# Introduction to Electromagnetism
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## Lecture - 37
## Vector Potential for Magnetic Fields

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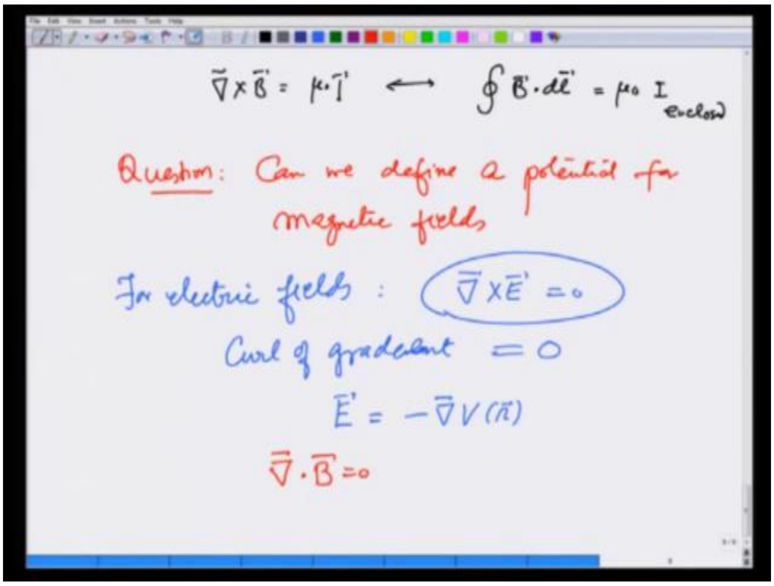


In the previous lecture, we talked about integral form of Ampere's law and used del cross B equals mu naught j or it is integral form which was B dot d l is equal to mu naught I enclosed by that loop to calculate magnetic field in certain symmetry situations. We also said there, the question we are traced was can we define a potential for magnetic fields. In this lecture we address this question, recall that for electric fields we had curl of E equal to 0 and we use this to define a potential.

Because curl of gradient is always equal 0 one could write, because of this condition at E is equal to minus gradient of V R, because this curl will always give you 0. Similarly, now we use in the context of magnetic field, this equation that the divergence of magnetic field is always 0.

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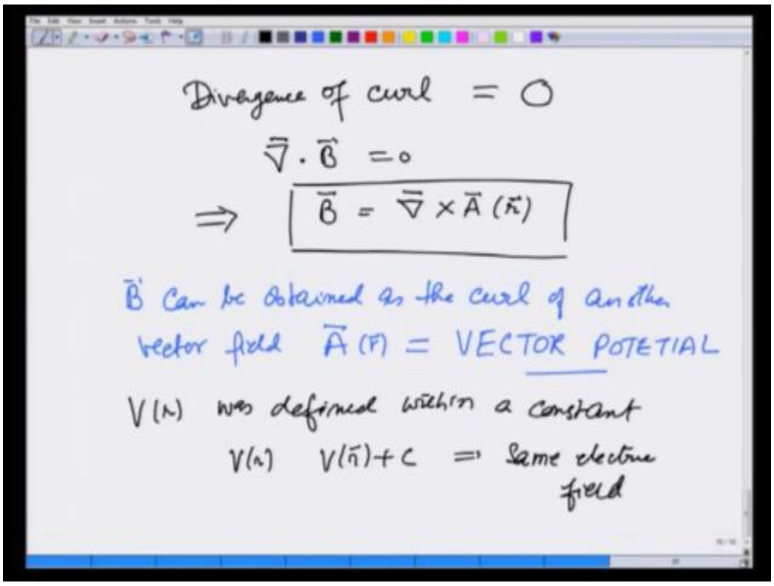


Now, one can show that divergence of curl of a vector field is always 0. So, divergence of B is 0 implies where I can write B as curl of A of r and therefore, now we can say that B can be obtained as the curl of another vector field A which I am going to call the vector potential. So, in the case of electric field we had a scalar potential V r which is also interpreter as the work done. In the case of magnetic field, it is the vector potential whose curl gives me magnetic field.

Now, just like the V r was defined within a constant. What is that mean? That means, I could have V r or V r plus C, I could had a constant V r it is still a give me the same electric field of same physical quantity.

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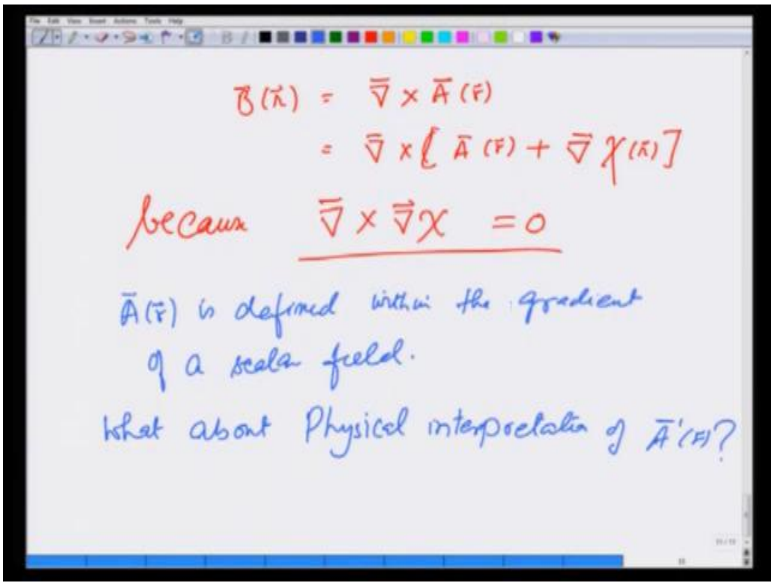


In a similar manner, when I have magnetic field B r as curl of A r I can also write this curl of A r plus gradient of some other quantity, let us called chi r, because curl of a gradient is always 0. And therefore, A r is defined within the gradient of a scalar field. So, just like the potential V had an arbitrary ((Refer Time: 04:55)) constant A r also has an arbitrariness up to the gradient of a scalar field.

What about physical interpretation of A? Can I ask what does A mean? Recall that, the potential energy in the case of electric field was the work required to move a unit charge in that electric field. Can I have a similar interpretation here?

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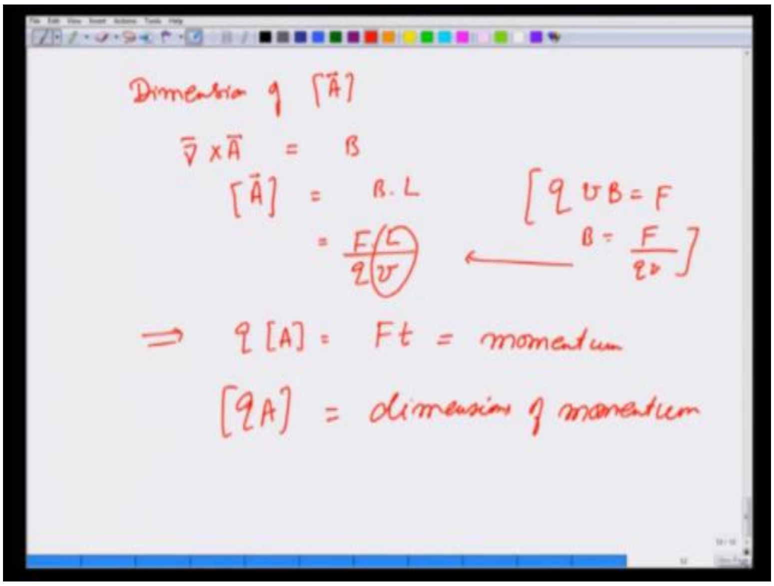


So, the answer is not simple, but we can get some inside side while looking at dimensions of a A. A curl of it is give me B and therefore, A has dimensions of B times the length, B recall that we have q v B equals force and therefore, B is force over q v. Let us substitute that here, then I get force times length divided by q v as a dimensions of A. And therefore, q A has a dimension of F, L divided by v I can write this time F times t is a momentum.

So, q A has dimension of momentum, although it is not directly momentum of particle or anything, but we let us see that this just play a role right, now will leave it at that. Let us now take examples and solve some certain problems where we calculate the magnetic electro potential.

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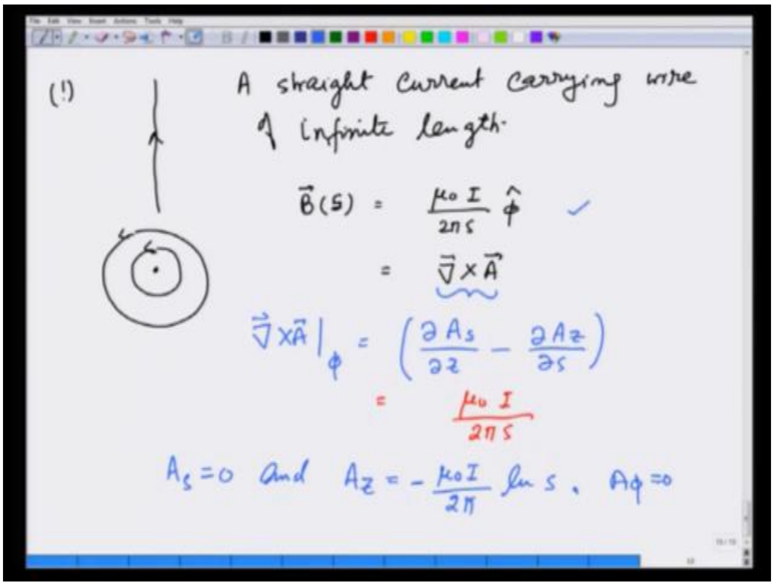


Example 1 I will take a straight current carrying wire of infinite length, in this case we just saw in the previous lecture that and we have calculated earlier also that be filled looks like going around in circles like this. And therefore, I can write B is equal to b at a distance s is equal to mu naught I over 2 pi as in the direction pi this is nothing but curl of A. Since, there is a cylindrical symmetry I will use the definition of curve in terms of cylindrical coordinates.

And cylindrical coordinate since I want only the five component, curl of A phi component is given as partial A s component z minus partial A z partial s and this in our case happens to be equal to mu naught I over 2 pi by s. This immediately suggests that I can take A s to be 0 and A z is mu naught I over 2 pi log s with the minus sign in front. You can immediately see this gives me the right answer for B, I can also choose A phi to be 0.

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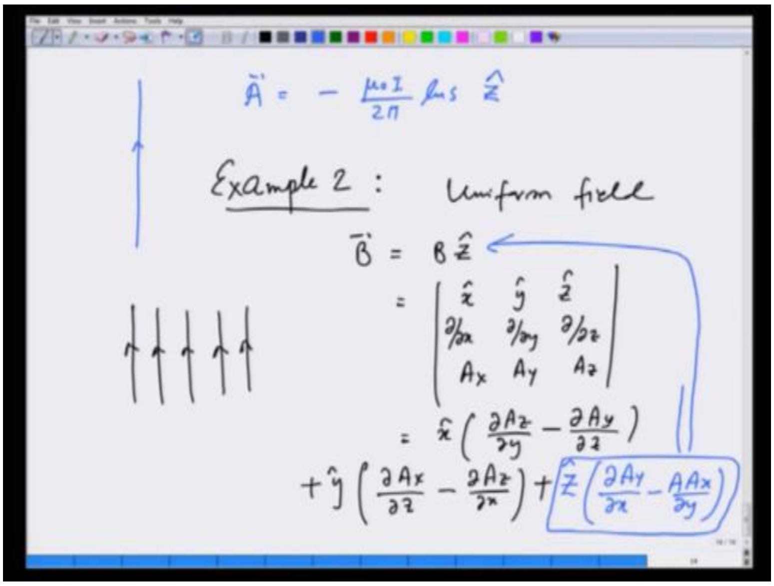


And therefore, I can write in this case of an infinitely long current carrying wire A vector to be minus mu naught I over 2 pi log s in this direction. Example 2, let us take a uniform field B equals B z, so that like this every where I am using Cartesian coordinates, so I am going to write this as x, y, z partial by partial x, partial y, partial z, A x, A y, A z I am interested in only the z components.

So, there will be some x component that is write x component which is partial A z partial y minus partial A y partial z plus y component partial A x partial z minus partial A z partial x plus z component, which I am really interested in partial A y partial x minus partial A x partial and this should be equal to this quantity.

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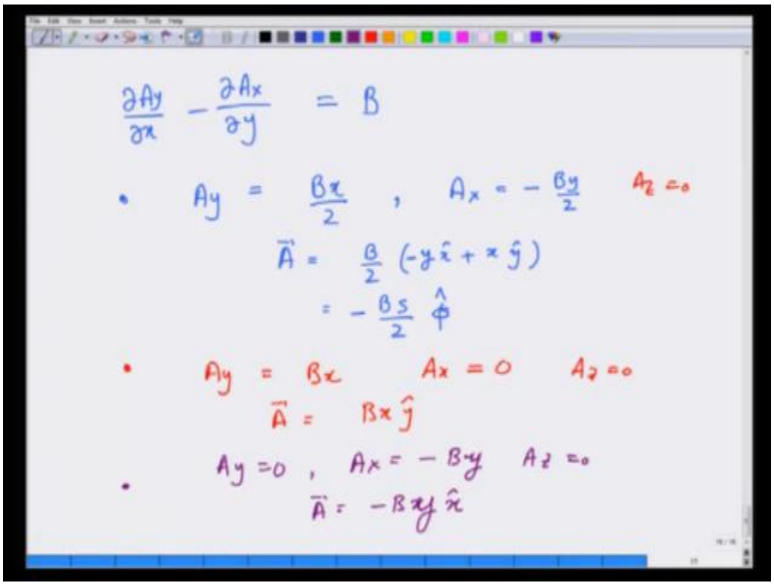


So, I want partial A x over partial y partial A y over partial x to be equal to B, you can immediately see that here three different possibilities I can take A y to be equal to B x by 2 and A x to be equal to minus B y 2. So, that A vector is equal to B by 2 y x plus x y direction which I can write as B s by 2 phi with a minus sign this one possibilities. Second possibility I can have A y equals B x A x equal to 0 I show you A z equal to 0 here and A z equal to 0.

So, I have A equals B x y or third possibility A y equals to 0 A x equals minus B y and A z equal to 0. So, that A is minus B x B y in the x direction. All these three possibilities give me the same answer for B, as I said earlier they will all differ by quantities which are gradients of one or the other quantity.

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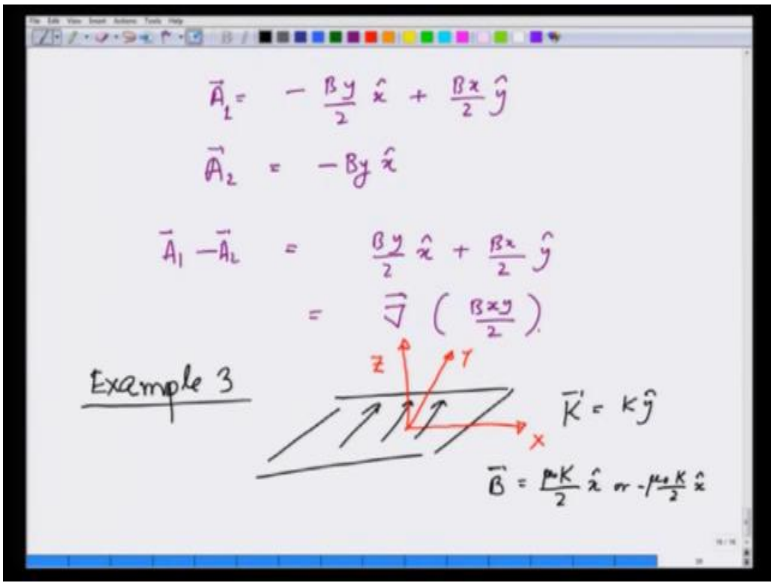


Let us now look at two particular cases I had A equals minus B y over 2 in the x direction plus B x by 2 in the y direction. And I also had A equals minus B y x let us take the difference, this call this A 1 A 2 A 1 minus A 2 is equal to B y by 2 x plus B x by 2 y and this you can clearly see as a gradient of B x y by 2. So, you shown that the difference between the two vector potentials is equal to gradient of a scalar field.

In third example, let us calculate the vector potential for this current carrying sheet with current surface current being K equals k y. Let me remind you for this current carrying sheet the previous lecture I had x in this direction y going along the sheet and z going up and the magnetic field B was given as K mu naught K over 2 x or mu naught K over 2 x with the minus sign.

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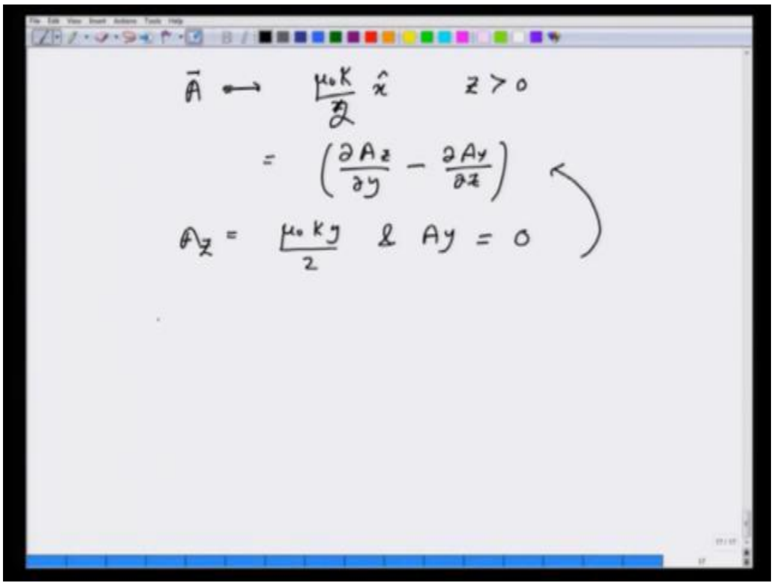

So, let us calculate A for this, A gives me as such that gives me field mu naught K over x, a K mu naught K over 2 in the x direction for z greater than 0. X component is partial A z over partial y minus partial A y over partial z. So, I can immediately write there either A z is equal to mu naught K y over 2 and A y is equal to 0 or any other combination that gives me this answer. If you calculate the difference between the different A's again that will come out to be equal to gradient of a certain scalar field.