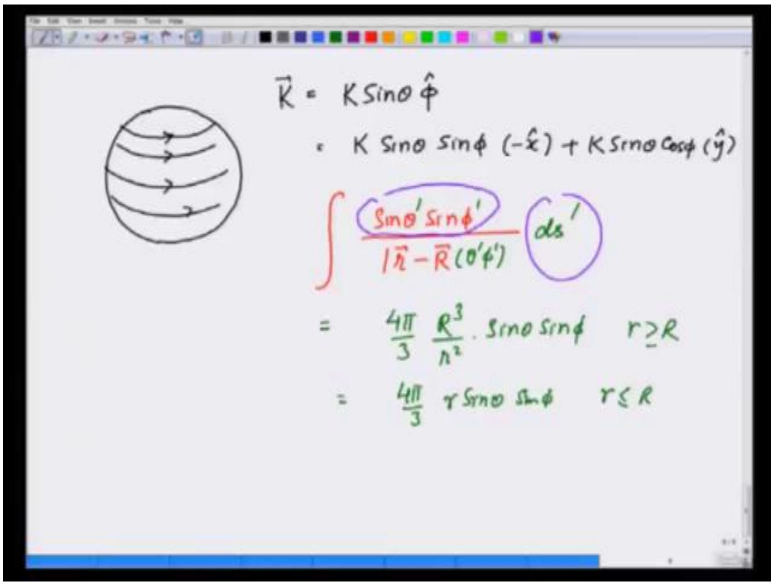

In the previous lecture, we calculated the vector potential of a current surface current carrying spherical shell, where the current density K is given as K sin theta phi unit vector, which I can write as K sin theta, sin phi in the minus x direction plus K sin theta cosine phi in the y direction. And we came across an integral like integral sin theta sin phi over r minus R.

We again write R as a function theta prime and phi prime, it should also be prime here d s prime. And I claim that, this is equal to 4 pi by 3 R cubed over r square sin theta sin phi for r greater than R and is equal to 4 pi by 3 r sin theta, sin phi for r less than equal to R and likewise for cosine term. I want to show, how this integral can be evaluated using the knowledge of electrostatics and potential there.

For this, we look for a charge distribution or surface charge distributions, since I am integrating over a surface. I look for a surface charge distribution that has this dependence over theta and phi and that is very easy to find.

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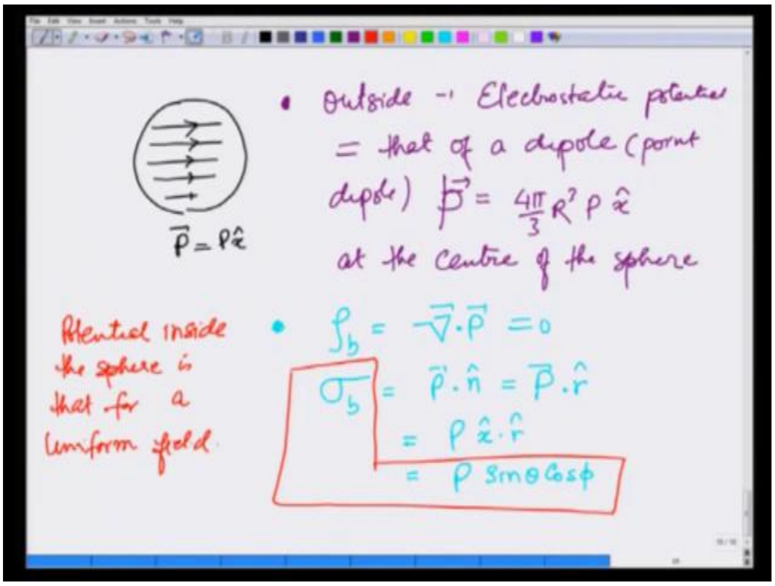

Let us take a sphere with a constant polarization P in the x direction, I can take it in the y direction, I will get some other dependence, so I will just start with the x direction dependence. Then, I know that outside the electrostatic potential is equal to that of a dipole, a point dipole of magnitude P is equal to or the point that P vector equal to 4 pi by 3 R cubed P in the x direction at the center of this sphere. So, that is facts number 1.

Fact number 2 is that row bound, which is given as minus divergence of P is 0. However, there is sigma bound which is going to be P dot n, which in this case is P dot r unit vector. So, I take P, which is P x dot r, which gives me P x dot r is nothing but sin theta, cosine phi. So, the potential outside in this case is due to the sigma bound, which has dependence, sin theta cosine phi and potential inside is that corresponding to a uniform field. So, let us write that also, potential inside the shell or inside the sphere is that for a uniform field.

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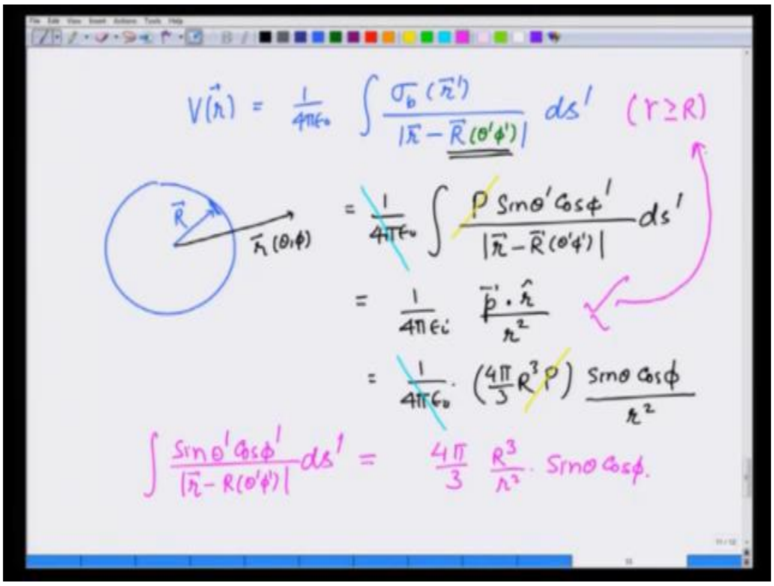

Therefore, I can write outside, I have V of r, which is 1 over 4 pi Epsilon 0, integration sigma bound at r prime is not happened to be a surface, divided by r vectors, it should be r vector minus R vector which is on the surface, which is a function of theta prime and phi prime d s prime. Let me remind you d s prime is an integration over this surface by a vector R here and R is a vector, where I am calculating. So, R somewhere r which is depends on theta and phi, R unit vector or capital R is theta prime and phi prime.

But, this is equal to than 1 over 4 pi Epsilon 0 integration P sin theta prime cosine of phi prime divided by modulus r minus R theta prime pi prime d s prime; this is the potential. But. I also know this equal to 1 over 4 pi Epsilon 0. P dot r unit vector over r square and P is in the x direction. So, this become 1 over 4 pi Epsilon 0, 4 pi by 3 R cubed capital P times sin theta cosine phi divided by r square, thus the potential.

Let us now sin theta terms, will see P cancel from both sides, 1 over 4 pi Epsilon 0 cancels from both sides. And what I am left with that then is, integral sin of theta prime cosine of phi prime over r minus R, which is the function of theta prime phi prime, d s prime is equal to 4 pi by 3 R cubed over r square sin theta, cosine phi. What about inside, so this is strictly valid for d s point out this is r for greater than equal to R, because we have taken this expression potential, which is valid for this.

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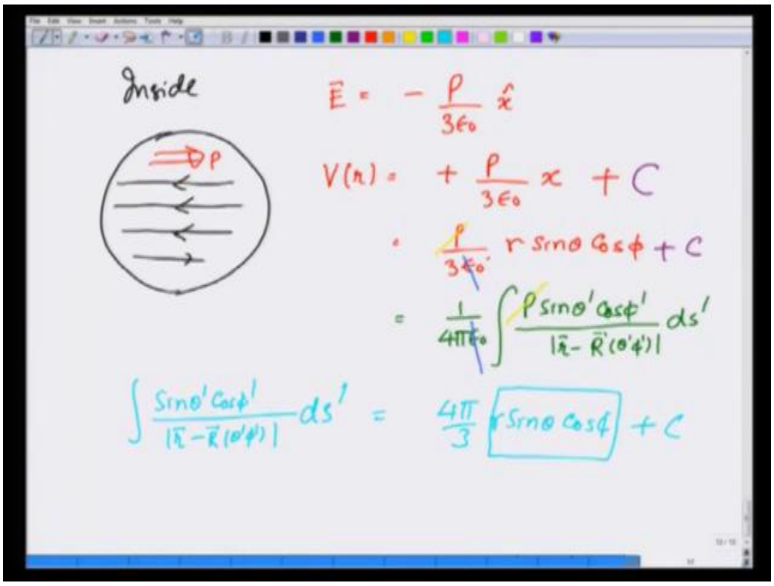

So, now, we go for inside, inside, I have uniform field, inside there is a uniform field. If the polarization P is in this direction and what is the value of this field, the value this fields E is nothing but minus P over 3 Epsilon 0 in the negative x direction. And therefore, the potential V r is nothing but plus P over 3 Epsilon 0 x, take this derivative respective x.

For that means, if I take it is gradient, it is give me a electric field going in the negative direction, which I can write as P over 3 Epsilon 0 r sin theta cosine of phi plus there could be a in conscious constant. But, the dependence or sin theta cosine phi which really matters for calculating the field is like this. And this should also be again equal to 1 over 4 pi Epsilon 0 integration sigma bound, which is p sin theta prime cosine theta of phi prime over r minus capital R, which is function of theta prime phi prime d s prime.

Again, let us cancel terms, this P cancel from both sides, this Epsilon 0 cancels from both sides. And what left with this then integration sin theta prime cosine phi prime over R minus r theta prime phi prime d s prime is equal to 4 pi by 3 r sin theta cosine phi plus C, where C will determined by what is a discontinuity at the surface. And if you want to make the V continues, then C will be determined by that. But, the dependence on theta and phi come to this r C sin theta cosine prime.

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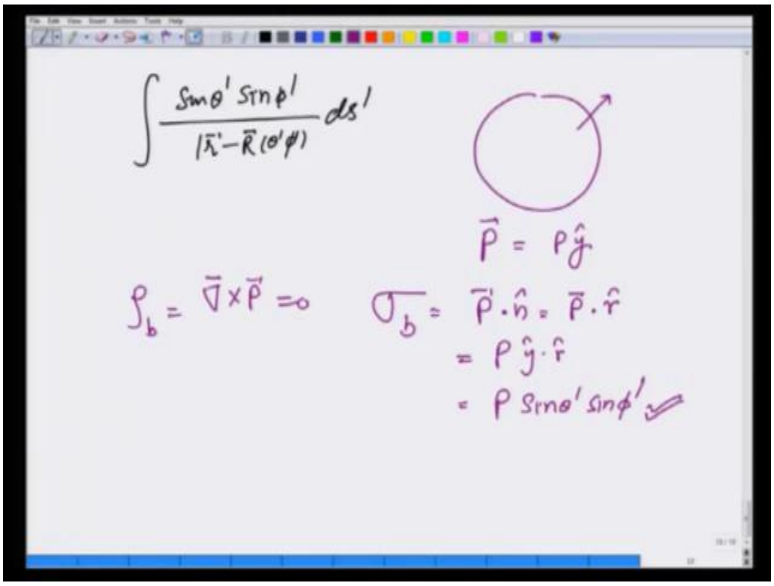

What about the other integrals? Now, if I want to evaluate the integral sin theta prime sin phi prime over r minus R, which is a function of theta prime and phi prime d s prime. What would I do, I would take this is sphere and have a polarization P, which is in the direction y. So, that again curl of P is 0, so row bound is 0, but sigma bound is going to be p dot n, coming out of the surface, which is P dot r which is P y dot r in this become P sin theta prime sin phi.

If I calculate the integrals corresponding to this and equate them to P dot r over r square, I will get the integral answers for integrals for this. So, I can always find these situations, where an integral corresponds to a particular expression for potential in the electro static case. And therefore, I can directly write, what my answer should be, more of this integral I will give you in your assignment.