Introduction to Electromagnetism
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Lecture - 38
Calculation of Vector Potential for a given Magnetic field

We have been talking about Vector Potential and how do we calculate it using the Magnetic Field.

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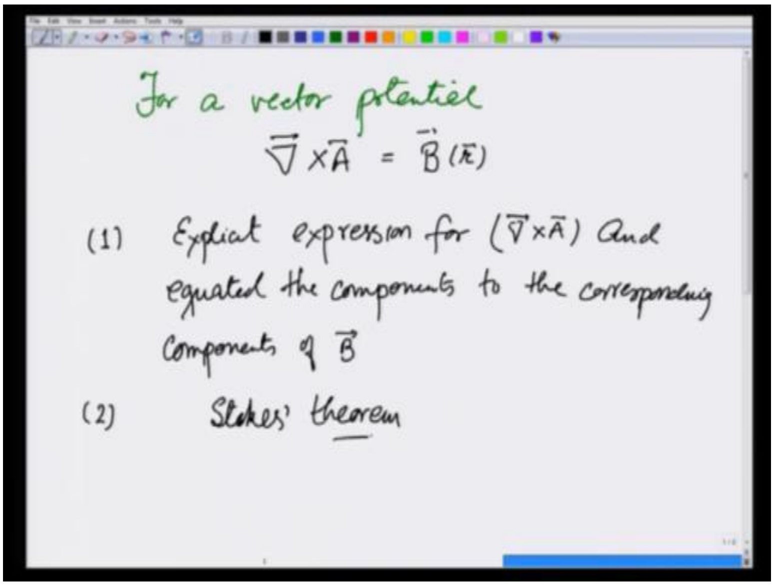

What we have done so far is that for a vector potential, we know that curl of A is equal to B at that point. And therefore, what we have done so far is calculated the explicit expression for curl of A and equated the components to the corresponding components of B. Of course, you may be wondering if we know the magnetic field B, why are we calculating A then. Because calculating a potential is supposed to make calculation of the field easier as we saw in the case of electric field.

The idea right now is to get a feel for the vector feel, vector potential for a given magnetic field. Later, we will see how we calculate a vector potential directly from a given current distribution. But, for the time being so far I will call this now method one that we have used there is another way of calculating the vector potential. So, we will be discussing that today making use of Stokes theorem.

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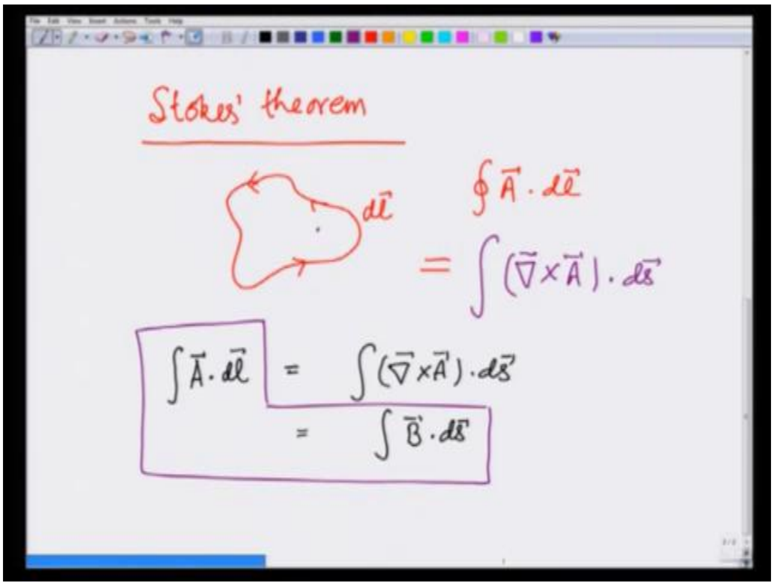

Recall that Stokes theorem relates the line integral of a vector around a closed path. So, this line integral is written as, so let us say this is A dot d l it relates it to the surface integral of the curl of this vector curl of A dot d s, where we take the direction of the surface element according to the right hand convention that is if my fingers go around d l my thumb gives the direction of d s. Let us now apply this to the magnetic field and in that case what we will see is, that A dot d l is equal to curl of A dot d s, but curl of A is nothing but the magnetic field and therefore, this s let me enclose this here.

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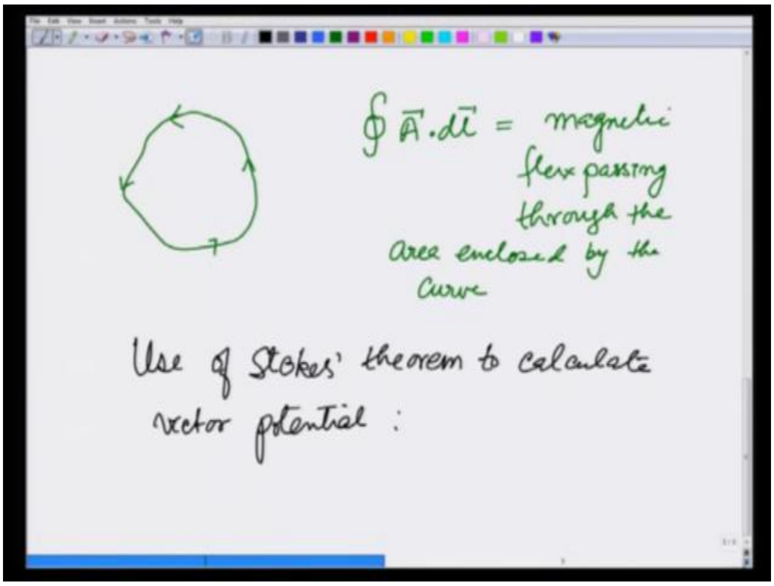

Therefore, what we learnt from this is that if I take around a close path, the line integral or vector potential A it is equal to the magnetic flux passing through the area enclosed by the curve and this we can make use of in calculating the vector potential particularly in those situations, where the there is some sort of a symmetry. We will of course be making use of Stokes theorem later in calculating other quantities and here we want to focus strictly on calculating. So, use the vector potential of Stokes theorem to calculate vector potential.

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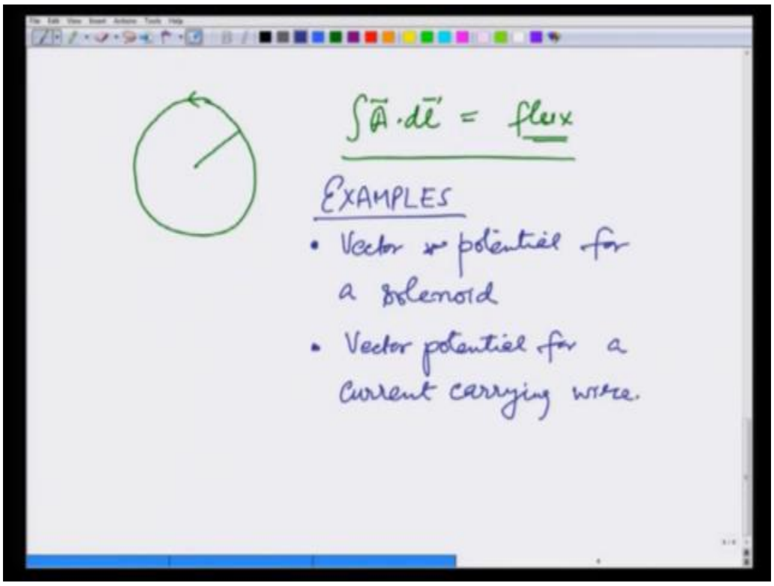

So, what we will be doing in this is given a situation take a path depending on if it depends only on the cylindrical coordinate s I will take a circular path or depending on the situation some other path calculate A dot d l and equate this to the flux passing through that the area covered by that path and that will give us the vector potential. I will be solving two examples with this one will be better vector, these are the examples two examples one will be vector potential for a solenoid carrying certain current and two vector potential for a current carrying wire.

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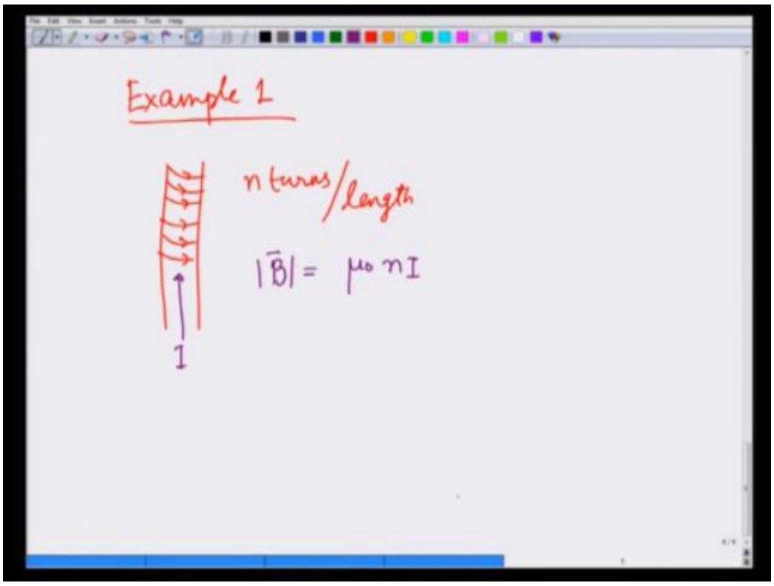

Example 1 I will be taking a solenoid which has n turns per unit length and is carrying current I. So, that the magnetic field for this B it is magnitude is given as mu 0 n I, so to calculate the vector potential for this solenoid.

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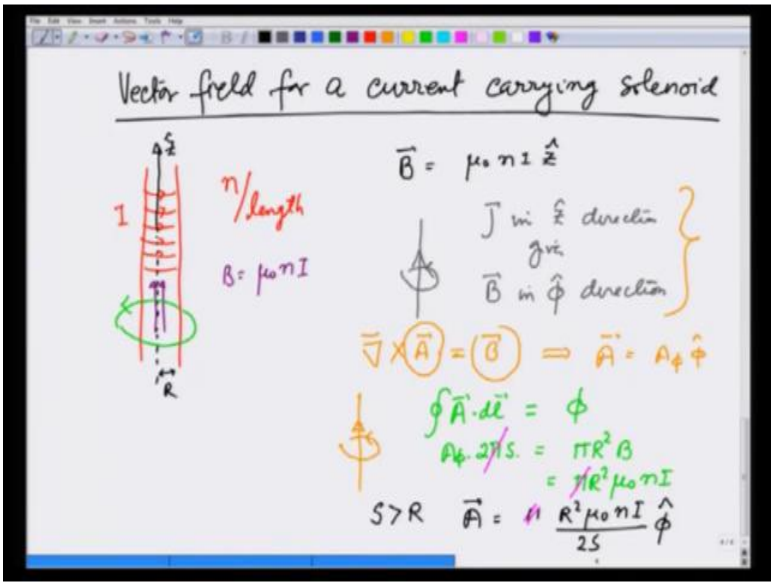

Let us now write these in terms of vectors, so I will take the axis of the current to be the z direction and therefore, this becomes solenoid of radius R phi directions. So, I have B is equal to mu naught n I in the z direction. Now, by analogy, the analogy that I was drawing if I have a current going in the z direction I have b which goes around in phi direction. So, j in z direction gives B magnet, not current magnetic field in phi direction I earlier said j in z direction gives current in phi directions j in z direction gives B in phi direction by analogy.

If I look at this equation del cross A equals B now j gets replaced by B and B gets replaced by A and B is in z direction again. So, I can anticipate by again same form of the equation that A will also be in the phi direction. So, what I anticipate now this implies by analogy with this that A would actually be some A phi in the phi direction. So, let me now choose a loop around the solenoid and calculate A dot d l on this loop and they should be equal to the flux passing through this loop.

A dot d l is going to be nothing but A phi times 2 phi s, where 2 pi s is the length since A is in phi direction I get only A phi, this is going to be equal to pi R square B which is equal to pi R square mu naught n I and therefore, for let me write this result in black now S greater than R the radius of the solenoid A vector is pi actually I can cancel this pi right here. So, I get R square mu naught n I over 2 s in phi direction that is a vector potential outside the solenoid.

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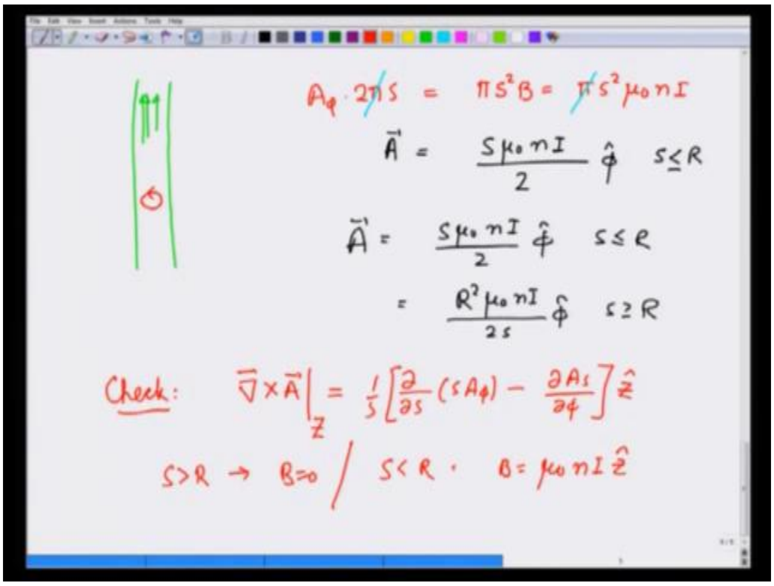

How about inside, let me make this again this is B for inside I will take a loop like this going in pi direction, again A will be in phi direction. Because, this is like a distributed source of current giving rise to B, here I have distributed B giving rise to A. So, I have A phi times 2 pi s and now the flux is going to be pi s square times B which is equal to pi square mu naught n I and from this again I am going to cancel pi and I get A vector is equal to s mu naught n I divided by 2 in the phi direction for s less than R you can put less than or equal to.

So, I have got my final answer that A is equal to s mu naught n I by 2 phi for s less than equal to R and is equal to R square mu naught n I over 2 s phi for s greater than or equal to R of course, at s equals R they are both equal. Let us check the answer it should give me correct A correct magnetic field B. So, curl of A since A is in only phi direction and I also know that the final answer is in the z direction. I can write only the z component of this.

The z component of this is given by partial s s A phi minus partial A s over partial phi 1 over s outside in the z direction. And you can see already that for s greater than R this gives B equal to 0 and for s less than it gives B equals mu naught n I z, you can take the other components and cylindrical coordinates and check for yourself that they are indeed 0.

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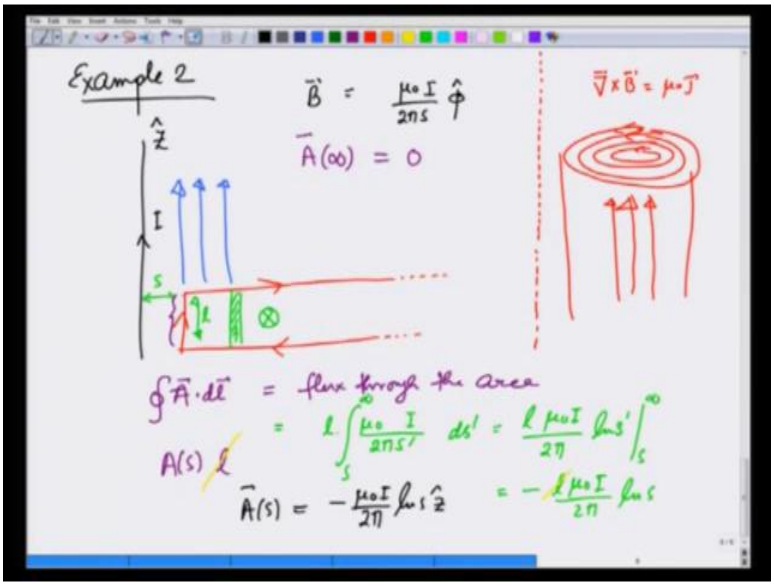

Let me take another example that of a current carrying wire, a current carrying wire long wire that carries a current I. In this case, we have already calculated B, if I take this direction to B the z direction in which the current is going B is nothing but mu naught over 2 pi s I in the phi direction. Again by analogy with the current producing a magnetic field, let me draw it on the right side of your screen.

When I take del cross B equals mu naught j and suppose I had a current going over a disc, what kind of feel do I expect, if it is all over in a cylinder the feel I expect in this case is nothing but B feel going up, we just now solved the solenoid case and that is precisely what this is. Similarly, in this case I should now anticipate that my A would be going up like this.

And therefore, I choose a path which is going like this and goes all the way to infinity and closes there. Of course, then I am going to assume that A at infinity is 0, if I choose at this path and now apply stokes theorem it gives me A dot d l over this path is equal to flux through the area, let me take the height of this to be l this is starting from s. So, that flux and since the right hand fingers are going clockwise, the area element is also going in.

So, B dot d s is positive and therefore, flux is positive this comes out to be l mu naught I over 2 pi s prime, then I am going to have this area element which is going to be l d s prime and this integrates from s to infinity which comes out to be l I have already taken care of. So, I will remove this l I will remove this also, this comes out to be l mu naught I over 2 pi log of s prime s and infinity discarding the infinite value this comes out to be minus l mu naught I over 2 pi log of s.

How about the left hand side? The left hand side the only contribution for a since it is in z direction comes from this particular vertical path. And therefore, this is going to be A at s times l and you can clearly see then by cancelling terms, if I cancel this l that final answer A s comes out to be the vector minus mu naught I over 2 pi log of s in the z direction, the answer we are obtained earlier.

So, I have done two examples of using the curl equation for B along with the stokes theorem to calculate B. This method is just to get a feel for what A looks like, it is not useful to calculate B, because right now we are calculating A from B. So, I already know the answer for magnetic field, what would be useful if I could calculate A directly in terms of the current and then A would be useful in calculating B.

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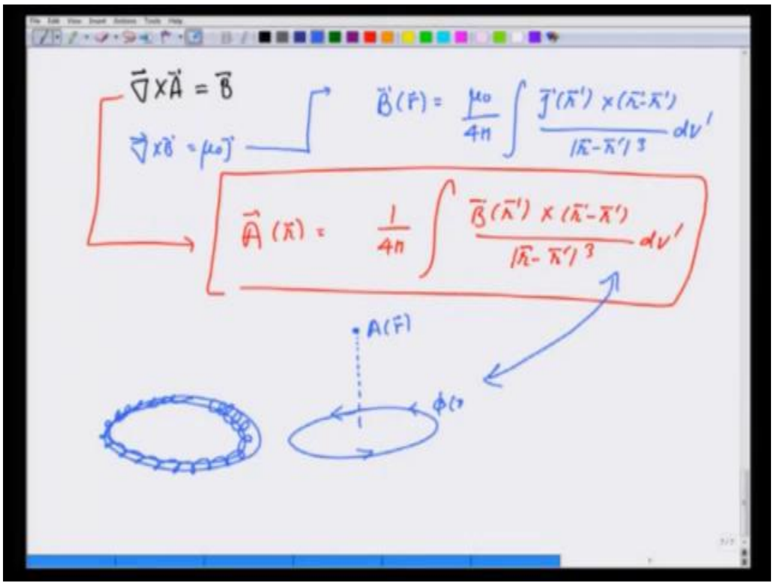

However, before I end this lecture I just want to push that analogy that I have been drawing between curl of A equals B and curl of B mu naught j a bit further and see that this equation actually in free space gives me the answer for B as mu naught over 4 pi integral j r prime cross r minus r prime over r minus r prime cubed d v prime, where d v prime is the integral over the current. Can I in a similar manner therefore, write A at r is equal to 1 over 4 pi integral B at r prime cross r minus r prime over r minus r prime cube d V prime.

Again emphasizing since the equations are the same only the symbols have changed the answers should be the same. For example, we have used this equation already by using the flux of B to calculate A, our case of this would be if I have a torrid which is like a solenoid went onto itself. So, it has these wires going around that carry current, if the solenoid is very thin suppose this is this solenoid or the torrid is very thin, it becomes like a flux carrying ring can I then calculate A on the axis just like I calculated.

Magnetic field for a current carrying wire can I then calculate A at this point by treating the flux phi going through this ring as a ring of flux and then applying this formula. This I will give as an exercise later and we will again see in coming lectures that this kind of similarity of equations these two, same answers or same expressions for the answers and that becomes a very useful way of visualizing different fields.