Introduction to Electromagnetism
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Lecture - 03
Force Due to Distribution of Charges

In the previous class, we calculated Force on a Charge Due to a Collection of Charges and we use principle of superposition.

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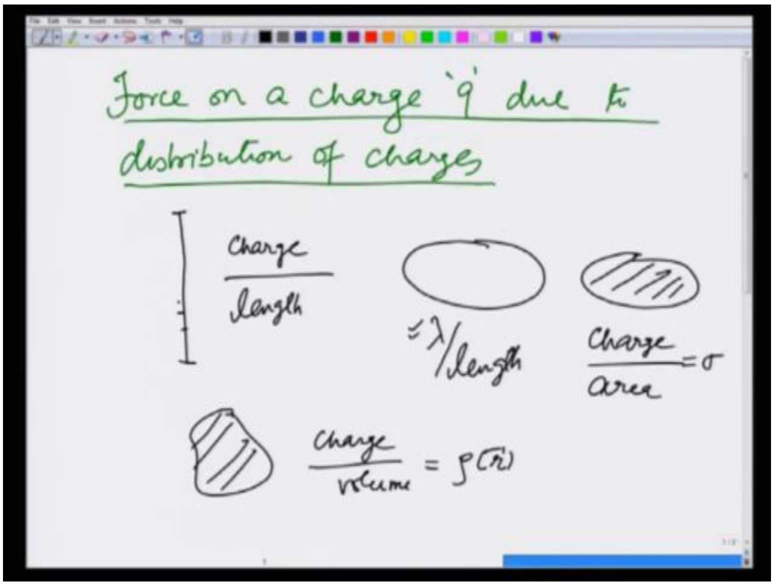

In this class, we will calculate force on a charge q due to distribution of charges. By distribution I mean it could be a charge distributed over a line, and this I will say charge per unit length will give me the distribution that charge could be dependent on which position and moreover it could independent. There could also be charge say on a ring or on a desk, here again I will say charge per unit length which is usually denoted by lambda or charge per unit area or charge in a volume, which will be charge density charge per unit volume. This is known as the charge density, this is known as surface charge density, this is linear charge density, so let us consider these cases.

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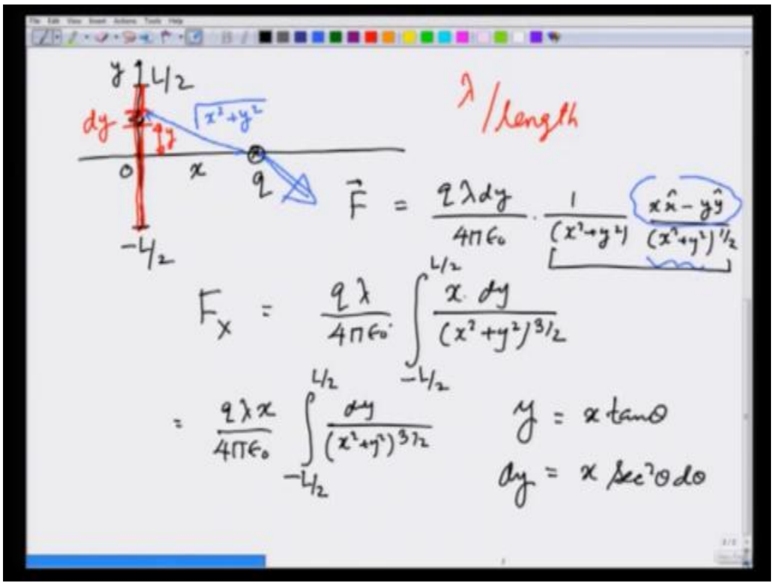

To start with, let us take a line charge extending from minus L by 2 to L by 2. So, that origin it is center is at the origin, let this direction be y and I want to calculate force on a charge q kept at a distance x from the origin. The line charge I will denote by different color, so let me write denote it by red, this is a line charge and let us magnitude be lambda per unit length.

Then, as we did in the case of collection of charges, let me take a small element of length d y at this point at a distance y from the origin and calculate the x and y component of the forces, of the force on charge q. In this case, the distance from this element to the charge is going to be square root of x square plus y square.

So, that by coulomb's law the force on charge q F vector is going to be equal to lambda d y is the charge in this element here, q over 4 pi Epsilon 0 1 over x square plus y square in the direction of x minus y y over x square plus y square raise to 1 half, where this vector that I have written here, you can check is this vector and that divided by the magnitude gives me the unit vector in that direction, this indeed is the force direction.

So, F x is given by the x component which is q lambda over 4 pi Epsilon 0 x d y over x square plus y square raise to 3 by 2, 3 by 2 comes by combining these two terms and this is integrated from minus L by 2 to L by 2, which will give me q lambda over 4 pi Epsilon 0 x will remain there. I am left with the integral to L by 2 d y over x square plus y square is to 3 by 2. You substitute y equals x tangent theta, so that d y becomes x secant square theta d theta.

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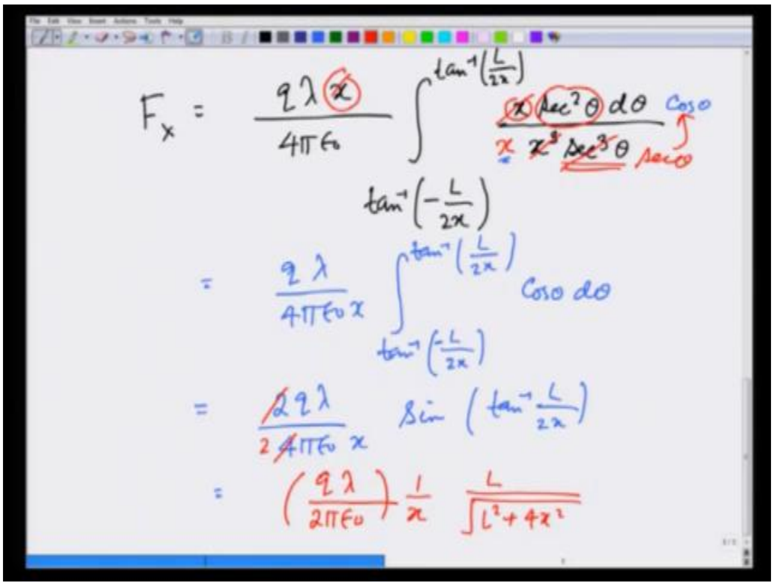

And substitute this in the integral and you are going to get F x is equal to q lambda x over 4 pi Epsilon 0 tan inverse minus L by 2 x to theta equals tan inverse L over 2 x, x secant square theta d theta divided by x cubed secant cube theta. I am going to cancel some terms, this x and this x cancels with this and gives me x only, this secant square theta divided by sec cube theta gives me secant theta which becomes cosine theta on top.

And therefore, this integral becomes q lambda over 4 pi Epsilon 0 tan inverse L over 2 x with the minus sign in front to tan inverse L over 2 x cos theta d theta and this immediately gives you answer which is 2 q lambda over 4 pi Epsilon 0, there was this x here also. So, this x here sin of tan inverse L over 2 x which actually becomes again I cancel this 2 with this and give you 2 to get 2 q lambda over 2 pi Epsilon 0 1 over x sin of tan inverse L over 2 x is going to be L divided by square root of L square plus 4 x square, that is the answer for F x.

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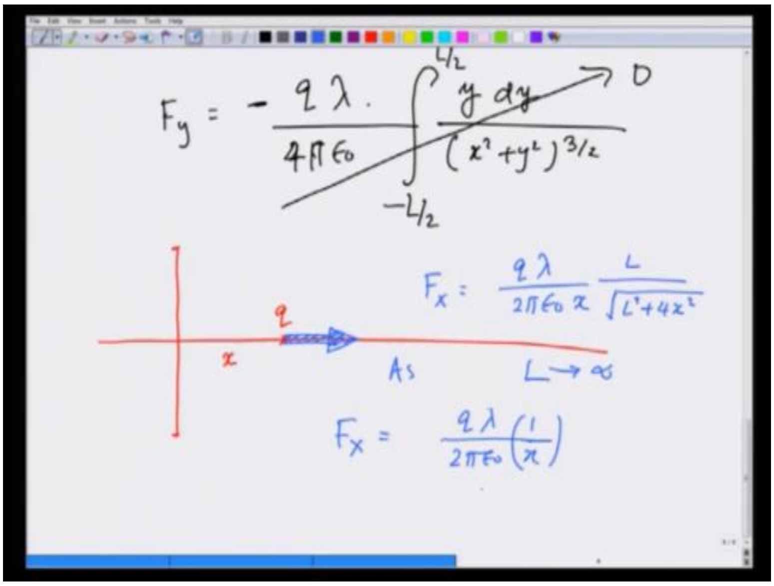

How about the y component? The y component is going to be F y equals q lambda, you can check that I am writing the expression from the previous slide 4 pi Epsilon 0 instead of x. Because, now I will be taking a component in the y direction, so I will have minus y d y over x square plus y square raise to 3 by 2 and this is going to be from minus L by 2 to L by 2, you can see this is an odd integral in y and therefore, this thing vanishes.

So, on the line passing through the middle of a line charge at a distance x from it, the force on charge q is in direction x only and it is value is F x equals q lambda over 2 pi Epsilon 0 x L over square root of L square plus 4 x square. As L tends to infinity; that means, the line becomes of infinite length, you can see that F x goes to q lambda over 2 pi Epsilon 0 l over x, a well known result.

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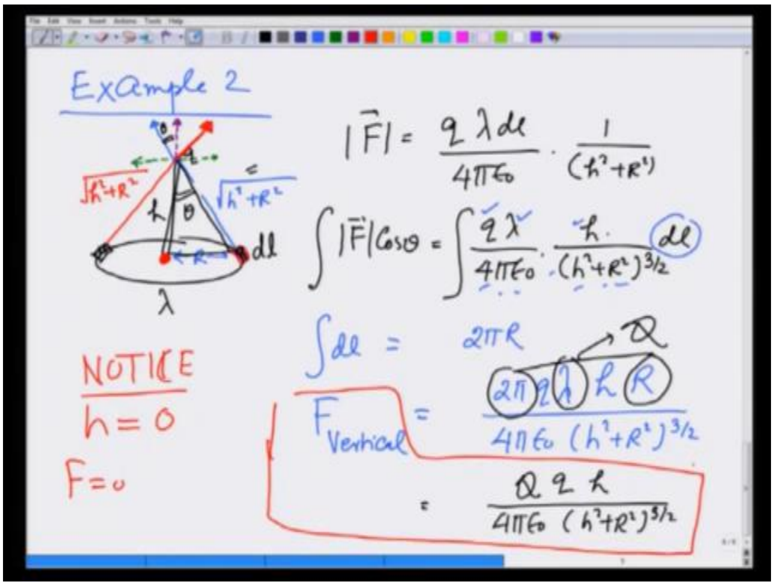

Let us take example 2, for the second example I am going to take let me write example 2. I am going to take a ring of charge, again having lambda charge per unit length and calculate force on a charge q kept at a distance h from the ring. Now, you see each element of this line charge is going to apply a force on this, let us take two particular elements on the opposite sides and I am doing it for a purpose, if I look at the force due to this element, it is going to be pointing this way.

And if I take the force due to this element is going to going to point this way, all the distances, this distance is going to be suppose this radius is R, this distance is going to be h square plus R square. Similarly, this distance is going to be h square plus R square, so the magnitudes of the forces of the same and by symmetry you can see, that they are horizontal components are going to be in the opposite directions and they are going to cancel and it is only the vertical component that will survive.

So, what I can do is, for each element I can take the vertical component add it up, let us see what the vertical component is going to be. So, the vertical component the force due to this small element of length d l is going to be on charge q, due to this charge lambda d l is going to be 4 pi Epsilon 0 force magnitude times 1 over h square plus r square. Now, I take it is vertical component, so if this let me erase this height now, now that it is clear what it is, if this angle is theta, so is this angle.

And therefore, vertical component is going to be F cosine of theta, which by this triangle you can see it is nothing but h divided by this distance. So, this is going to be q lambda over 4 pi Epsilon 0 h divided by square root of h square plus r square which will give me h square plus R square raise to 3 by 2 and there is a d l. Now, if I integrate this force that gives me the total force, this integration all the quantities q lambda, thus all these are fixed. So, only integration comes over d l and integration d l is really the length of the ring which is 2 pi R.

And therefore, the net force answer comes out to be only in the vertical direction is going to be equal to 2 pi q lambda h R over 4 pi Epsilon 0 h square plus R square raise to 3 by 2. You can actually see that 2 pi, R is the length times lambda, it is going to the net charge on the ring. So, I can also write this as net charge on the ring times Q times h divided by 4 pi Epsilon 0 h square plus R square raise to 3 by 2.

Notice, I am writing it on the left of the screen that has h tends to 0 or it becomes 0, force is 0 which makes sense, because the charge is right here, all the forces from all around pushing are pushing the charge by equal amount. So, this is the answer. I can now use this result to calculate force on charge q on the axis of a disc and that is going to give my example number 3.

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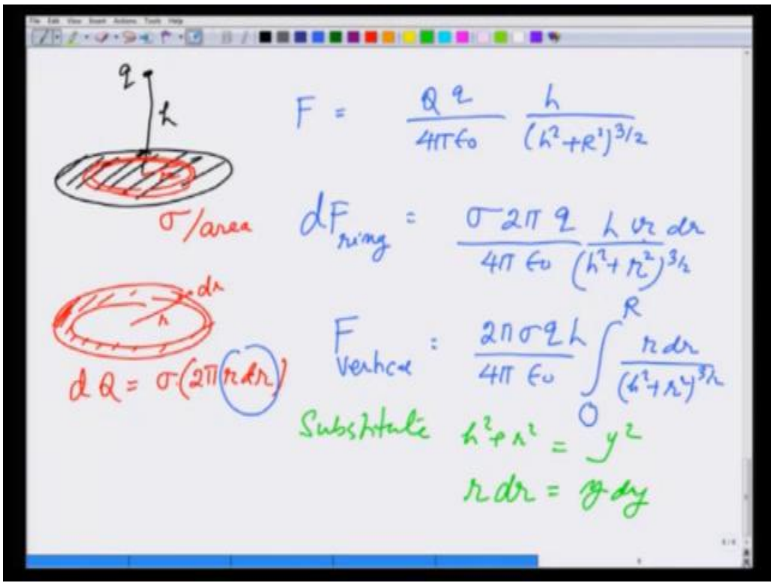

In example number 3, I am going to take this disc and put a charge at height h on the axis, I am going to use the result of previous example. In the previous example, what we saw is that the force on the charge is in the vertical direction, it is magnitude is given by Q q over 4 pi epsilon 0 h over h square plus R square raise to 3 by 2. What I am going to do now is, divide this entire disc into small rings.

So, let us take a ring of radius R, so I am taking a ring of radius R width d r and if the disc carries charge sigma per unit area, then on this ring the charge Q is going to be or let us say small differential charge d Q is going to be sigma times 2 pi r d r, 2 pi r d r is the area of this ring. And therefore, the force due to this small ring or let us write differential force is going to be d Q which is sigma times 2 pi Q over 4 pi Epsilon 0 h over h square plus now I am going to write small r square raise to 3 by 2 and there is a r d r which is this term which I have written out here to facilitate integration.

So, therefore, the net force which is again going to be vertical will be 2 pi sigma q over 4 pi Epsilon 0 h integration r varies from 0 to capital R r d r over h square plus r square raise to 3 by 2, and now if I make the substitution h square plus r square equals let us say y square.

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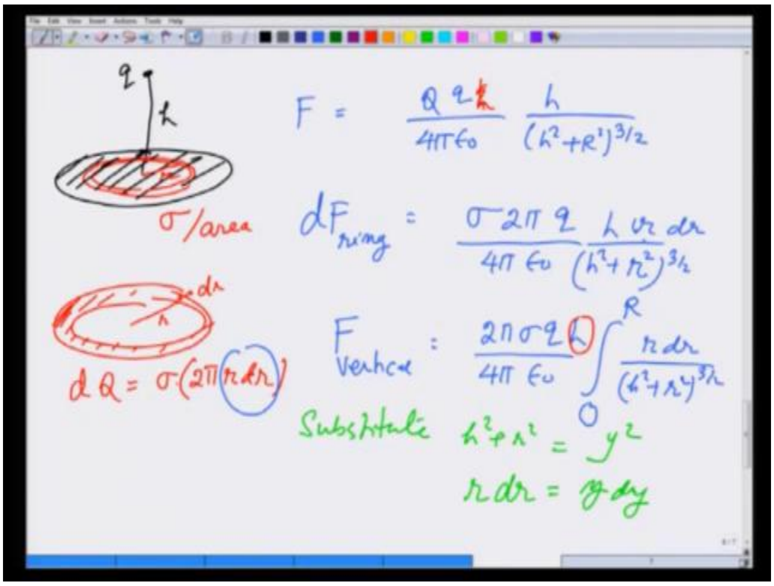

So, that r d r is y d y I get the vertical force to be 2 pi sigma q over 4 pi Epsilon 0 integration this integration is now going to be from h to square root of h square plus R square y d y over y cubed this gives me y square and I get to 2 pi sigma q over 4 pi Epsilon 0 1 over y with a minus sign h square root of h square plus R square and that answer comes out to be, let us cut this also two sigma q over 2 Epsilon 0 1 over h minus

1 over h square plus R square square root and this is going to be the vertical direction. Notice, if R tends to infinity this force goes to sigma q over 2 pi Epsilon 0, if I go to the previous page there was not this h also which I missed. So, there is going to be 1 h out here.

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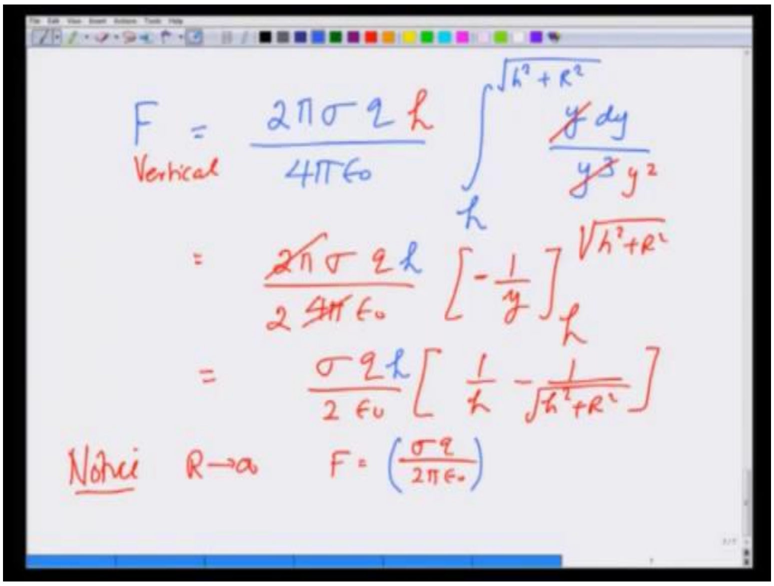

Next page, there is going to be an h out here, let me show it by different color mistake the mistake I made h out here, h out here. And therefore, the force becomes sigma q upon 2 pi Epsilon 0 and that is the answer. So, I have solved three examples simple examples of forces due to charge distribution.