**Introduction to Electromagnetism**
**Prof. Manoj K. Harbola**
**Department of Physics**
**Indian Institute of Technology, Kanpur**

**Lecture - 36**
**Ampere's Law for Magnetic Fields**

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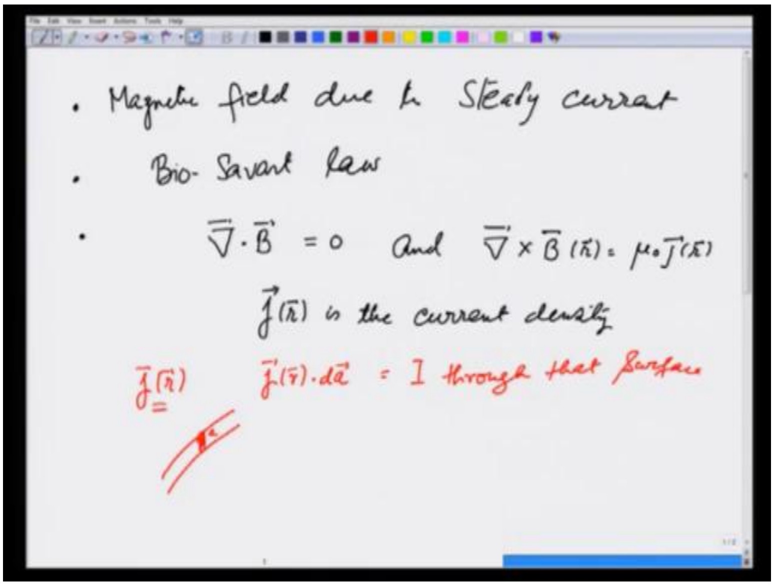


We have focused on magnetic field due to steady current. Learnt about Biot-Savart law; and on the basis of this, derived the divergence of field to be 0 and curl of field at any point to be mu naught j r; where j r is the current density. Let me remind you j r is a vector quantity that, current density j r is such that j r dot d a across any surface a gives the current I through that surface. And its direction is in the direction of the current. This is how we build it out.

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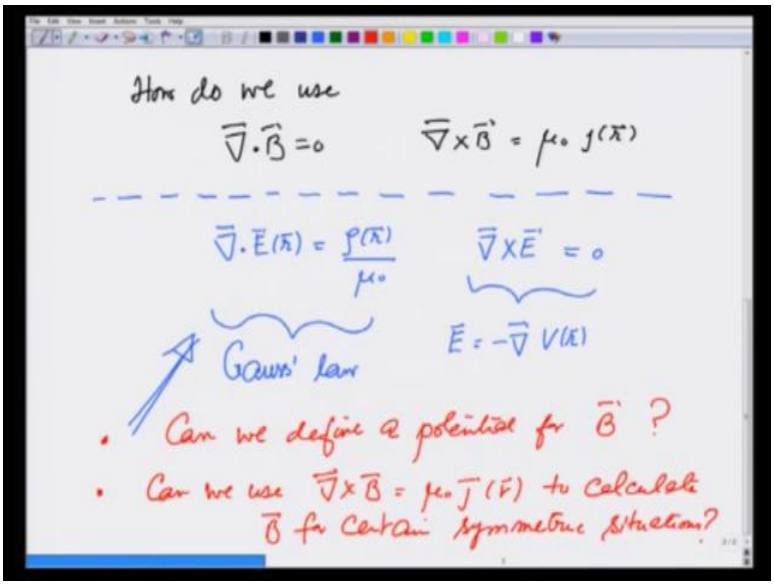


Now, what we want to now ask is how do we use the equations that, divergence of B is 0 and curl of B is mu naught j r in calculating the fields. Recall that, the parallel equations for electric field were like this that, we had divergence of E r to be the charge density divided by epsilon 0. And curl of E everywhere is 0. For example, the curl point allowed me to define a potential V r such that E was minus gradient of V r. This equation is known as Gauss's law. Its differential form in particular or Poisson's equation; and in certain symmetric situations, one could use this equation to get the electric field.

For example, for a point charge or for a uniformly charged sphere or for a uniformly charged cylinder, one could use this equation – Gauss's law here to calculate the electric field. So, the question we now ask is can we define? So, let us ask this question – can we define a potential for B? That is one question we ask. The other question we ask is – can we use del cross B is equal to mu naught j r to calculate B for certain symmetric situations. I will postpone the question of defining a potential for B for later lecture. In this lecture, we will focus on the second question; and that is – we use the curl of B equal to mu naught j r to calculate B for certain symmetric situations.

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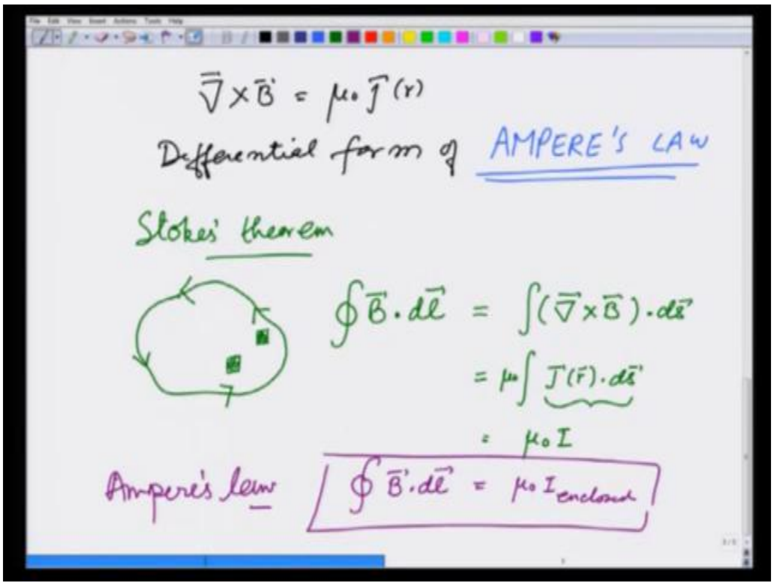


So, let us just first give this a name – curl of B equals mu naught j r, is the differential form of… I am going to write it separately – Ampere’s law. Let us write its integral form; and that is like this. Recall that, Stokes’ theorem for a vector field tells me that, if I have a closed path and I traverse it like this; then B dot dl across this path; and I am going to put a circle here over the integral to show there is a closed path. It is going to be equal to integral curl of B dot ds; where ds is that small area element. And the direction of s is taken with the right-hand rule. If I turn my fingers towards dl; then the thumb gives me the area element direction. This immediately tells me that, B dot dl is going to be equal to mu naught j r dot ds. And j r dot ds as we said in the previous slide is equal to the current. It is therefore, mu naught I. So, let us write the integral form of Ampere's law; that is, if I take a closed path; then the line integral of B over this path is equal to mu naught times the current enclosed passing through the area enclosed to that path. So, let us write this enclosed. This is similar to the integral form of Gauss's law.

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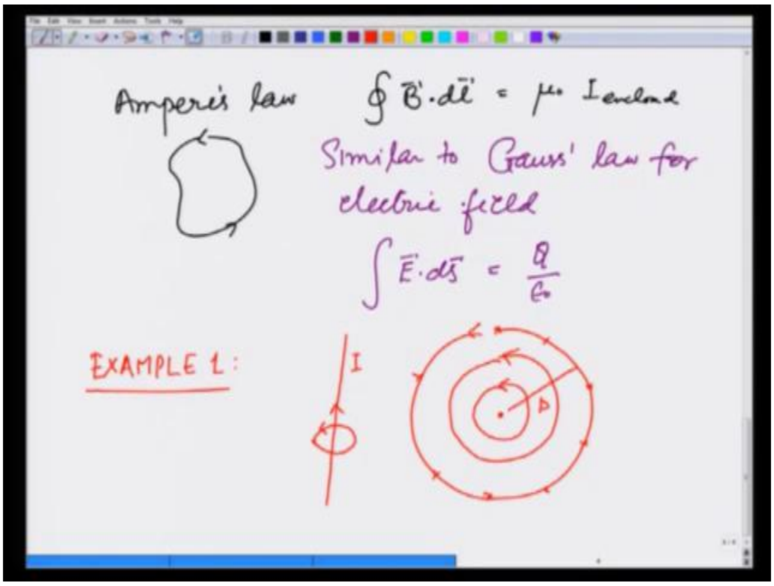


So, let us write this again. We have Ampere's law that says integral B dot dl is equal to mu naught I enclosed by that path. Let me make the path also. This is similar to Gauss's law for electric field that said that, over a surface E dot ds was equal to charge enclosed divided by epsilon 0. So, just like we use Gauss's in certain symmetric situations, we can also use Ampere's law in symmetric situations to calculate the magnetic field. Let us now take examples. We start with the simplest example whereby I have a long wire carrying current I. I anticipate by Biot-Savart law that, magnetic field around this is going to be in the circular direction. This we have already calculated once. But, I emphasize it again that, if I looked at it from the top, the current is coming out of the paper. Then the magnetic field lines look like this. They go around the circles. So, they depend only on the distance s and not on the point where phi here I am at, that is, by symmetry. So, this is a symmetric situation, where there is a cylindrical symmetry. Let us now calculate the magnetic field due to this applying Ampere’s law.

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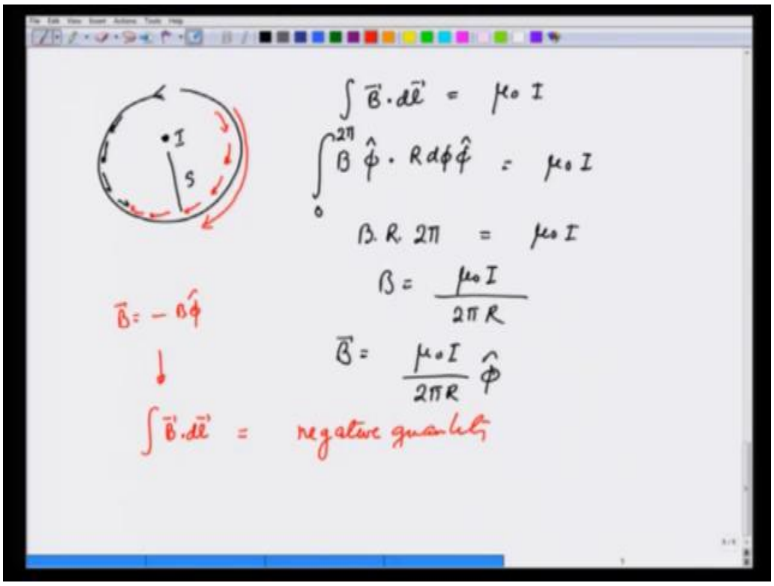

Drawing it again; there is a current I and there is a path at distance s. So, I have integral B dot dl equals mu naught I. If I take dl along the path like this; it is in the same direction as B. So, B magnitude comes out; or, I can write B as B phi dot dl is nothing but R d phi in the phi direction – integration phi from 0 to 2 pi is equal to mu naught I. And this phi dot phi is 1. And therefore, I get B R times 2 pi is equal to mu naught I. And B therefore is mu naught I over 2 pi R. And B vector is going to be in the phi direction mu 0 I over 2 pi R phi. I just want to make one point.

Suppose we had decided to go the other way. And we assumed that, this is how we are going to take dl. Then I would have gotten B dot dl; and thinking that these are also in the direction, B dot dl – I would have gotten to be in negative quantity, because j and ds would be in the opposite directions. I am assuming here that, B is in the negative phi direction and then I get a negative answer. And therefore, again I will get correct answer for B that, B would be going in the positive phi direction. So, this is the… If I am consistent with my direction of line elements and direction of surface area, then I will get the correct direction for B also.

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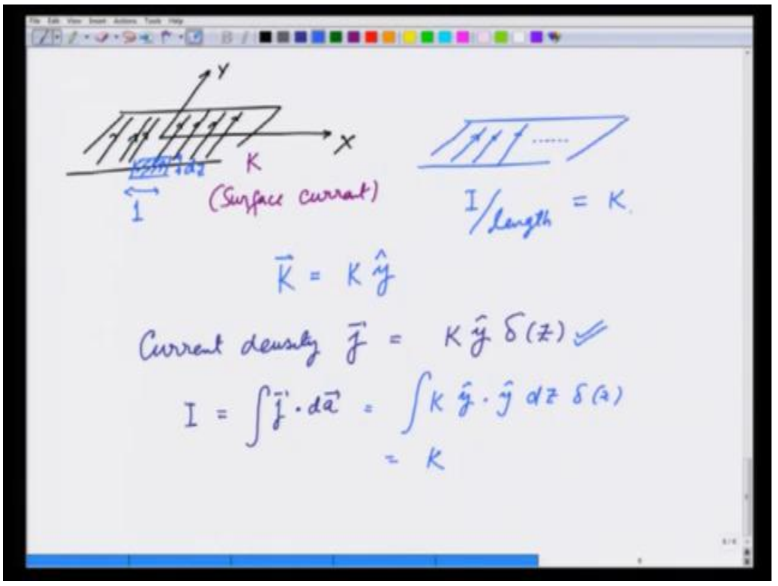


As the second example, I will take a sheet of charge in the xy-plane carrying a current let us say in the y direction – surface current. This is a sheet of current. So, its thickness is 0. I need to carry the current K, which I will call surface current. Before I solve this problem, let me spend a few minutes on surface current. Surface current is of thickness 0 because this is on a sheet of current. So, this is a current, which is per unit length; it gives me I per unit length is equal to K.

And since here K is in the direction y, I can write K vector. This is a current. So, it is a vector quantity is equal to K magnitude in y direction. If I insist on writing this as the current density j; then this will be equal to K y delta of z, because the sheet is in the xy plane. Let us check that. According to the definition, I should be equal j dot da. And in this case, the area would be across the plane. The height of this will be dz. And let us take this to be unit 1. So, this is going to be equal to K y dot area y – 1 times dz times delta z integral over z. It gives me K. So, this is consistent. j for a surface current is like a delta function; delta goes as 1 over z. So, you can see this is already current per unit area. K surface current is per unit length. This is similar to the charge density and surface charge density that we use in electrostatics.

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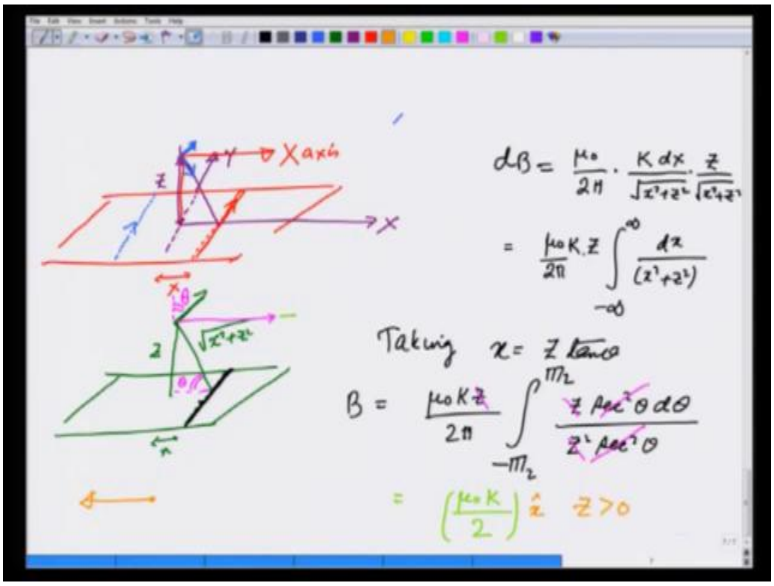


So, now, what we have is we have the sheet. So, let me now make it like this. In x direction like this, y let us say is going in; x cross y in z direction is going up. If I were to calculate his field using our line – the field due to a line current; then I would do something like this. I would take at height z, field due to line at a distance… This is y; this is x. Then I would take field at height z. This height is z. The current is going in this direction. So, I cross the vector to the point where I am is going to give be a mere field as shown by this blue arrow. Let me now erase this; all right? After all field goes in a circular line. So, this is going to be like this.

What about if I take a symmetrically placed wire on the other side? Again in this, the field lines go in a circular direction. So, this will be in this direction with the result that, net field is going to be in the direction parallel to the x-axis . So, all I need to do is take the x component of this field due to a wire at a distance x and add it up. So, let us now again make this figure neatly. This is a sheet; I am talking about a distance height z. A distance from here at a distance x is going to be a square root of x square plus z square; and I am going to take the component in the x direction. If this angle is theta; so is going to be this angle theta out here. And therefore, the x component is going to be whatever small B is coming from this current here, which I am showing in black, which is mu naught over 2 pi times I, which is going to be K dx; per unit length is K divided by square root of x square plus z square; and its component in the x direction, which is sin theta, which is going to be z over square root of x square plus z square. So, this comes out to be mu 0 over 2 pi K z integral dx over x square plus z square raised to 1. And this is from minus infinity; x square is from minus infinity to plus infinity.

Taking x equals z tangent theta, this integral can immediately be change to mu naught K z over 2 pi minus pi by 2 to pi by 2 z second square theta d theta over z square second square theta. Let us do some cancelations; this cancels; z-z cancels z square and you are left with the final answer, which is mu naught k over 2 in the x direction. So, this is the field. If you calculate the field on the other side, the lower side here – this will be in the opposite direction. You could have anticipated that by looking at the current and the direction of the field that it is giving rise to. So, this is in the x direction for z greater than 0 and minus x direction for z less than 0.

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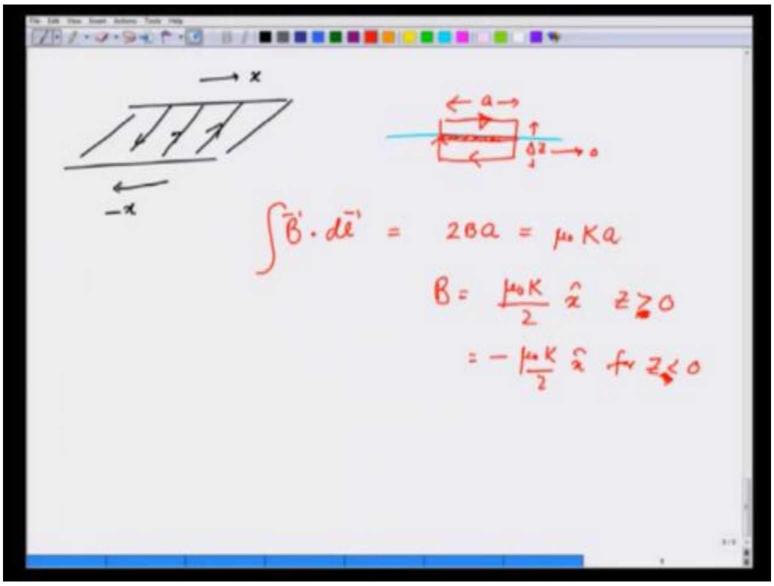

Let us do the same calculation using Ampere's law. If I look at the sheet by looking at the current, which is flowing in this direction, I can already anticipate that, field in the upper direction is going to be in the x direction and the lower direction is going to be in minus x direction; again a symmetric situation. And if I take a loop like this, where this distance is very very small delta z – almost 10 into 0; then B dot dl across this loop will give me if I take the length of this loop to be a; it will give me 2Ba. And this should be equal to the current enclosed by this loop, which is going to be I since this length is a is going to be by definition of surface current K a times mu 0. And this immediately gives you B equals mu 0 K over 2 – x for z greater than 0; and is equal to minus mu naught K by 2 x form z less than 0. So, you can see that, in symmetric situations, I can use the integral form of Ampere’s law to calculate the magnetic field.