Introduction to Electromagnetism
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Lecture - 35
Divergence and Curl Magnetic Field

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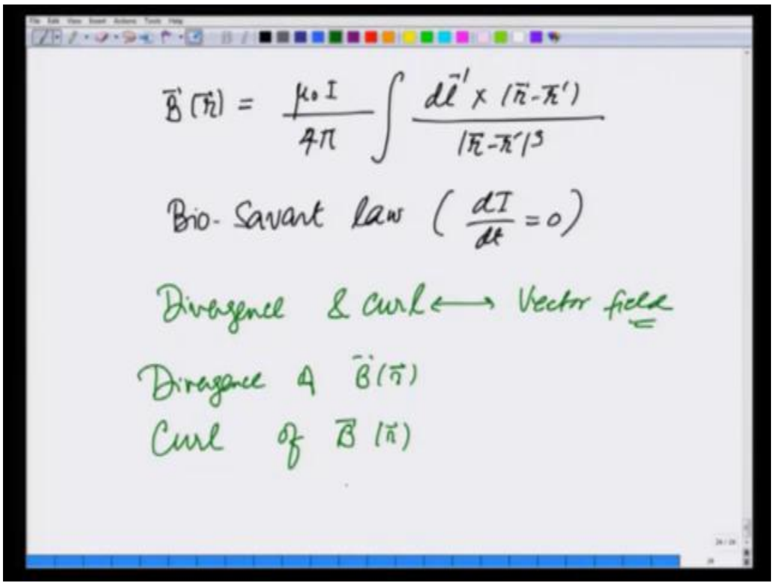

We have seen that the magnetic field is produced by current and for steady currents. That means, these currents which are not changing with time is given as d l prime cross r minus r prime over r minus r prime cubed. This also now tells us the properties of magnetic field. So, this is known as Bio-Savart law and this for those cases, where d I d t is 0. Now, remember when we were discussing electric field, we said that any vector field can be calculated by it is divergence and curl together give a vector field.

So, now if you want to work with magnetic field, it is important to know what the divergence of B s and what the curl of B s, this lecture is going to be concerned with these two quantities. I am going to use a little bit of vector calculus, but I will keep explaining it as we go along that is necessary and I would also request you to master this technique, because this will make life very easy while dealing with these fields.

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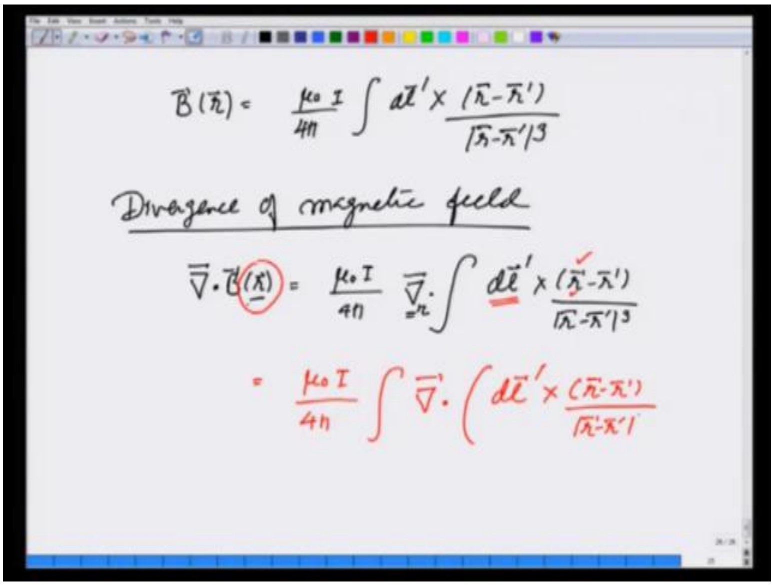

So, when I have B r equals mu 0 over 4 pi I d l prime cross r minus r prime over r minus r prime cube, let us calculates it is divergence directly. Divergence of B is going to be mu 0 I over 4 pi, divergence of this quantity d l prime cross r minus r prime over r minus r prime cubed. Notice, this is divergence with respect to r, because this is divergence with respect to the r out here which is where the field is being determined.

So, this is with respect to r of this quantity, integration is with respect to the prime quantity. So, I can actually take it inside and write this as mu 0 I over 4 pi, integration of divergence of this vector product d l prime cross r minus r prime over r minus r prime cubed.

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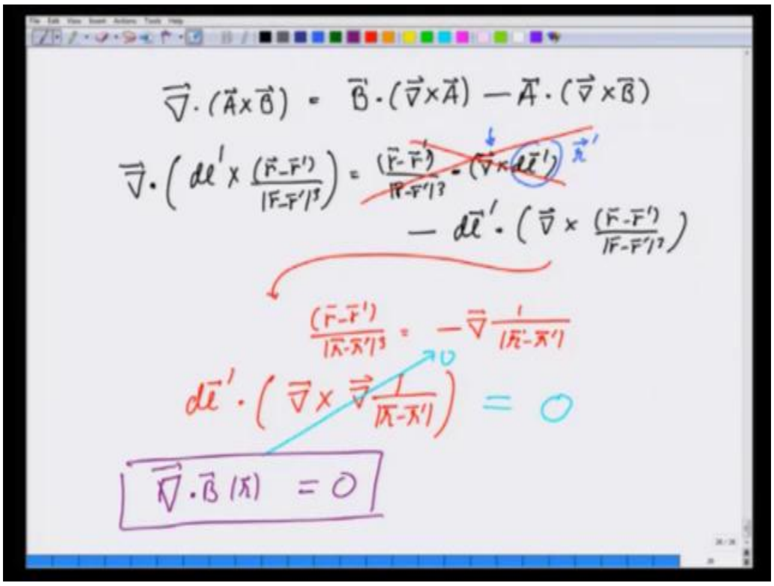

Now, from the vector identity the divergence of A cross B is equal to B dot curl of A minus A dot curl of B, we find that divergence of d l prime cross r minus r prime over r minus r prime cubed is going to be equal to r minus r prime over r minus r prime cubed dotted with curl of real prime minus d l prime dot curl of r minus r prime over r minus r prime cubed. Notice that d l prime is with respect to very well r prime and this differentiation is with respect to r, so this term goes to 0.

For the second term, recall that r minus r prime over r minus r prime cube is nothing but minus grad of 1 over r minus r prime. So, this term is d l prime dot curl of grad of 1 over r minus r prime, what is curl of grad curl of grad is again 0. So, this whole term comes out to be 0 and therefore, what we find is that divergence of B r is nothing but 0. This is a result we have anticipated in one of the previous lectures by looking at that there are no mono poles, there are no single sources of magnetic field.

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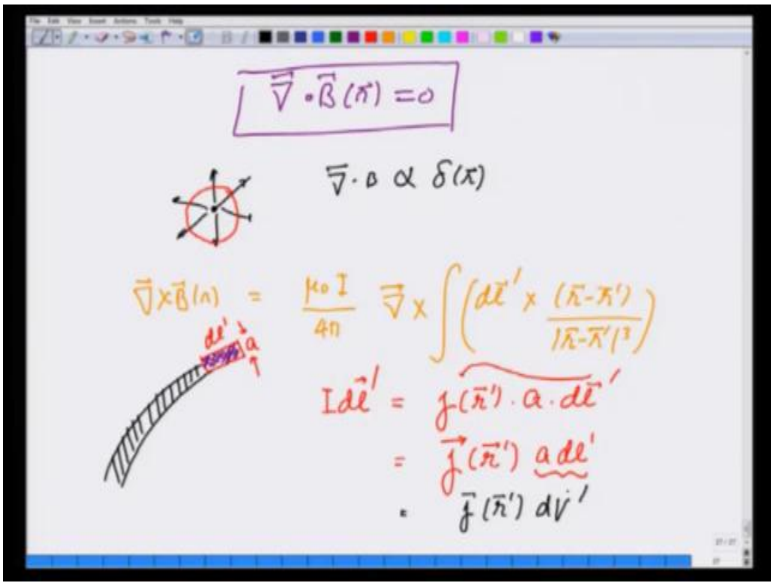

So, we find from the Bio-Savart formula that divergence of B at any point is always 0, this also means there are no independent sources or sings of B. Let us see how does that happen, because in the independent source and fields will be coming out of here and this divergence or B proportional to delta r or equivalently by divergence theorem flux to the surface to be non zero telling me the divergence cannot be 0.

Next let us calculate curl of B directly, so this is also going to be mu 0 I over 4 pi curl, curl is with respect to r, integration of real prime cross r minus r prime over r minus r prime cubed. To calculate curl I am going to play a little check, if I take this current loop, a current carrying wire assure is likely wide at any point. Suppose I take this point, where I am taking d l prime and this cross section is a, then I can write I d l prime vector as j at that point, where j is the current per unit area times a times d l prime vector, d l prime is in the direction of current.

So, I am going to bring this vector over to j and define by current density is a vector quantity j r prime a d l prime. What is a d l prime? A d l prime is the volume of this smaller element which I am not showing by the shaded purple lines therefore, I can write this whole thing as j vector r prime d v prime.

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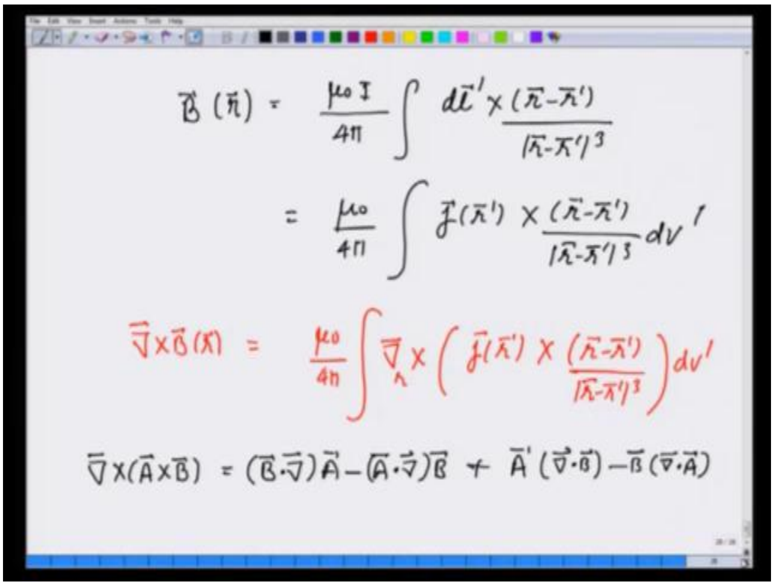

So, therefore, I can write B r which is mu naught I over 4 pi integration d l prime cross r minus r prime over r minus r prime cube as equal to mu over 4 pi integration j r prime cross r minus r prime over r minus r prime cube d v prime. Obviously, the integration over j is not zero and that precisely what how we transfer form that I d l from to j d v prime formula.

Now, let us take a curl, so curl of B r is going to be mu 0 over 4 pi curl of again I can take curl inside, because curl is with respect to r the and integration is respect to r prime. So, j r prime cross r minus r prime over r minus r prime cubed d v prime, now I am going to use vector identity which is curl of A cross B is equal to B dot del operating on A minus A dot del operating on B plus A divergence of B minus B divergence of A what it means is let me explain in next slide.

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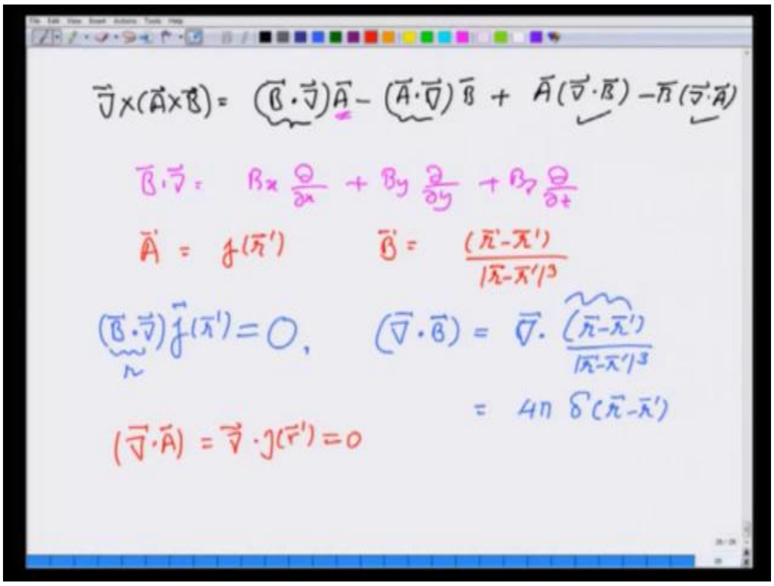

I have written curl of A cross B as B dot del A minus A dot del operating on B plus A divergence of B minus B divergence of A, you have familiar with this term and this term or this term or this term means is B dot del minus B x D by D x plus B y D by D y plus B z D by D z and this operates on this vector A given a vector quantity. Now, let us substitute for A I have j r prime for B I have r minus r prime over r minus r prime cubed.

Therefore, B dot del operating on j r prime this differential with respect to r, j is with respect to r prime is gives me 0 del dot B is divergence of r minus r prime over r minus r prime cubed and that is recall you did time and again is 4 pi delta r minus r prime. Again a physical way of looking at it is that this quantity r minus r prime over r minus r prime cube is nothing but represents electric field due to a charged 4 pi Epsilon not in magnitude. And therefore, the charge density is the divergence is a charge density which is nothing but a delta function, so that gives you that very quickly. Let us similarly del dot A which is divergence of j r prime is also 0.

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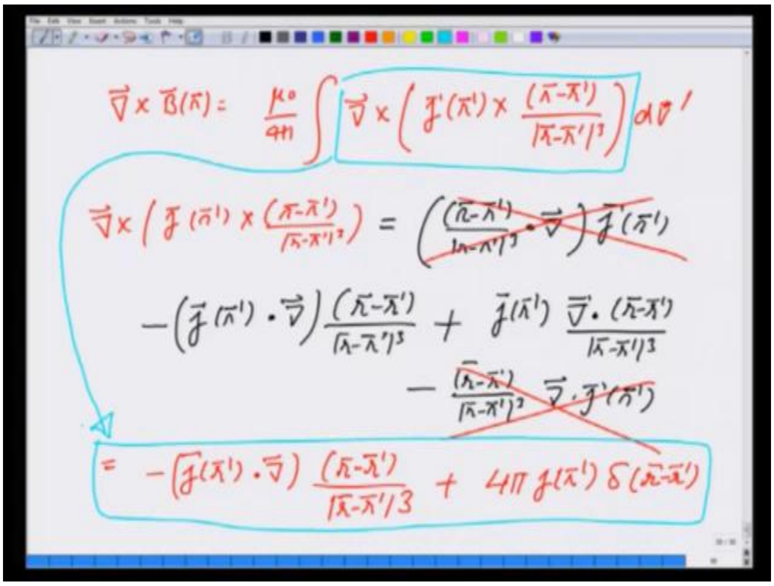

So, when I take the curl of the which is mu 0 over 4 pi integration curl of j r prime cross r minus r prime over r minus r prime cubed d v prime curl of j r prime j cross r minus r prime over r minus r prime cubed is nothing but equal to r minus r prime over r minus r prime cubed dot del operating on j r prime minus j r prime dot del operating on r minus r prime over r minus r prime cubed plus j r prime time's divergence of r minus r prime over r minus r prime cubed minus r minus r prime over r minus r prime cubed divergence of j r prime. And we have fond this term is 0, this term is 0 and we left with minus j r prime dot dell r minus r prime over r minus r prime cubed plus 4 pi j r prime delta r minus r prime this is this term.

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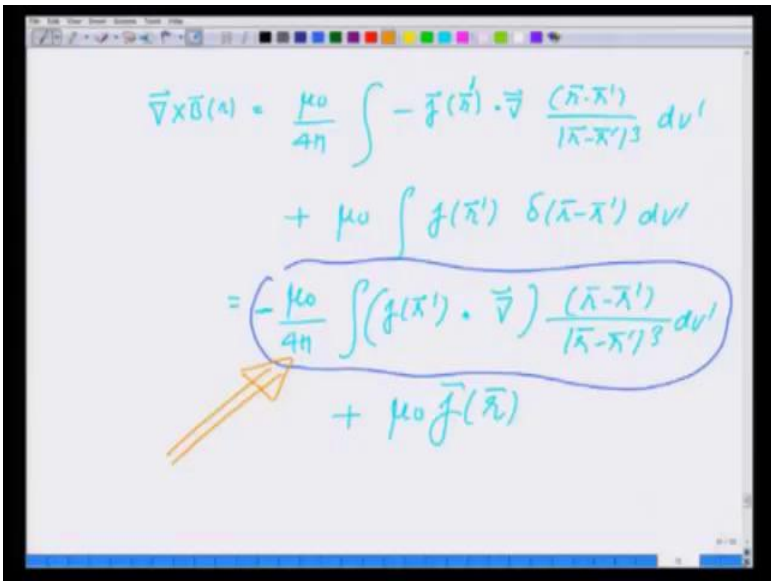

And therefore, I have curl of B at r is equal to mu 0 over 4 pi integration of minus j r prime dot del r minus r prime over r minus r prime cubed plus mu 0 integration j r prime the 4 pi 4 pi cancels delta r minus r prime d v prime which is equal to mu 0 over 4 pi minus outside j r prime is a prime ((Refer Time: 15:39)) dot del operating on r minus r prime over r minus r prime cubed B prime plus mu 0 j r, j r is a current density as we already discussed the direction as there that of the current and this current per unit area, the only term we are now left evaluate is this term to do this again I will to a check.

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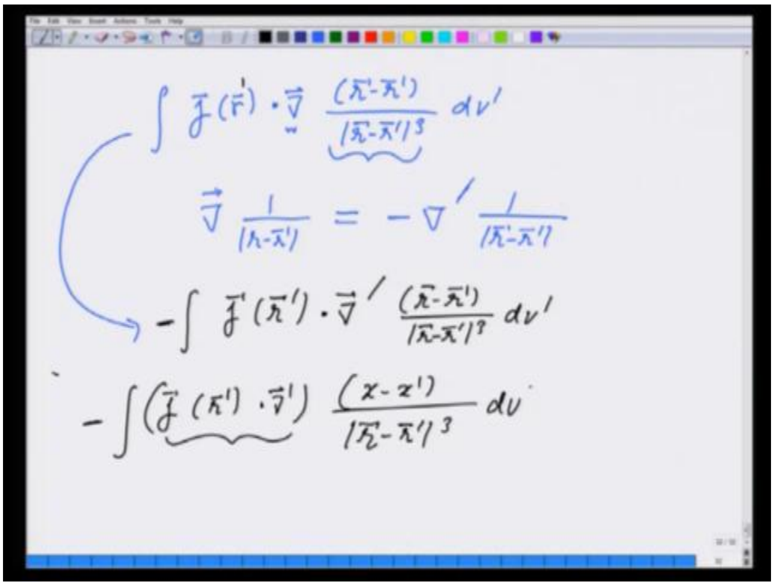

So, I am interested in evaluating j r prime dot dell r minus r prime over r minus r prime cubed d v prime. When del operates on r minus r prime it is equivalent to over del operating on any function of r minus r prime it is equivalent to minus del prime operating on 1 over r minus r prime. So, I can write this you can clear on with many function as you see that this is true. So, I can write this as integral j this is r prime r prime dot del prime acting on r minus r prime over r minus r prime cube d v prime to the minus sign.

To evaluate this I calculate this x, y and z components separately if I do x components is 0 and then you can check the same thing for y and z components. So, x component of this is going to be the minus sign j r prime dot del prime, since this is a dot product already taken. So, there is no x component in this all three components come here I have x minus x prime over r minus r prime cube d v prime let us go to the next page.

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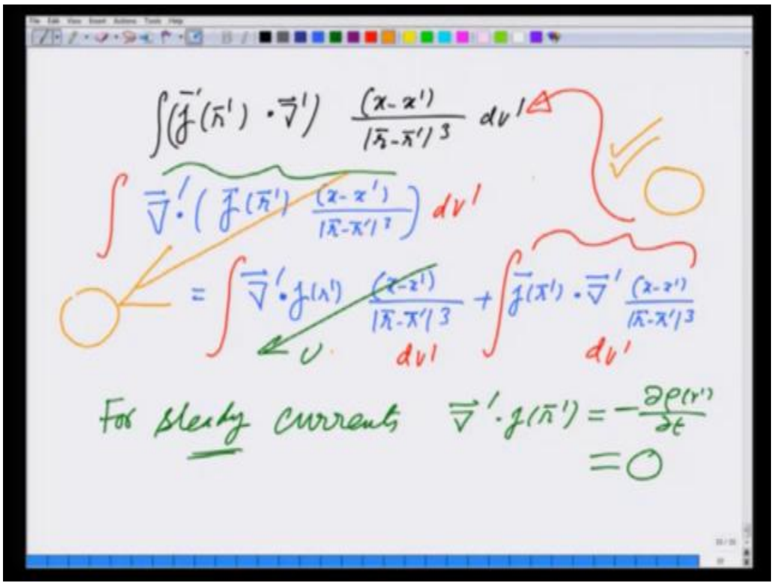

So, that I am evaluating is integration j r prime dotted with the del prime component I take only of the vector quantities x minus x prime over r minus r prime cubed d v prime. Now, let us look at the divergence of this quantity with respect to del prime j r prime x minus x prime over r minus r prime cubed, this is equal to divergence of j r prime x minus x prime over r minus r prime cubed plus j r prime that del prime x minus x prime over r minus r prime cubed with divergence.

If I integrate both sides with respect to d v prime, this is what I get, notice that this term it preciously what I want. Now, for steady currents del prime dot j r prime which otherwise is minus d row r prime d t is 0. Otherwise, it will be minus d row r prime d t right now is 0 for steady current only if the current time wearing this time does contribute and we will see later that this ((Refer Time: 19:36)) size of something called the displacement current for the time being for Bio-Savart law for steady currents this term is 0.

Another currents are localized; that means, for away from the system they go to 0, this divergence can be changed into a surface integral and that surface integral will also go to 0, because far away j has really becomes 0 on that surface. So, what you are left with this the last term, this term here since left hand side is 0, this term is 0, this term is also 0. So, what I have shown you now ((Refer Time: 20:11)), if I go back is that this term here is 0.

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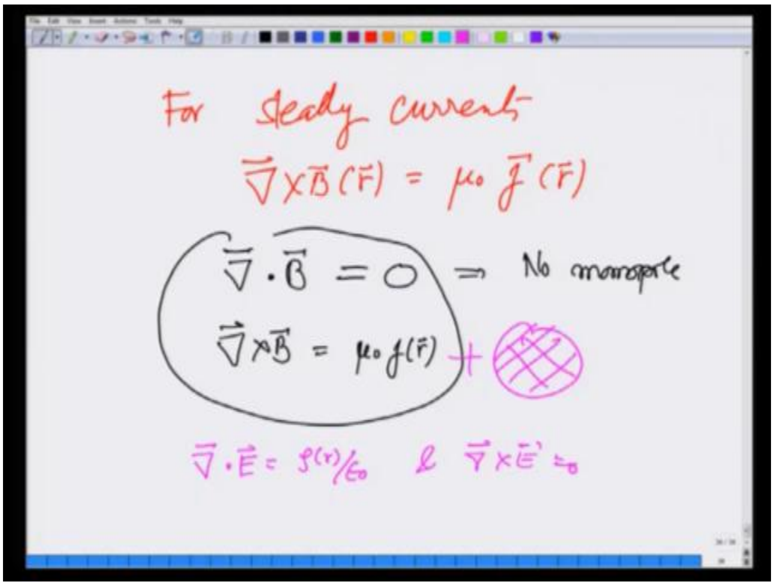

And therefore, for steady currents we find that curl of B is mu 0 j the current density r. So, what we found in this lecture through Bio-Savart law that divergence of B is 0 and this physically means no mono pole and curl of B is mu 0 j of r, compare these equations with that for the electric field, where I had divergence of E as row r over Epsilon 0 and curl of E 0. So, here curl is B is non zero and divergence of B is 0 I remember this is true only for steady current, if this time dependent current then there will be a negation term here which will deal with data right now will focus only on magnitude statics and therefore, magnetic field you to steady currents.