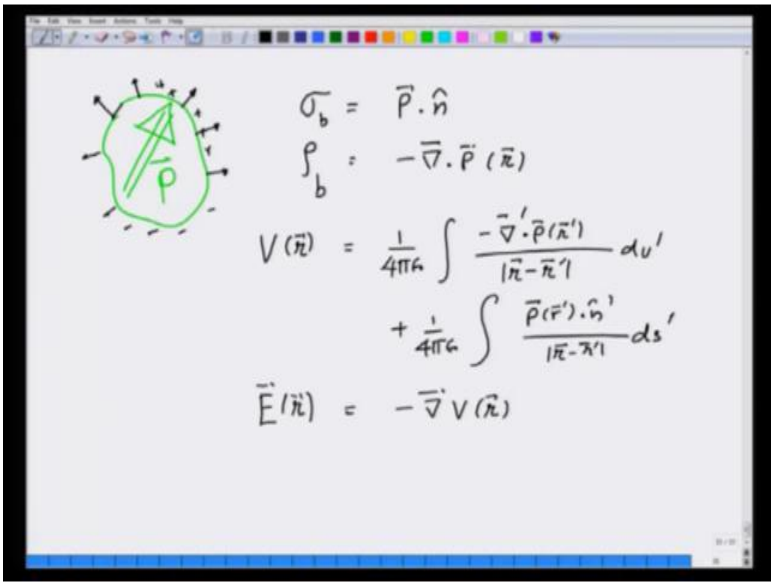
Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 31
Electric Displacement

(Refer Slide Time: 00:12)

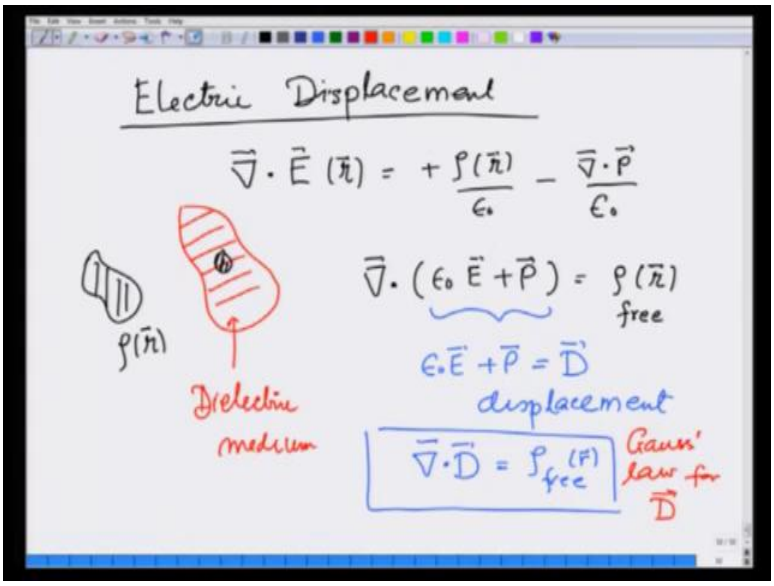


We have seen that, if there is a material that is polarized or carries polarization P. We can think of this as if for electrical problems, there is a sigma, which is P dot n over the surface. So, I can write some plus charges here, minus charges here, where n is the normal going out of the surface or out of the material. And charge density which is minus divergence of P, let me explicitly write a function of r here and this, we call bound charge, because these are bound charges, they are not free to move.

And we what we saw is that, we can calculate the potential at some point or the electric field from these charges. For example, the potential was given as 1 or 4 pi Epsilon 0 and integral minus del prime dot P, r prime, I am writing r prime. Because, r prime I am taking as source point, r minus r prime, d v prime plus 1 over 4 pi Epsilon 0 integration, P r prime dot n prime over r minus r prime d s prime. And the corresponding electric field E will be given as minus grad of V r. So, this is a general strategy.

However the materials that we are going to deal with are going to be polarizable; that is when they are put in external feel, if I apply a field to it or put it in presence of other charges, they are going to create a polarization. And what we would like to know is, develop a general method to calculate electric fields or the effects of these media, these media polarizable medium or presence of charges and the polarizable medium together what kind of fields or potential does it give rise to.

(Refer Slide Time: 02:36)



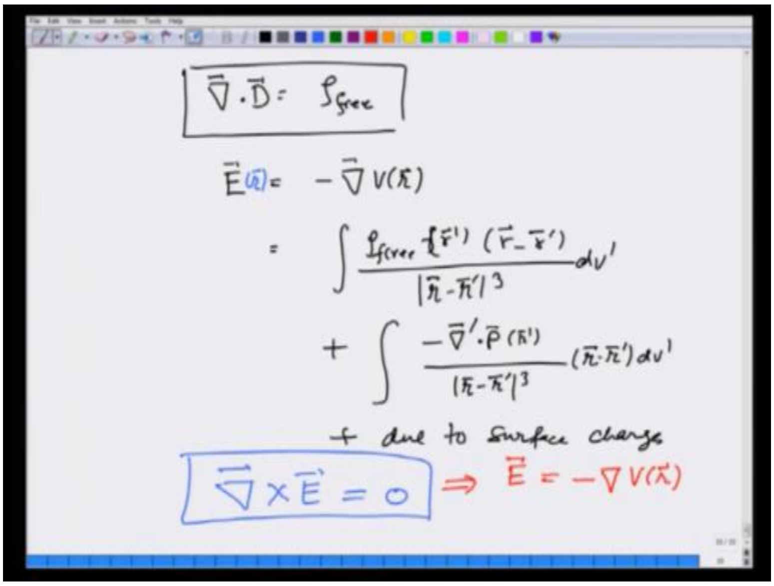
So, in this context, we are going to do introduce something called the electric displacement that facilitates solving of such problems. To understand displacement, let us then write, that if I have electric field E due to the presence of some charge distribution, which I will call rho r and a dielectric medium, which I am showing by red, this is the Dielectric medium. This charge distribution could be inside it; this is different from the bound charges.

Then, del dot E, divergence of E is going to be minus this is rho r upon Epsilon 0. Additionally, there is field due to the bound charges, so that is going to be given as minus divergence of P; that is a bound charge over Epsilon 0. I can rewrite this equation as divergence of Epsilon 0 E plus P is equal to, this is plus rho of r and this to distinguish it from the bound charges arising out of polarization, I am going to call it rho free.

And this quantity in brackets Epsilon 0 E plus P, I am going to call D or displacement and this satisfies the differential equation that divergence of displacement is equal to rho free. If you like this is the Gauss's law for D. Now, what is the advantage of taking displacement, you see charge free is something that I know, what I have supplied or something that we bring in.

So, it is good to have a quantity that is related to that charge, rather a quantity, which is induced. For example, if I bring a medium like this one here and put it in front of an electric field, there is going to be induce charges on this and I do not know, what those induce charges are. And therefore, I would like to develop theory in terms of charges, which I do know and which are rho free. So, we have got one quantity displacement, which is related to the free charge like this.

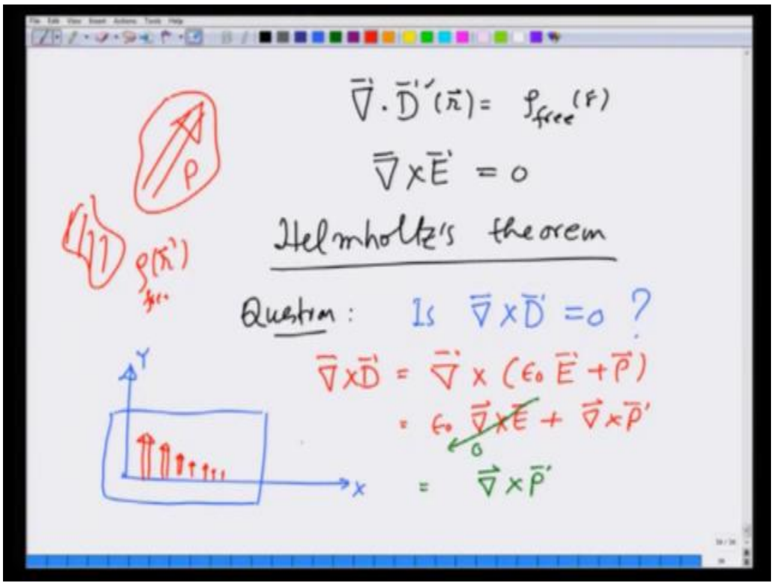
(Refer Slide Time: 05:35)



So, del dot d is equal to rho free. On the other hand, the electric field is still given rise to by charges, whether bound charges or free charges. And therefore, as we saw earlier can be written as minus V r, I could have also explicitly written this as integration rho free at r prime r minus r prime divided by r minus r prime cubed d v prime plus integration minus divergence of P r prime. That is the bound charge divided by r minus r prime cubed r minus r prime d v prime plus due to surface charges.

The point is that it is coming out of certain charges. So, none the less, what we are seeing is that, electric field at r is arising out of these static charges and therefore, curl of E is 0. What that also means, that we can calculate E from a potential V r, which we have already seen.

(Refer Slide Time: 07:16)

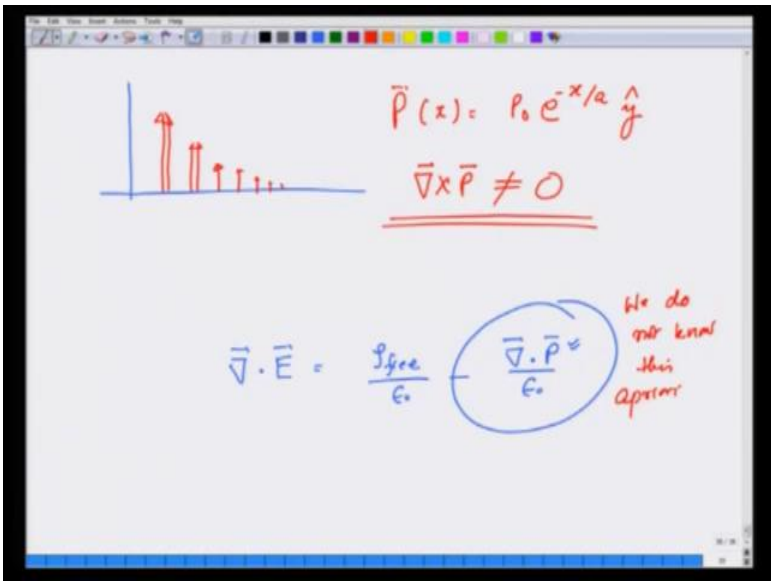


So, I have two quantities, whenever I have a medium with polarization, whenever I have a medium both polarization. So, this is polarization and some free charges rho r prime. I have two equations, one is with the displacement, divergence of displacement is equal to rho free r and curl of E is 0. Now, recall Helmholtz theorem that says that to calculate any quantity, I need it is curl as well as divergence.

Here, I have two quantities D and E, I know divergence of D and curl of E. Let us ask a question is divergence of curl of D is 0. If curl of D was 0 and it was known, it becomes like a problem of calculating electric field. Then, I know the divergence and curl of D, I can write D as a gradient of a potential and do all the calculations the way we did earlier. But, this is not necessarily true, because curl of D will be equal to curl of Epsilon 0 E plus P, which is Epsilon 0, curl of E plus curl of P, this is 0. So, I am left with curl of P.

Look at the picture on the left of your screen, suppose I have a material, let us take this to be Y direction, this to be X direction and I have a polarization, which is in the Y direction and changes with X. So, that P is given as I made a polarization, which is in the Y direction and changes.

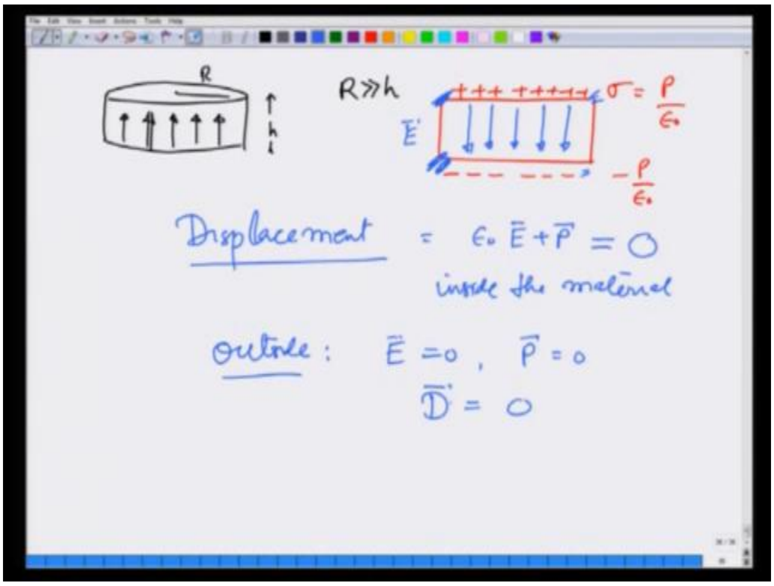
(Refer Slide Time: 09:26)



So, suppose I have P of x is equal to some P naught, E raise to minus x by a, y unit vector. You will immediately see that curl of P is not 0. The point I am trying to make is, curl of P need not be 0. And therefore in general, I can write divergence of E rho free bound charges del dot P over Epsilon 0. But, if there is a polarizable medium, where P itself depends on E, I do not know this.

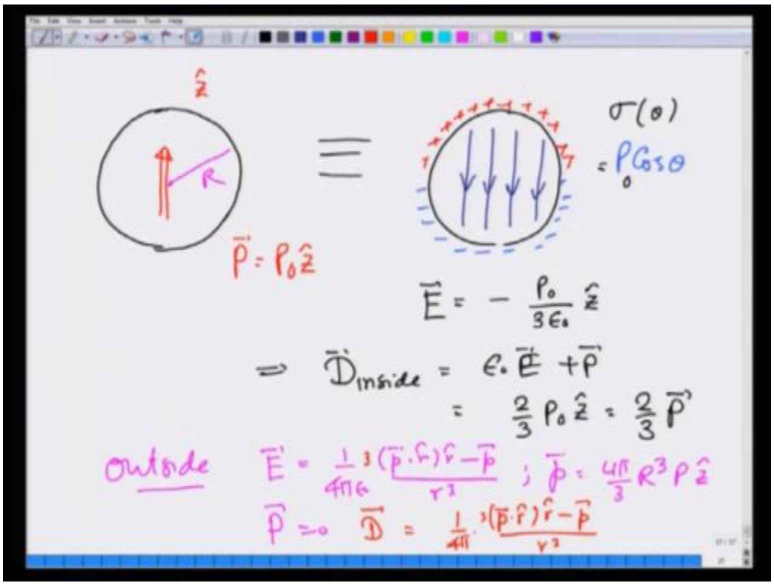
So, let me write this, we do not know this a priory and therefore, now we have to develop techniques to solve problems in presence of a dielectric medium. Before, we do that; let me now do some calculations for electric displacements. So, that you have an idea about, what it is like.

(Refer Slide Time: 10:49)



Remember earlier, we had solved a problem, where I had given you a dielectric cylinder with height H radius R, R much, much, much better than H and P going up, this way. In this case, we showed that this was equivalent to the cylinder with positive charges on top, which sigma equals P over Epsilon 0, negative charges on the bottom with charge being minus P upon 0 and the electric field therefore is in this direction. This is E, if we neglect fringe effects on the side, which I am showing here, if I neglect fringe effects E outside, anyway is 0, P is 0. So, displacement D will also be 0.

(Refer Slide Time: 11:57)



As a second example for displacement, let us take a polarized sphere that we had talked about earlier with polarization being uniform and let us say this Z direction. And what we showed was that this is equivalent to a sphere with positive charge here and negative charge on the lower side with this being given as P cosine of theta and this gave an electric field inside, which is going down. So, if P is P naught z, then sigma theta was equal to P naught cosine theta and the electric field is minus P naught by 3 Epsilon 0 z.

And this implies that displacement inside is going to be Epsilon 0 E plus P, which is 2 3rds P 0 z or 2 3rds P outside E is that of a dipole, where p is 4 pi by 3 R cubed, R is the radius P z and polarization outside is 0. And therefore, D is simply 1 over 4 pi P dot r, there is a 3 here R minus P over r cubed. So, in this lecture, we have introduced a new quantity called electric displacement, which is Epsilon 0 E plus P. It is divergence is equal to three charge in the system and it is curl is not 0 and it is related to and we have calculated some displacement for certain examples.