Introduction to Electromagnetism

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Lecture - 30
Electric polarization and bound charges-II

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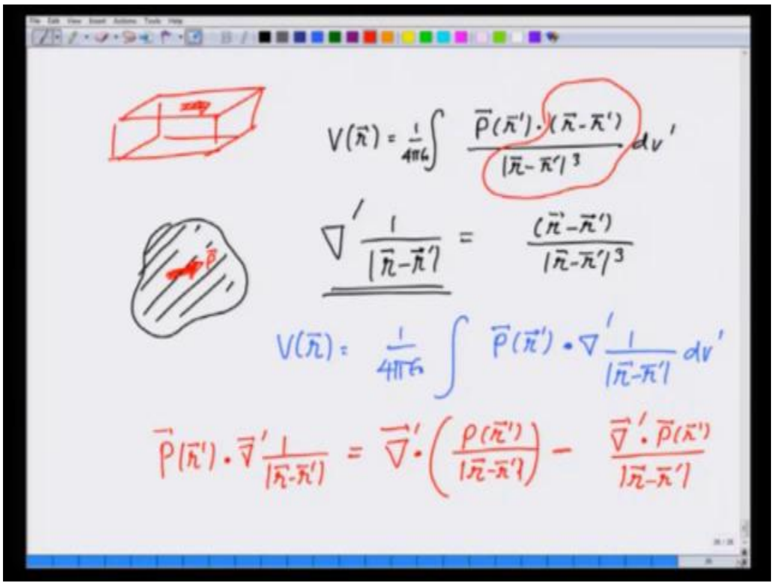

We were looking at in the previous lecture a medium or material, that has some polarization density, and therefore a net dipole moment. And we looked at physically and said that gives rise to surface charge as well as bulk charge. Exactly what this charges are, what do they produce, is what we want to see in this lecture. Finally, the effect that we want to see due to these charges; the field they produce or the potential they produce. So our part of learning about, what kind of charges give rise to, is going to be through this quantities only, because this are quantity we are interested in. So, let us take some material which is polarized, and it carries polarization p. Then the potential we saw in the previous lecture at r, due to this polarization, is going to be given as 1 over 4 pi epsilon 0 p at r prime dot r minus r prime over r minus r prime cube d v prime. D v prime means I am integrating over the volume over which this charges is given.

Now to understand this, let me use this trick. If I take gradient, with respect to the prime variable of r minus r prime, this you can easily show. I will leave this an excise for you, come out to be r minus r prime divided by r minus r prime cubed. And therefore, I can replace this quantity here in circle by red, by this quantity here, and write v r as 1 over 4 pi epsilon 0 integration p r prime dot gradient with respect to this prime variable 1 over r minus r prime d v prime. Now let us do some manipulations; p r prime.dot gradient of 1 over r minus r prime. I am going to write as, divergence of p r prime divided by r minus r prime minus. This is divergences to primed variable. Primed, divergence of p r prime divided by divided by r minus r prime. This is a vector identity in 1 d it is very easy to see, in 3 d also it is very easy to see, and you can just work it out.

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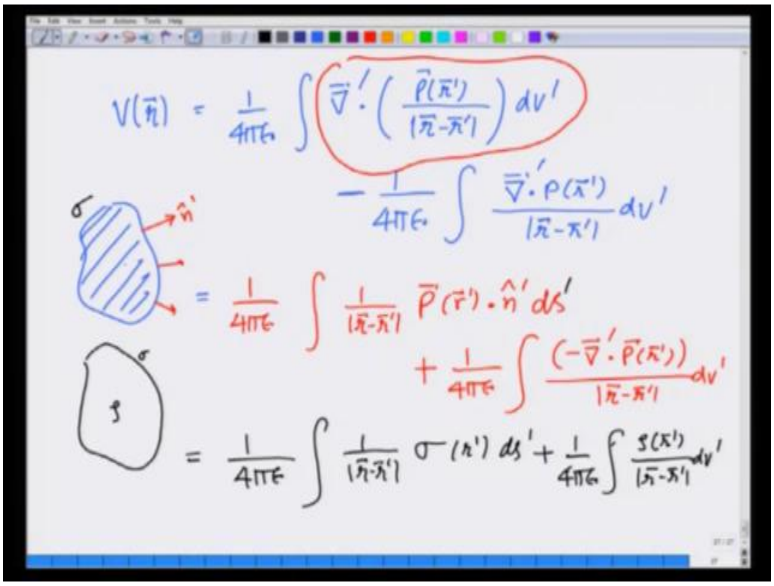

And therefore, I get the potential v r to be 1 over 4 pi epsilon 0 integration to divergence with respect to prime variable p r prime divided by r minus r prime d v prime minus 1 over 4 pi epsilon 0 integration divergence with respect to prime variable of r prime divided by r minus r prime d v prime. Remember we are looking at this polarize material. I can use divergence theorem for this first term, and write this as 1 over 4 pi epsilon 0, integration 1 over r minus r prime p r prime dot n prime; n prime is this unit vector out of the volume, which is the direction of the surface element d s plus 1 over 4 pi epsilon 0 integration. I will take the minus sign inside minus dell prime dot p r prime divergence of population density divide by r minus r prime d v prime.

Let me write this in slightly different form and you see the, what this quantities mean, 4 pi epsilon 0. If there was a surface charge sigma on a similar volume, if there was a surface charge sigma, and bulk charge row and sigma. I will write the charge density as 1 over r minus r prime sigma r prime integrated over the surface plus 1 over 4 pi epsilon 0 integration row r prime over r minus r prime d v prime d s prime. What does this tells you. This tells you that, as for as electrostatic properties, electrostatic potential, or electric field are concerned this charge density sigma is equivalent to this quantity, and the bulk density row is equivalent to this quantity here, the divergence of p. So, I am going to interpret in terms of, as far as electricity or electrostatic properties are concerned

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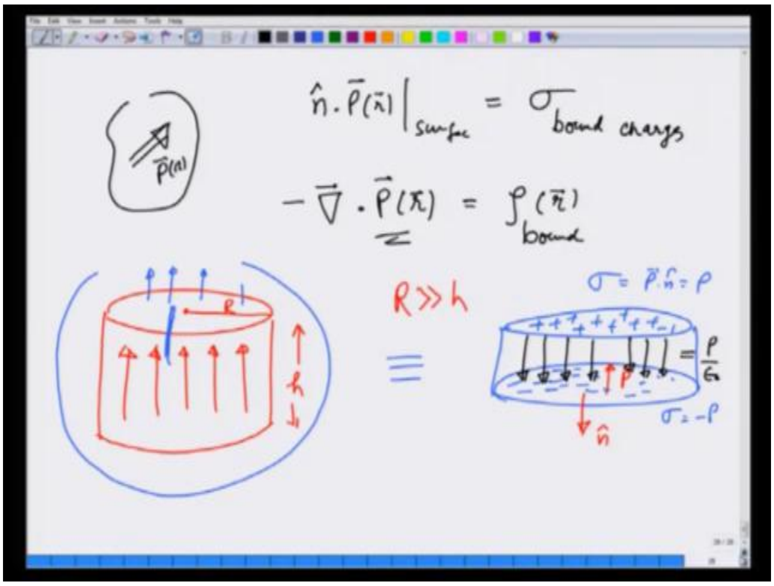

That given a bulk material, with some charge polarization density p r n dot p at the surface, is like a surface charge density. And since these charges due to polarization or not free to move, I am going to call this, bound charges. And similarly the divergence of p r, I do not have to have prime here, is equivalent to bulk charge density row r. Again these are not free charges. I am going to call this bound. You see this is consistent with the physical picture we developed earlier, that the polarization gave rise to a surface charge, as well as bond charge, and you can see divergence of actually how p varies in space, if it varies then gives rise to bound charge. If p was a constant there will be no bound bulk charge, but there is still be surface charge.

So, as the electrostatic potential is concerned, p is equivalent to bound charge surface, as well as bulk. And therefore, the electric field will also be, which is very inter potential will also be equivalent to as if being rise to this bound charges. Let us look at some example; suppose I have a cylinder of height h radius r, such that r is much greater than

h. So, it is like a very thin disc, and it carries polarize, a constant polarization p. Then as far as electrostatic potential or field is concerned, this is going to be equivalent to a surface on a bound charge. Since p is a constant, there's no bound charge inside. However on the upper side there is n coming out. So, p dot n will give me positive charge over the upper surface, with sigma being sincere p and r the same direction, sigma b is equal to p dot n, which is p. And the lower surface will give negative charge, because here n in this direction, and p is in this direction. So, it will give me negative charge minus sigma equals minus p. So, this, thing shown in red out here, is equivalent to two large plates with plus and minus surface charges, and therefore, the electric field inside, is going to be, in the direction opposite of p, the magnitude being p over epsilon 0; that is example one.

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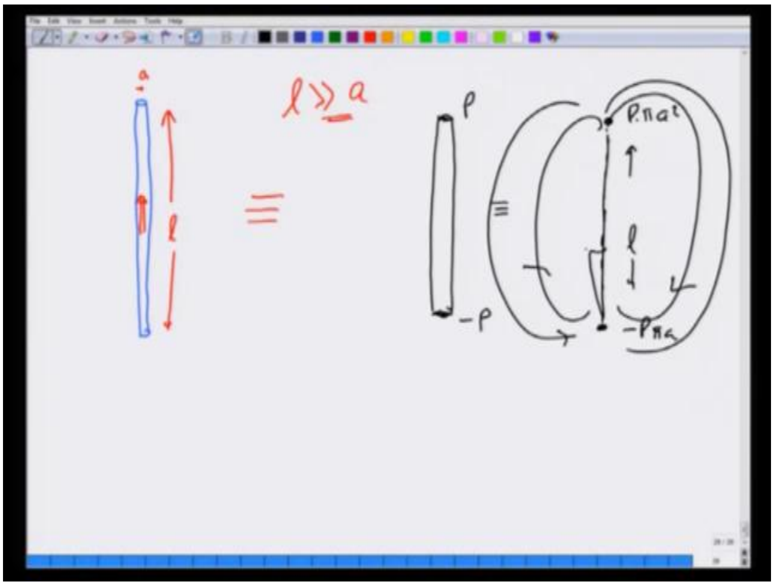

Let us take example two; suppose instead I have a thin rod, again with p being given inside, with this length being much larger than the radius a here. Then I can consider this equivalent to against p, is a constant, there is going to be no charge inside ,no bulk charge inside, but the upper and lower ends again give me p as surface charge density. And since this is very a small compare, the area on the sides are very small compared to the length, I can consider this to be equivalent to two charges; p times pi a square, separated by a distance pi a square l, and the field lines are going to be look like this, in this approximation.

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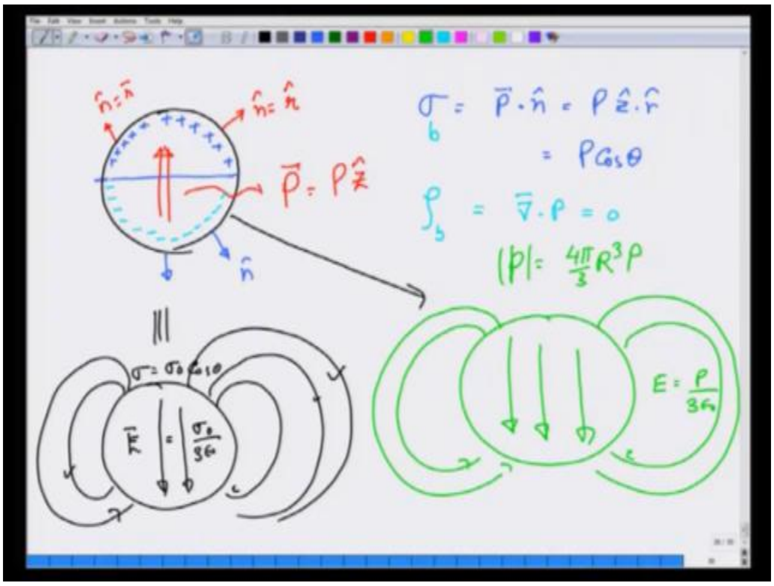

As at third example, let us take a sphere, which carries a constant polarization. For a sphere, the n is same as unit vector r in spherical coordinate; n is same as unit vector r, and polarization p is p z. Then the surface charge sigma, is going to be p dot n which is p z dot r. And if you call from the lecture on this spherical coordinates, this is p cosine of theta. On this side n is like this lower side. So, what you see is that, sigma is positive as far as cosine theta is positive, it goes smaller and smaller as you go towards cosine theta equals 0, becomes precisely zero on the equator, and then becomes negative on the lower side, with the dependence p cosine theta. How about the bulk is bound, bound row which is divergence p which is 0, because p is a constant.

Now this is equivalent if you recall, equivalent to a problem we solved earlier in which I had sigma equal sigma 0 cosine theta, and what did that give me, that give me an e field in the opposite direction, whose magnitude was sigma 0 over 3 epsilon 0, and outside it becomes the field of two, slightly separated charged sphere positive and negative spheres, and become like a dipole filed which I did not do and solve that example, but now you can see, that the way we derived it, it was two opposite we charge spheres slightly shifted which for outside sphere this becomes a dipole. So, this, will also give me the previously the same field. It will give me a constant e inside, which is equal to p over 3 epsilon 0, and outside will give me a dipole field, of a dipole p, dipole movement p, which is 4 pi by 3 r cubed b. So, what we have seen in this lecture is, how I can think of the polarization as a charge distribution, as divergence give me the bulk charge distribution, bulk charge density, and stock product with n, where n is the normal going out of the dielectric gives me the surface charge density, and form that I can calculate potential and field.