Introduction to Electromagnetism
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Lecture - 32
Electrostatics in Presence of Dielectric Materials-I

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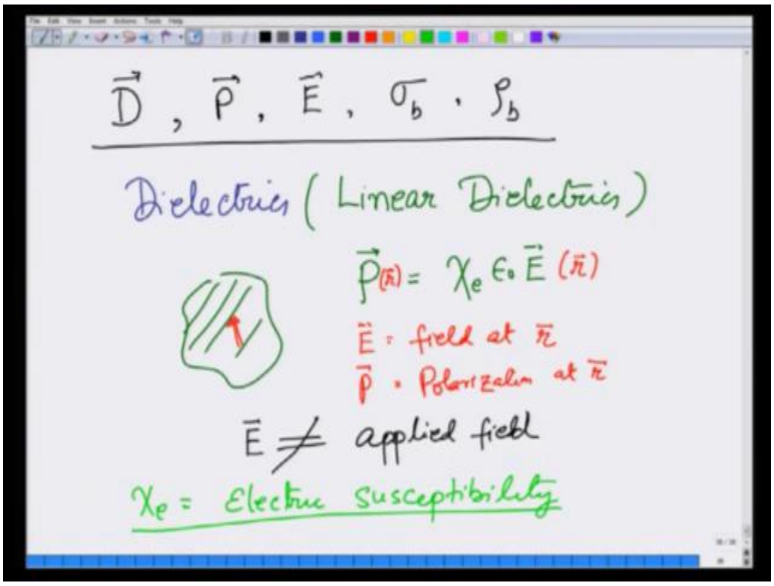

Looking at dielectric materials, we have introduced polarizable materials. We have introduced a quantity – electric displacement; we have talked about polarization; we have talked about electric field; we have talked about bound charges. How do we solve problems using these quantities in electrostatics? We are now going to concentrate on something called the dielectrics; and in particular, linear dielectrics. These are materials, which do not have any polarization by themselves; but if they are put in a medium in an electric field, not in the medium; if they are put in an electric field, then they develop a polarization P, which is equal to… I am defining a quantity chi e epsilon 0 E; where E is the field at that point. So, E is field at r; let me write this explicitly – r P at r.

And P is polarization at r. I want to just sound a note of caution that, E is not equal to the applied field; it is equal to the local E at that point, which will be a combination of the applied field and the field, which is generated due to these induced charges, which come because of the polarization. E is not equal to applied field; it is the field at r. Chi e is known as the electric susceptibility; that is, how susceptible, how responsive the system is to the applied the electric field or to the field locally at that point. Chi images that.

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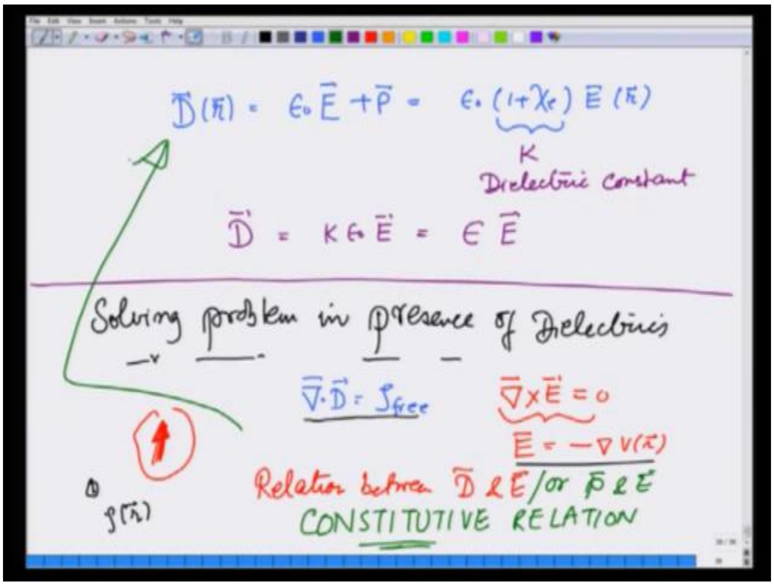

So, displacement therefore, at r, is given as epsilon 0 E plus P; which I can write as epsilon 0 1 plus chi e E at r. This quantity – 1 plus chi e – we are going to call it K or dielectric constant. So, D – dielectric material is going to be equal to K epsilon 0 E. And sometimes, we write this as E – epsilon E; where epsilon is the permittivity of that material. Now, let us see what all do I need to solve in a problem. Solving problem in presence of dielectrics – what do we have? We have equations for D – divergence of D is equal to rho free; that is one equation I have. The other equation I have is curl of E is 0. And this tells me that, I can write E as gradient of a potential. Now, let us look at this dielectric material given somewhere and some charge distribution – rho r. It produces an electric field, so that divergence of D is rho E – minus V r. But, now, I need to relate what the polarization inside this material is. Let me erase this point. What the polarization inside this material is to D or E. And that relation between D and E or P and E is called constitutive relation. And we have already written this relationship for linear dielectrics. This is given for linear dielectrics right here.

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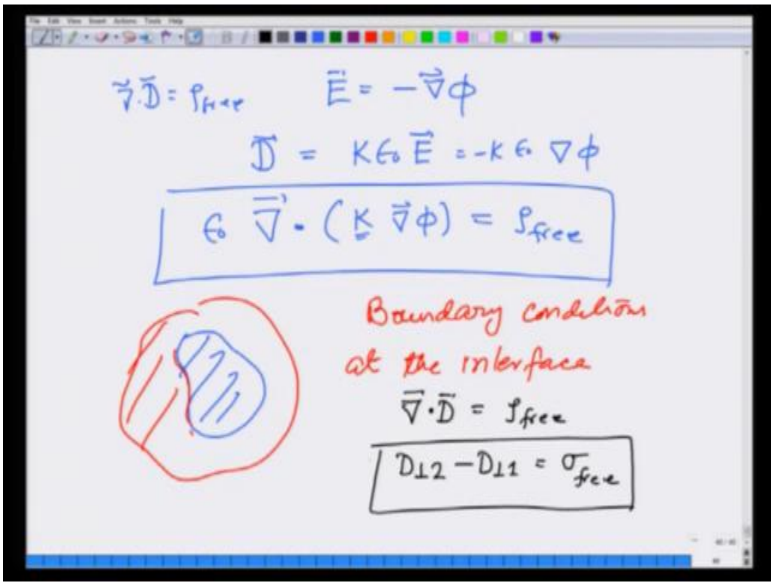

So, what we have is we have divergence of D – rho free. We have E equals minus grad phi. We have D is equal to K epsilon 0 E; which is K epsilon 0 grad of phi with the minus sign. So, if you like, we have divergence of K epsilon 0; I can write outside – grad of phi is equal to rho free. And what I need to do now is this K if you like; if I think of the whole thing as a dielectric medium – one medium here; maybe outside the second medium, K is changing. What do I do at the interface of the boundary. So, third thing I need is boundary conditions at the interface. And these are easy to derive from divergence of D equals rho free. I can write by Gauss's theorem that, D perpendicular in medium 2 minus D perpendicular in medium 1 is equal to sigma free. Let us see that what it means.

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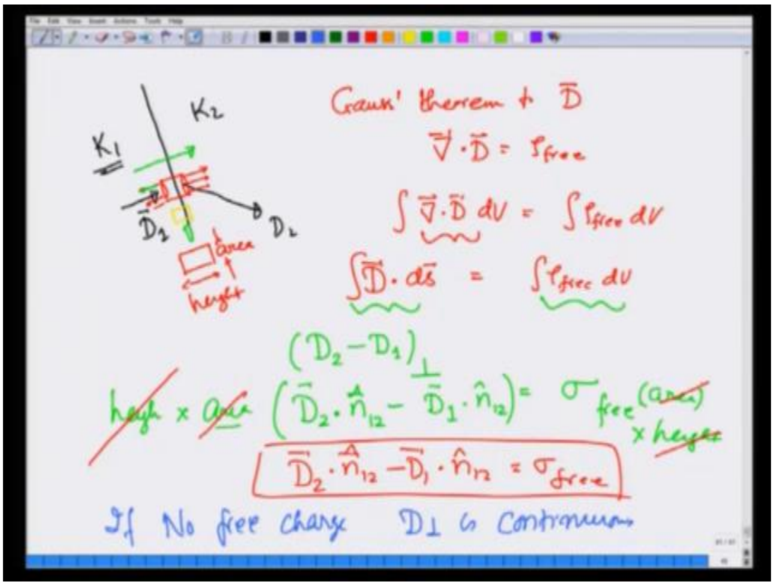

So, suppose there is an interface here; this is medium 1, this is medium 2. This medium could even be vacuumed. And there is a D 1 here; there is a D 2 here. If I take a small ((Refer Slide Time: 07:16)) box here; let us take in normal here like this – normal here like this; and, apply Gauss's thereom to D; which satisfies divergence of D is equal to rho free; which gives me divergence of D – d volume equals rho free d volume. This I can write it as divergence of D dot ds rho free dv. Now, I let the size of this box becomes smaller and smaller. Then this gives me D 2 minus D 1 perpendicular if I take the direction of D to be the same all over. So, in any case, I can write this also as D 2 dot n from 1 to 2 minus D 1 dot n from 1 to 2, because n from 1 to 2 is opposite of this, is equal to… As I make box smaller and smaller, the only charge that will be left here will be sigma free times the area of this box. This will also be area times this box. Let us do it little more carefully; area times the height of the box. I can write the height of the box here. Let me show it here. This is the box; this is the area; and, this is the height. These things cancel. And I get D 2 n from 1 to 2 minus D 1 dot n from 1 to 2 unit vector is equal to sigma free. If no free charge; then D perpendicular is continuous. So, that is one boundary condition.

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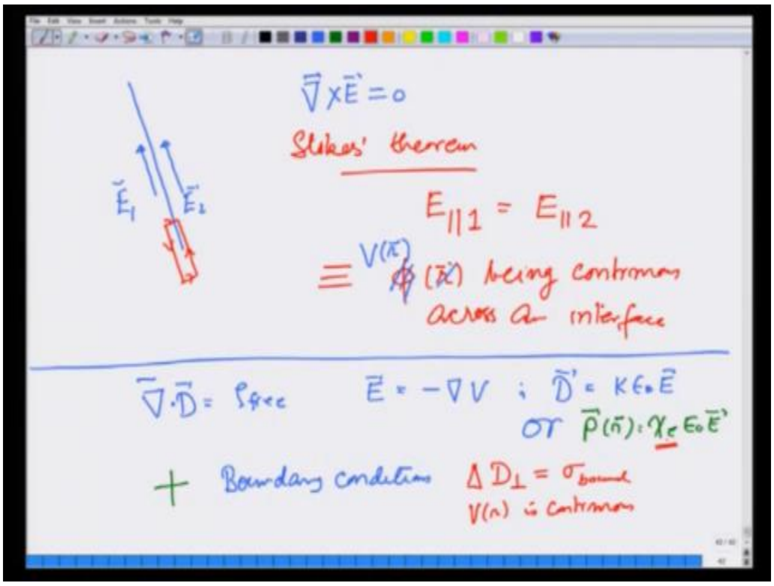

In other boundary condition, is that, again look at this surface; look at E here. E on side 1; E on side 2. The curl of E is 0. And therefore, if I take a loop like this and apply Stokes' such theorem to this, I get E parallel on side 1 is equal to E parallel on side 2, which is equivalent to phi r being continuous across an interface. So, let us summarize all these. What do I have? I have to solve the problems – divergence of D is equal to rho free. I have E equals minus grad of V. I wrote phi for the potential; let this be written as V r. I also have constitutive relationship – D is equal to proper K epsilon 0 E or equivalently the local polarization P of r is equal to chi e epsilon 0 E. And then I add to all these boundary conditions. And the boundary conditions are that, change in D perpendicular across an interface is equal to sigma bound and phi of v r is continuous. With these, we can solve all the problems in linear dielectrics, because I know this constitutive relationship.

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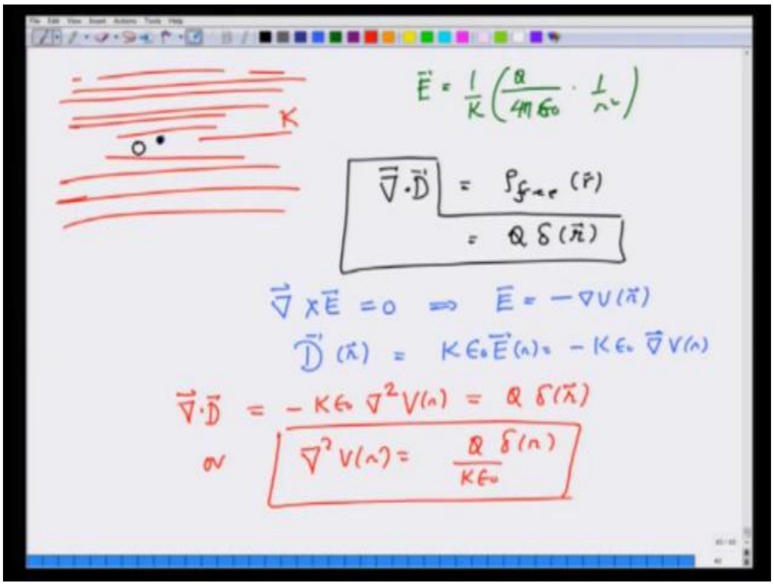

Let us solve one example. A simplest example would be I have a point charge, which is in an infinite dielectric medium of dielectric constant K. You already know the answer from your twelfth grade that, this is electric field is given as Q over 4 pi epsilon 0 1 over r square. It gets reduced by K. Let us now get this answer by applying our mathematical machinery. I have divergence of D is equal to rho free at r. Let us take this point to be the origin. Then I can write this as Q delta of r. That is my equation 1. I also have curl of E is equal to 0 and that gives me E is equal to minus of V r. Now, divergence or linear dielectric is equal to K epsilon 0 E of r. And therefore, this is minus K epsilon 0 grad of V r. This immediately gives me divergence of D is equal to minus K epsilon 0 Laplacian of V of r. And this is equal to Q delta of r; or, divergence of V r is equal to Q delta of r over K epsilon 0. This is the equation that will give me the answer. But, I already know the answer of this.

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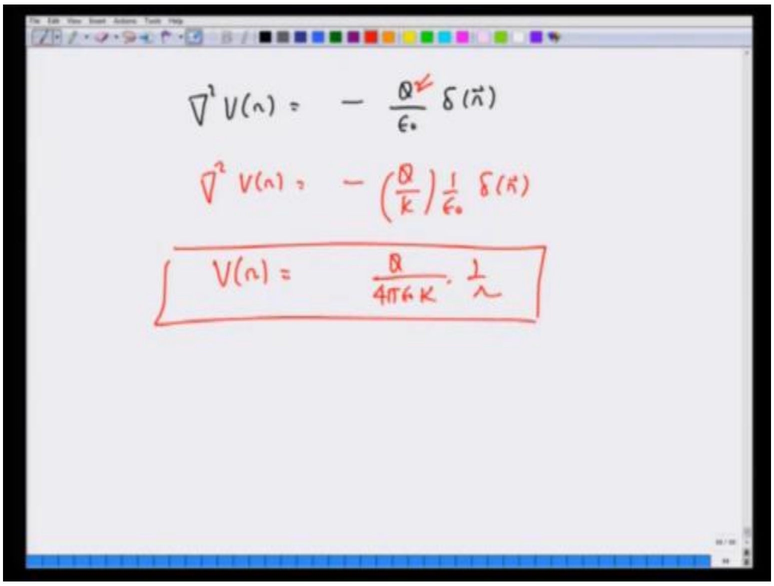

This is equivalent to a point charge equation, which tells me this is equal to minus Q over epsilon 0 delta of r and there is a point charge of each centre. Instead, what equation I have now is del square V r is equal to minus Q over K 1 over epsilon 0 delta r. So, all this tells me that, V r therefore, is going to be 1 over K times the original free charge solution. And therefore, Q over 4 pi epsilon 0 K 1 over r. Rest of the answer is then easy to get. In the next lecture, we will take a few more examples of solving for electrostatic potential and the related field when linear dielectrics are present along with some charge distribution.