Introduction to Electromagnetism
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Lecture - 33
Electrostatics in presence of Dielectric Materials – II

We have been looking at the electric field and associated quantities in Dielectric medium.

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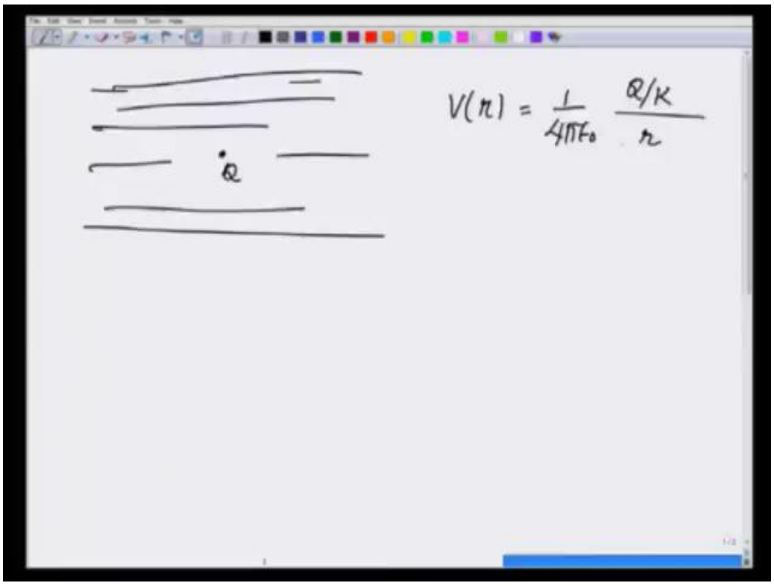

The last problem that we solved was, if I have an infinite dielectric medium and a charge Q in it. Taking the position of the Q at the origin we found that V at r was equal to 1 over 4 pi Epsilon 0 Q over K over r.

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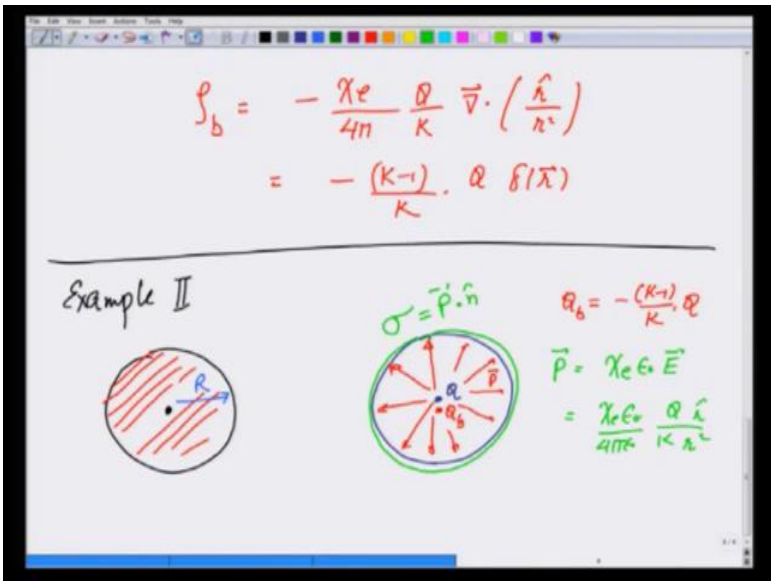

To recall rho bound comes out to be minus chi e over 4 pi Q over K divergence of r over r square which is nothing but minus chi e is K minus 1 over K times Q delta r. Precisely what we found earlier, but now we have seen it mathematically corresponding to this charge of this dielectrics sphere not infinite dielectric, but dielectrics sphere of radius R.

If I see what happened in the previous example, what I will imagine this as there is this charge Q, in addition there is a bound charge Q b which is equal to minus K minus 1 over K Q. And now, recall what this electric field does, the electric field produced by these charges, produces a polarization p and what is the value of this p, p will be equal to chi e Epsilon 0 E which is nothing but chi e Epsilon 0 Q over K r r square.

So, there is this p and what would this p do, if I take this make this sphere of radius R, it will give me a sigma or the surface charge here which is p dot n. So, let us see the picture that is imagine now for this case.

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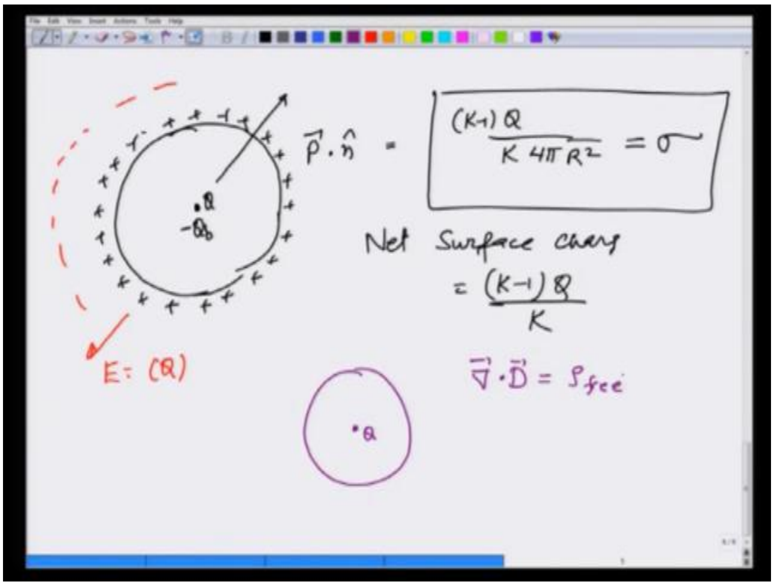

Picture that imagines now in this case is the finite size sphere of dielectric with the charge Q inside, there will be a Q bound and there will be this positive charge. Because, p is coming out all over which will be p dot n and in this case, if you plug in the value of p, it comes out to be Q over K K minus 1 which is chi e R square is equal to sigma.

So, net surface charge is going to be K minus 1 Q over K which is exactly opposite of the bound charge which is negative of K minus 1 over K cube at this center. And therefore, the effect outside of these bound charges is going to cancel and I should have an E out here due to Q alone. Let us now see this little more rigorously to see this, if you make this sphere again charge Q in the middle and I have divergence of D displacement is equal to rho free.

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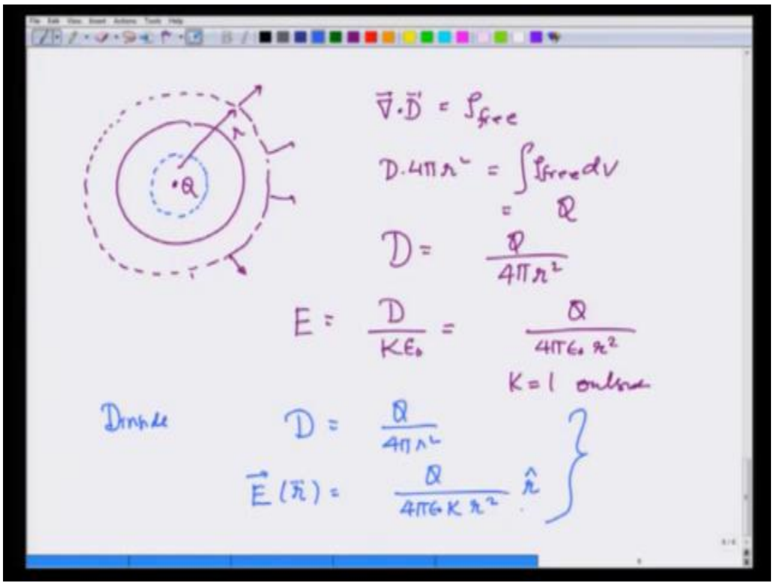

And I am going to use Gauss's law for this, so I have this sphere charge Q at the center and divergence of D is equal to rho free. Since, everything is going to be spherically symmetric there is no curl. So, now, I can use Gauss's theorem outside and that will give me D times 4 pi r square, if the radius of this outside sphere is r is equal to rho free times d v which is Q free. So, that is my Q and therefore, D comes out to be Q over 4 pi r square, E will be D over K Epsilon 0 which is going to be equal to Q over 4 pi Epsilon 0 r square, because K is equal to 1 outside and the direction is; obviously, in this spherical in the radial direction.

So, this is the same answer as we anticipated earlier by saying that the net bound charge is 0, when I include a surface and the point bound charge. How about D inside? D inside is still the same Q over 4 pi r square, because if I take this Gaussian surface inside, this is a D I am going to get. But, now E is going to be different and let me write E vector now is going to be Q over 4 pi Epsilon 0 K r square in the r direction.

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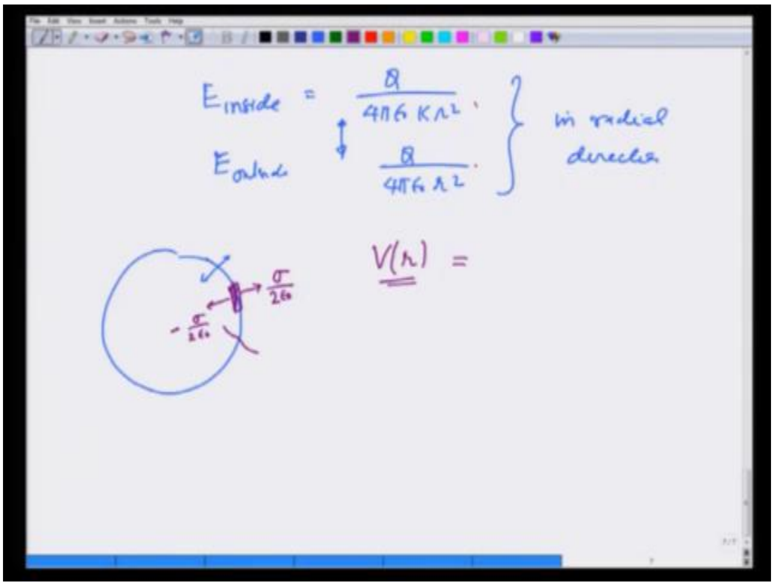

So, E inside is Q over 4 pi Epsilon 0 K r square and E outside is Q over 4 pi Epsilon 0 r square in radial direction. So, notice there is a change sudden change in the electric field magnitude as you cross the surface and that is because there is a surface charge. Whenever there is a surface charge, suppose there is a surface charge sigma out here by Gauss's law it gives sigma over 2 Epsilon 0 outside and sigma over minus 2 Epsilon 0 inside.

And therefore, field inside is going to be slightly less, field outside is going to be slightly more than field inside and this is the effect of polarization. How about the potential v r? Remember, we said potential should be the same everywhere, because of the interpretation that this is the work done and I will leave this is as an exercise and give it in the assignment for you to complete. Needless to say, since the electric field inside and outside is slightly different, the potential is going to have a discontinuity in the slope at the surface.

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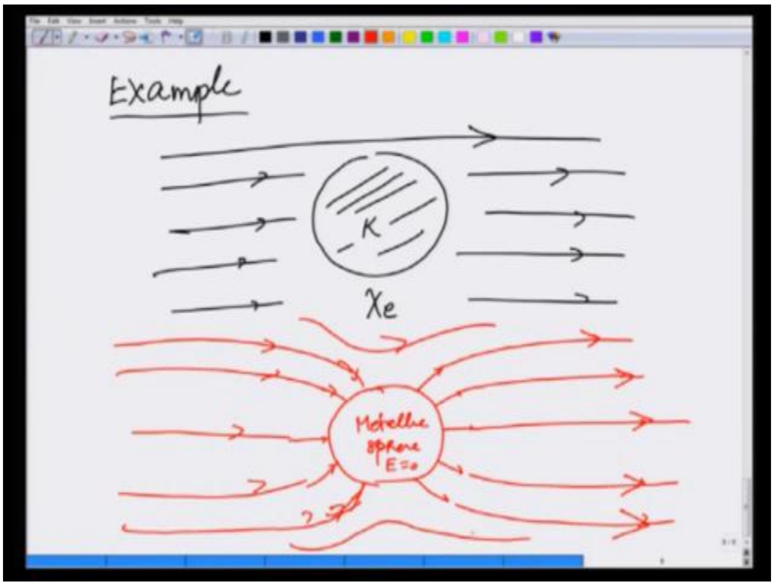

Continuing the examples on electric fields around dielectrics, let us take next example where I put a sphere of dielectric material of constant K or susceptibility chi e in a uniform field. And now I ask what will happen, let us first compare what happens in the case if this was a metallic sphere, suppose this was a metallic sphere. In a metallic sphere, because that is made of a conductor the field inside would be 0.

So, E inside would be 0 and E at the surface would always be perpendicular and far away the effect of this sphere would were of and therefore, I should have the same field as the applied field. So, this is what I would expect in a metallic sphere and field out here would go something like this. What would happen in a dielectric? Dielectric let us the electric fields penetrate a bit. Because, they are though are no free charges only bound charges.

Therefore, although the overall picture may look the same, but the magnitudes are going to be different. In addition, we saw in the boundary condition that E parallel out here is also continuous whereas, there is no E parallel in conducting sphere. So, let us see how do we solve this problem.

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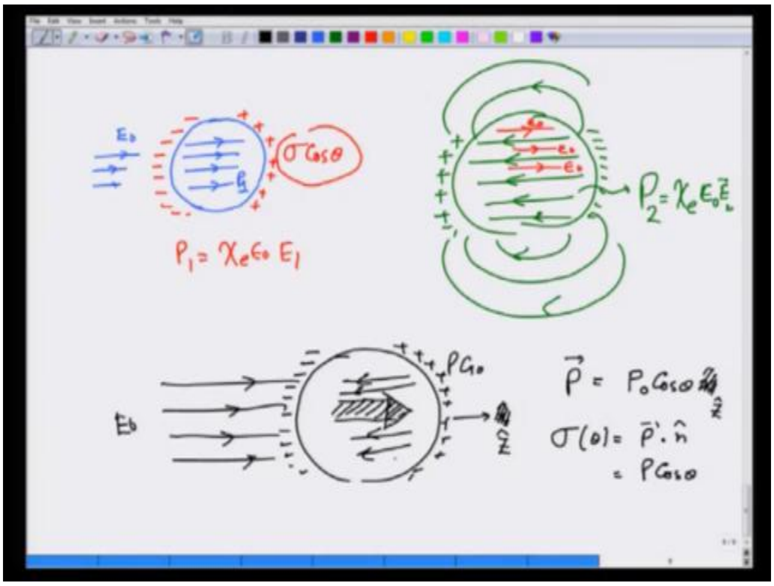

There are several ways we will go the simplest possible way, so let us take this E 0 field and what we expect this E 0 would do is that produce a polarization P. Let us call this P 1, because I am going to work step by step. What would this P 1 do? P 1 would produce surface charge on this side, as well as on this side it will be negative on this side and what kind of distribution I expect, because P is a constant p let us call P 1 equals chi e Epsilon 0 E 1. So, it is a constant p in z direction, I would expect on this side field to be like surface charge to be sigma cosine theta.

And you recall that sigma cosine theta gives me a field which is uniform field inside this sphere and outside it will be like a dipole field. I am right now not so worried about the outside field as field inside and inside remember the applied field is E 0. So, you see that because of this green field which has been given rise to by this sigma cosine theta, field inside will reduce a bit in metals or conductors it becomes 0.

But, here it reduces, what is E, mu E this mu again will give rise to a P 2 which is chi e Epsilon 0 some E 2, which in turn will give me positive charges on the left hand side of this sphere and negative charge on the right hand side and so on it will keep on happening. I could go on solving this problem by doing this iterative method or assume right in the beginning, that an effect of this applied field E 0, this is the applied field E 0 is that, it produces a net polarization P in the same direction in this sphere and this P...

So, it produces a P which is some P 0 cosine theta, let us call this direction x in the x direction for purposes of understanding or easier mathematics. So, it does not make difference physically let us call this z direction, because that will make life simpler. Now, you can see that this will give rise to sigma theta is equal to P dot n which is nothing but P cosine of theta. So, now, I have this positive charge on this side which is P cos theta and corresponding negative charge on the other side and this gives rise to a field inside this all net.

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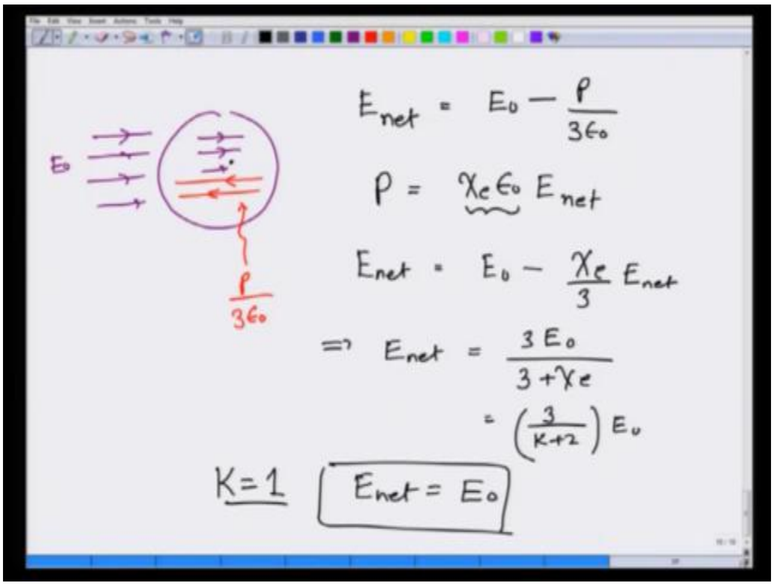

So, what has happened now is that when I put this sphere in this field E 0 there is this E 0 inside and there is a field backwards, which is P over 3 Epsilon 0, this we have worked out already. So, E net inside is going to be E 0 minus P over 3 Epsilon 0, now remember what P is, P is equal to chi e Epsilon 0 E there. So, it is e net whatever the final value of e out here is that multiplied by chi e is Epsilon 0 is the P and therefore, I have e net it is all in the same direction.

So, I am not really putting a vector on top is equal to E 0 minus chi over 3 e net and this gives e net is equal to 3 e 0 divided by 3 plus chi e or 3 over K plus 2 E 0. So, field inside is a still in the same direction, but reduces in the amount, you can easily see if K equals 1; that means, there is no dielectric sphere when E net is same as the E 0 as must be the case.

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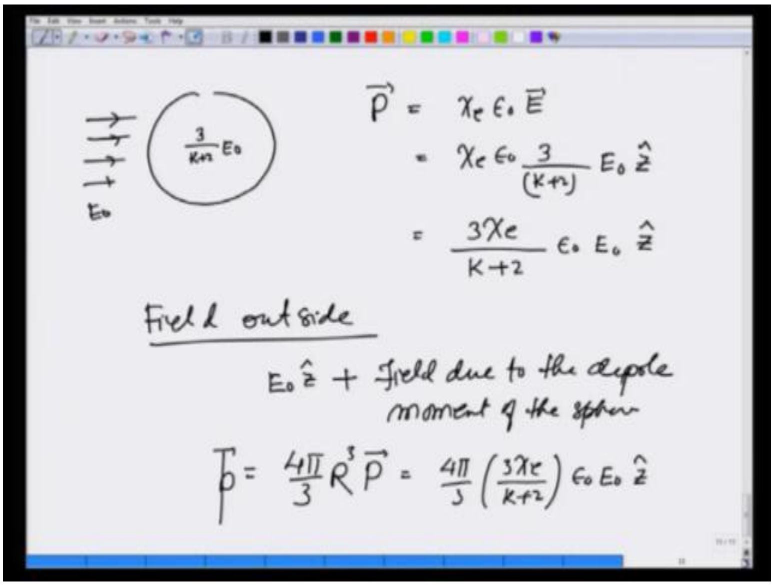

So, what we have found is that in this sphere when I put it in this field E 0 field inside becomes 3 over K plus 2 E 0. What about the polarization inside? Polarization will be equal to chi e Epsilon 0 E at that point and therefore, this is going to be chi e Epsilon 0 e which is 3 over K plus 2 E 0 z 3 chi e over K plus 2 Epsilon 0 E 0 z that is the p or the polarization.

And therefore, the field outside is going to be E 0 z plus a field due to the dipole moment of the sphere. What will be the dipole moment? Dipole moment p will be equal to 4 pi by 3 R cubed times the polarization which is 4 pi by 3, 3 chi e over K plus 2 Epsilon 0 E 0 in the z direction.

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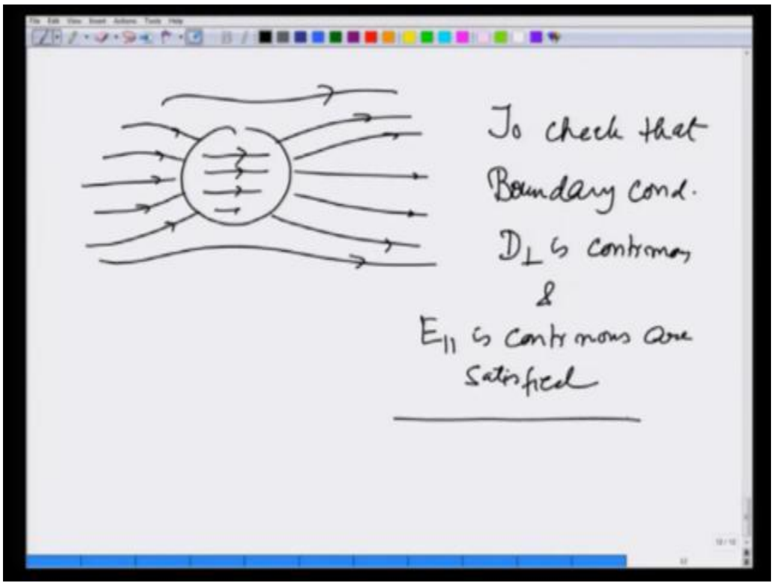

You add that field and final picture that emerges is that this field is weak inside and outside it is going to be a filed like this. Because, this is dipole field is going to be added to the applied field. So, this is one way we have solve the problem I will leave it for you to check that boundary conditions a D perpendicular is continuous and E parallel discontinuous are satisfied.

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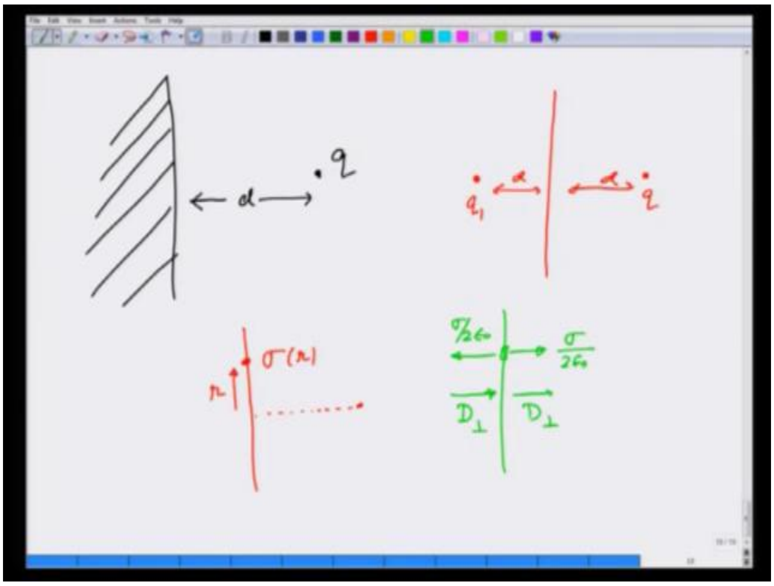

Next example I am going to take in this is going to be suppose I have a dielectric slab and I put a charge Q in front of it at a distance D. What happens to the field inside the dielectric, remember if I had a metallic slab on the left there will be no field inside. But, for a dielectric the field does penetrate inside and we want to calculate how much is this field. A standard way of solving this is that you take this q put a q inside which is q 1 at the same distance d and then calculate the field and satisfy various boundary conditions we are going to do it slightly differently.

What we are going to say is that by symmetry this charge produces a sigma which depends only on r from the line joining the plane and that point. Now, what will this sigma r do in this sigma r out here will give me a field in this direction sigma over 2 Epsilon 0 and in this direction sigma over 2 Epsilon 0. And the boundary condition should be that D inside should be same as D here the D perpendicular to the surface and we will use that to find sigma let us do that.

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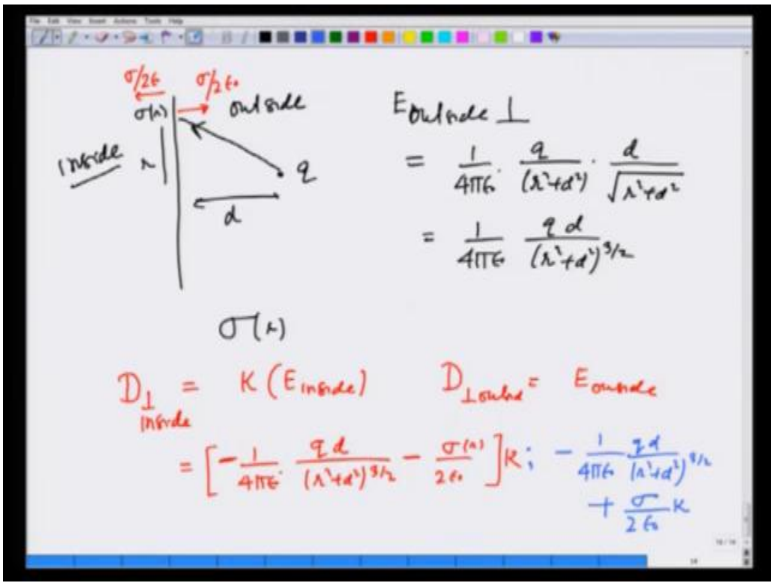

So, if I take this surface this is d, this is q at distance r, this is sigma r, then E perpendicular let us call it outside, on the right hand side inside on the left hand side E outside is going to be this q gives a field in this direction, this perpendicular component is going to be 1 over 4 pi Epsilon 0 q over r square plus d square is the field and it is z component with the perpendicular component is going to be d over r square plus d square which is equal to 1 over 4 pi Epsilon 0 q d over r square plus d square raise to 3 by 2 and E inside and E outside are the same.

Now, sigma r as I said earlier gives me a field sigma over 2 Epsilon 0 going to the right and sigma over 2 Epsilon 0 going to the left. Now, if I apply the condition that D perpendicular is the same, D perpendicular is going to be K times E D perpendicular inside is going to be K times E inside and D perpendicular outside is going to be E outside, because outside K is 1. So, this is going to be equal to 1 over 4 pi Epsilon 0 q d over r square plus d square raise to 3 by 2 in the negative direction.

In the same direction sigma over 2 Epsilon 0 sigma r times K and the right hand side is going to be minus 1 over 4 pi Epsilon 0 q d over r square plus d square raise to 3 by 2 plus sigma over 2 Epsilon 0.

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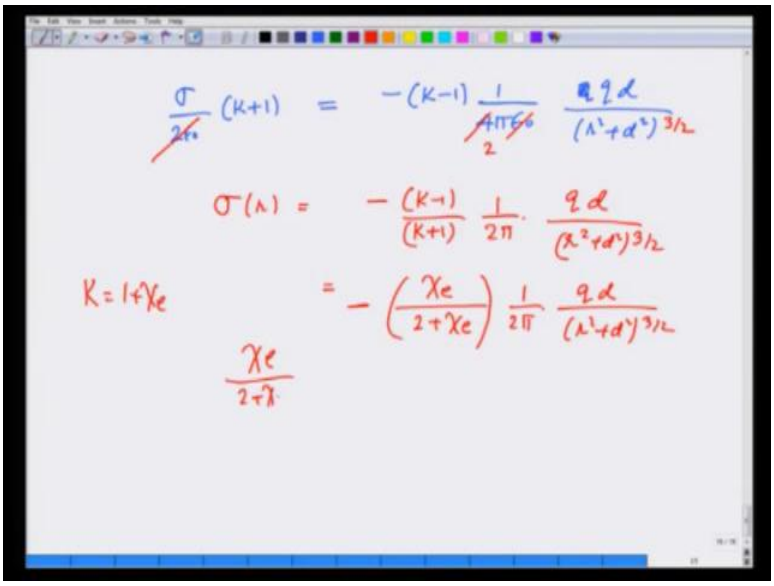

Let us now do our algebra bring this sigma to this side and take this to the other side and you get sigma over 2 Epsilon 0 K plus 1 is equal to minus K minus 1 1 over 4 pi Epsilon 0 q d over r square plus d square. And therefore, now this 2 Epsilon 0 cancels with this here you are left with 2 sigma at r is going to be equal to minus K minus 1 over K plus 1 1 over 2 pi q d over r square plus d square raise to 3 by 2 there is a 3 by 2 here which is nothing but chi e over 2 plus chi e, remember K is 1 plus chi e 1 over 2 pi q d over r square plus d square raise to 3 by 2.

If you recall this is precisely the form of this is precisely the form of image charge distributed over metallic surface, except that now the charge is reduced by this factor chi e over 2 plus chi e. So, one could also have solved this problem using the image charge plot, I will that what the image charge should be behind the dialectic as an assignment problem and send it over the assignment.