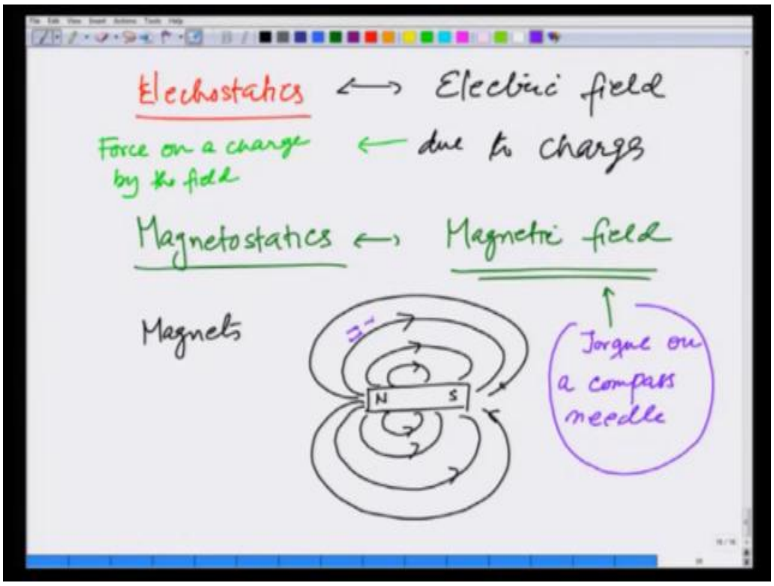
Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 34
Introduction to Magnetostatics; The Bio-Savart law

We have so far talked about electrostatic that mainly concerned with electric field due to charges.

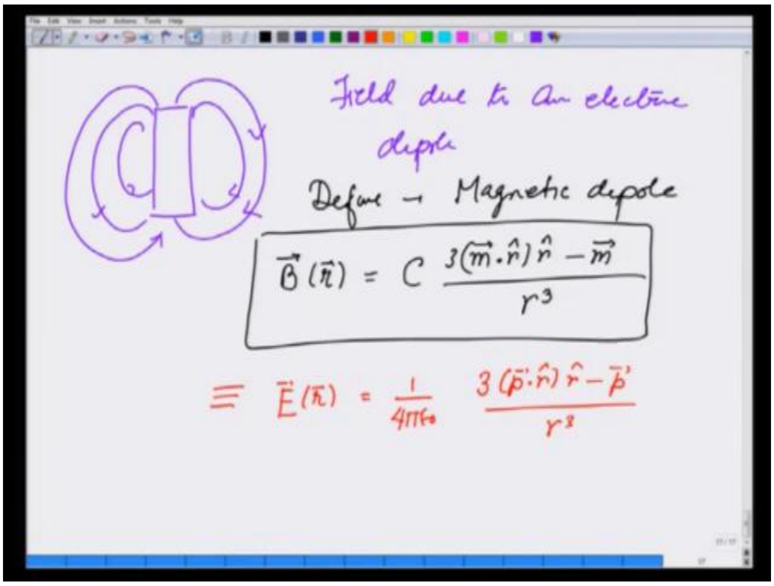
(Refer Slide Time: 00:13)



We are now going to change here is and go to discuss magnetostatics and see how the magnetic field can be dealt with just to give you a little background, the way magnetic field came about was through magnets. And if one took a bar magnet, you may have done it in school, take two sides N and S and if I plot it the field lines around I they look like this.

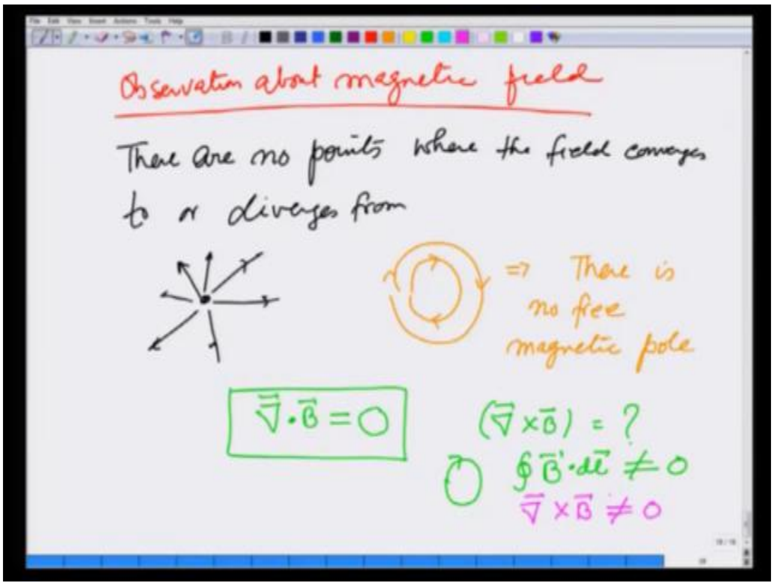
How do you plot the field lines? Remember, in electric field represented force on a charge by the field. In magnetostatics magnetic field is identified by torque on a magnetic compass needle. So, that if you put it in a magnetic field turns around, that is how I know there is a magnetic field. So, to plot these magnetic lines one puts a compass and draws arrows going from south to north of the compass.

(Refer Slide Time: 02:24)



What is found is that these magnetic lines pretty much follow the same pattern as the field due to an electric dipole. So, it makes sense to define a magnetic dipole and I can say that magnetic field which I am going to denote by B, B r is going to be some constant which will depend on the units. The net magnetic dipole moment dotted with r r 3 minus m divided by r cubed, this is in parallel with the way we defined electric field for an electric dipole which was 1 over 4 pi Epsilon 0 which I am representing by c for the case of magnetic field 3 p dot r r minus p over r cubed, so that is one.

(Refer Slide Time: 03:53)

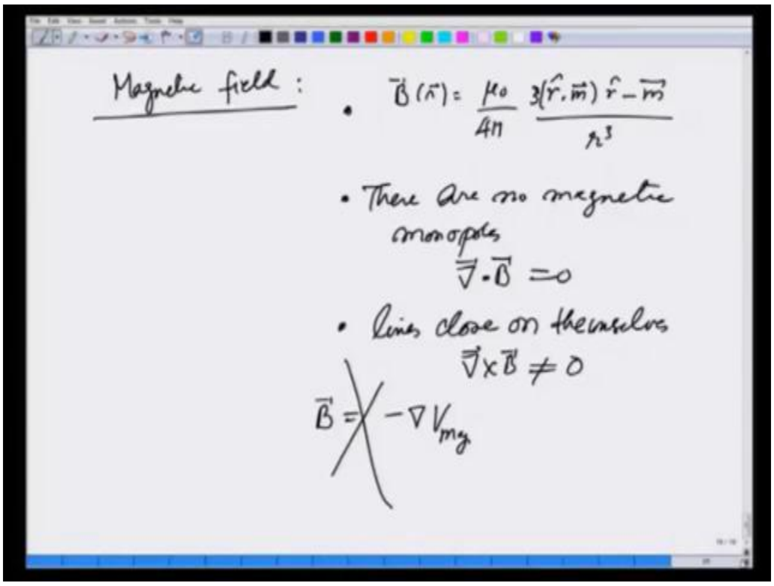


The other thing which is observed about magnetic field is that there are no points where the field converges to or diverges from. That means, although in the electric field you saw the charge q gave you free line like this, in magnetic case you never find a point where it happens, magnetic field lines always close on each other and this means that there is no free magnetic pole unlike the charges.

So, that is one difference although the formula for the magnetic field may look similar to the formula for the electric field of a dipole. But, there are no free magnetic poles, one thing you notice from this is therefore, the divergence of B will always be 0. Because, divergence refers to source and in this case happens to be no independent source from where all the lines are coming from or where all the lines are converging.

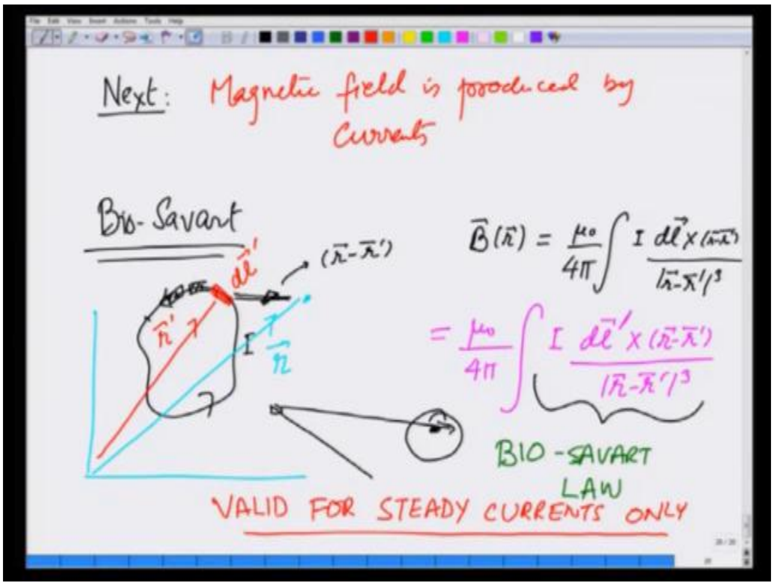
What about curl of B, again if I look at the lines it appears that they never close on themselves, if certain field lines do not close on themselves. That means, that B dot d l will not be 0 and therefore, by stokes theorem del cross B may not be 0.

(Refer Slide Time: 06:09)



So, from these observations what we see is that for a magnetic field or dipoles, I can write B r as some constant as mu 0 over 4 pi 3 r dot m r minus m over r cubed, there are no magnetic monopoles. And therefore, divergence of B is 0, lines close on themselves therefore, curl of B is not equal to 0. That means, I also cannot write B as minus grad some V magnetic I cannot do that. This is where the study of magnetic field would have stopped, if a new connection was not found and that connection was found by Oersted who found that magnetic field is produced by currents.

(Refer Slide Time: 07:26)



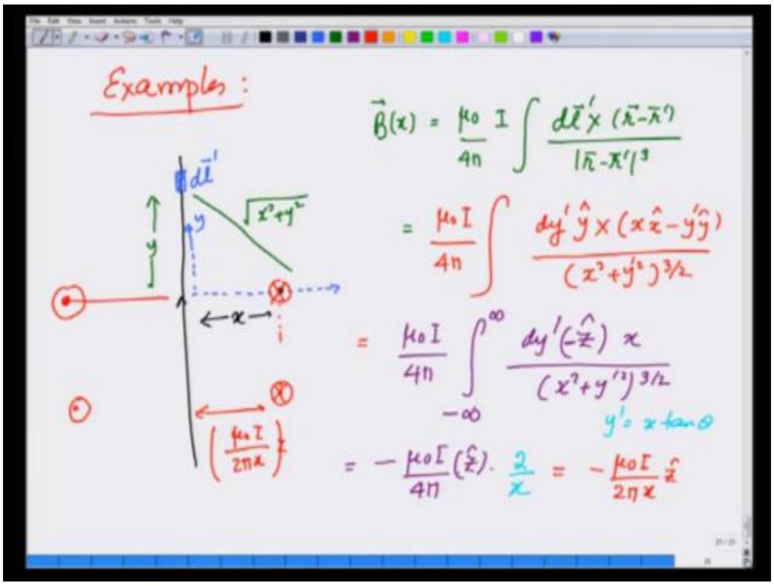
So, next experimental observation is that magnetic field is produced by currents, not only that one could find the formula of how this is. So, if I have a current loop carrying current I or a large current element. So, remember it always closes on itself, then the total B at point r can be written as mu 0 over 4 pi, these constants were fix later according to units I d l cross r minus r prime over r minus r prime cubed, let me write it again neatly here, this is equal to mu 0 over 4 pi I d l let me also put d l prime. So, that it denotes the point of integration cross r minus r prime over r minus r prime cubed.

So, let us see what does it mean, suppose I look at the first case, first loop here and I want to see the field at this point, which is let us say at distance r. What I will do is, I will take a small element here in the direction of the current d l. So, direction of the current or d l prime, direction of the current decides which directions that d l is, this is at r prime. The vector from this to my position, where I want to find the magnetic field is r minus r prime.

I will calculate add all these small, small things and add them together and that gives me the net magnetic field. So, at this point if I take d l which is in the direction of the plane of this paper in this direction, this is the direction of r. So, d l cross r if I take that cross product it will give me something going in. So, this will produce a field going in and I add these small, small contributions due to the entire loop and that gives me the net magnetic field, this is known as the Bio-Savart law.

One thing I must point out here that this law is valid for steady currents only, by steady means the current is not changing with time. If current changes with time new terms come in which we will see later for the time being for a steady state current, you find that the magnetic field is given by this formula. The moment this happens now you can have in lot and lot of more studies, because now you have a formula from which you can derive various properties of magnetic field. So, this gave a jump to the steady of magnetic fields quite a bit.

(Refer Slide Time: 11:11)



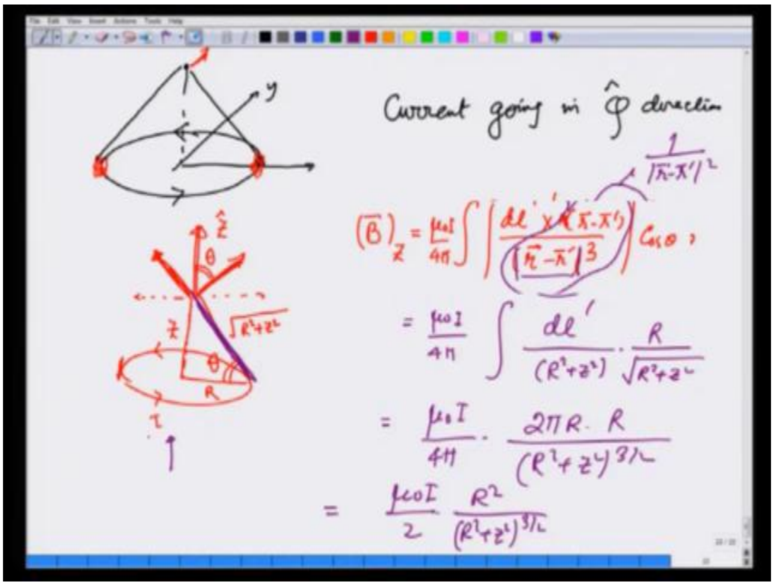
Let us take some examples of Bio-Savart, law as a first example I take a long wire it may close from outside coming from a distance. So, that will not affect things and want to calculate the field at a distance, let us say x by taking this direction to be x and direction of the wire to be y I want to calculate the field at a distance x from the wire, we are taking this plane to be x y plane. If I take a small element d l prime it is distance from x is going to be and this is at distance y is going to be square root of x square plus y square.

So, B at x vector is going to be mu 0 over 4 pi I is outside d l prime cross r minus r prime over r minus r prime cubed which I can write as mu 0 I over 4 pi integral d l prime is nothing but d y in the y direction cross r is nothing but at distance x in the x direction minus y prime is y in the y direction divided by x square plus y square raise to 3 by 2.

Notice that I did not have to write prime out here, but if you like I can write prime here, prime here denoting that prime is actually where I am integrating and prime here and no unprimed variables are those, where I am calculating the field. This can be further written as mu 0 over I over 4 pi y prime varies from minus infinity to infinity d y prime y cross x gives me z with a minus sign x y cross y gives me 0 divided by x square plus y prime square raise to 3 by 2.

So, this gives me minus mu 0 I over 4 pi z you can do the integration very easily by substituting y prime equals x tangent theta and you will get an answer of 2 over x here. So, that the final result is minus mu 0 I over 2 pi x z, so at this point a field goes in the minus z direction x cross y z is coming out of the paper. So, it is going into the paper, if you calculate at this point it will be coming out, since this is an infinitely long wire this point where it is it does not really matters. So, all over the field is going in coming out on this side and this magnitude is mu 0 I over 2 pi x.

(Refer Slide Time: 14:46)



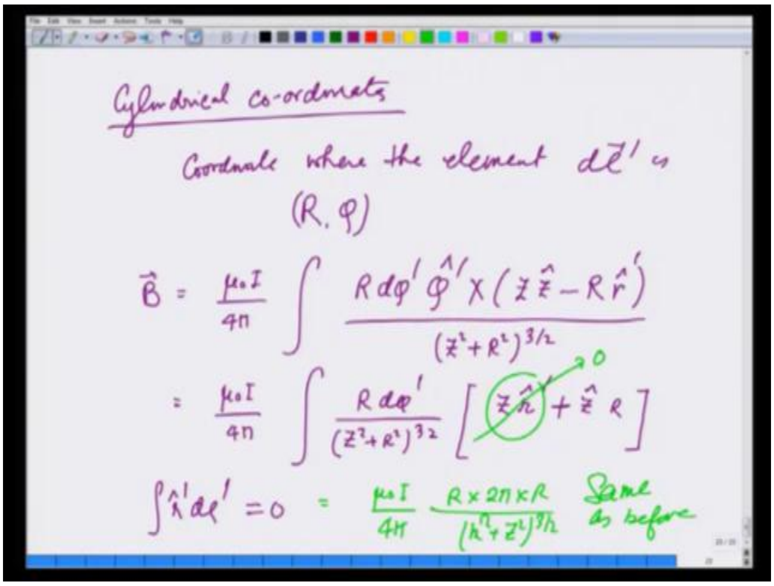
As a second example let me take a ring of wire, a ring of current let us say in the x y plane. So, this is x, this is y and I am calculating field at some point z at the height z of it, this has a current going in phi direction, since this is in phi direction it is easy to use the cylindrical coordinates. Now, let see if I calculate the field at point z due to the line element here and a line element here, you can easily see that d l cross r will give a field going like this.

Let me make it here this point will give me a field in this direction and this point the current is coming out, this will give me a field in this direction. So, that two opposite ends will keep cancelling this horizontal component and net field is going to be in the vertical direction along the z axis. So, let us calculate that, what I will do is, I will calculate the z component and add it, this distance is r current is I and this distance is going to be square root of R square plus z square this height is z let this angle be theta, so that this angle is also theta.

So, you can see that B z component is going to be integration d l prime, since the direction of current and the vector r minus r prime is perpendicular. So, this is going to be over r minus r prime cube r minus r prime cross and I am taking it is magnitude and cosine theta component and I will add it up mu 0 I over 4 pi. So, let me say it again I am taking the field due to these small elements and adding it up taking only the z component, this can be written as mu 0 I over 4 pi integration, this distance is nothing but r minus r prime cubed modulus r minus r prime this is also modulus.

But, we have already said this is going to give me d l prime over square plus z square, because this together gives me 1 over mod r minus r prime square, cosine theta is going to be R over square root of R square plus z square. So, this gives me mu 0 I over 4 pi times 2 pi R times R over R square plus z square raise to 3 by 2 which is mu 0 I over 2 R square over R square plus z square raise to 3 by 2 and it is in the z direction the upper half plane and same direction here also as you can see easily by taking the vector product.

(Refer Slide Time: 18:41)



Let us do this in cylindrical coordinates, in cylindrical coordinates they coordinate where the element d l prime is nothing but R phi. So, remember you have B is equal to mu 0 I over 4 pi integration d l prime is going to be D d phi prime in the phi direction or phi prime direction cross r minus r prime is going to be z, z direction minus R r direction divided by z square plus R square raise to 3 by 2 which is r minus r prime cubed which gives me mu 0 I over 4 pi integration R d phi prime over z square plus R square raise to 3 by 2 inside phi prime cross z phi prime cross z gives me r prime unit vector minus phi prime cross r prime phi prime cross r prime gives me minus z unit vector. So, this will become plus z R that is the answer.

Now, integration r prime d phi prime is 0, because r prime vector has cosine prime and sin phi prime and the other integration gives you 2 pi and therefore, the final answer comes out to be mu 0 I over 4 pi, this first term as I already said gives you 0 R times 2 pi times R over R square plus z square raise to 3 by 2 same as before. So, we have to solve two examples of getting magnetic field using Bio-Savart's law, where we calculate one the field due to a long wire and two the field due to a ring of wire. Remember these formulas are valid only if the current is steady.