Introduction to Electromagnetism
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Lecture - 28
Reciprocity theorem for conductors – II An example

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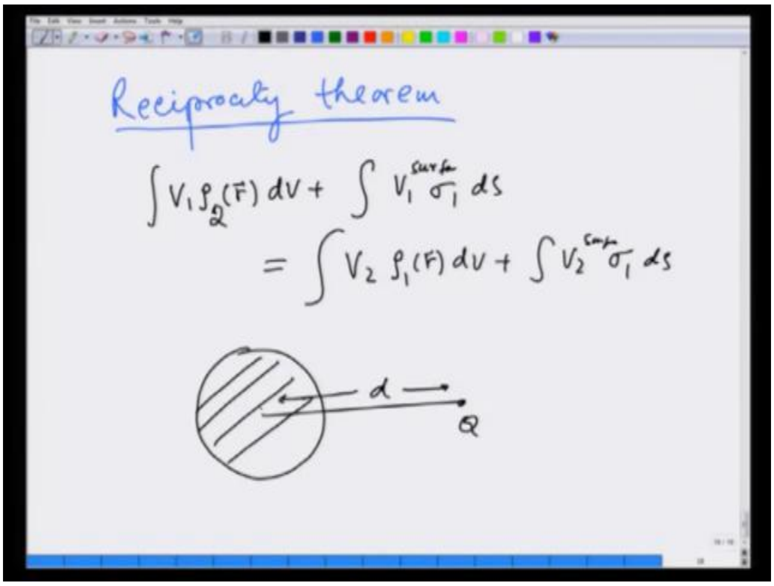

We have been talking about Reciprocity theorem in the previous lecture and solved one example using Reciprocity theorem. Situation that I deal with the Reciprocity theorem is, where I know the answer in one case and I want to know the answer in the other charge distribution. The theorem says that, given the same geometry of conductors, a charge distribution rho 1, which gives rise to potential V 1 d V plus surface term V 1 surface, sigma 1 d s is same as integration V 2 rho 1 r d V plus V 2 surface sigma 1 d s.

So, it relates the charge distribution or potential one situation to the other. The example we solved in the previous lecture was taken a conducting sphere and put charge Q at a distance d from it is center. In this lecture, I will take another example which we talked about part of it will talked about earlier is that.

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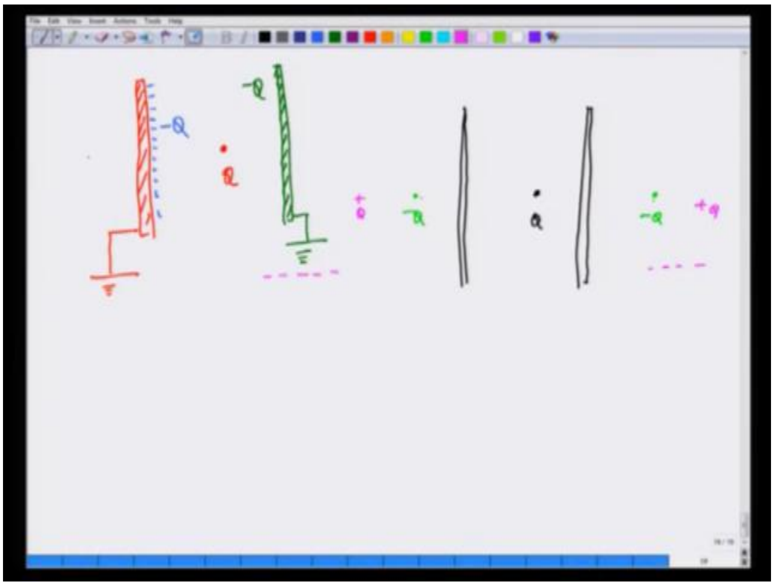

If I take a conducting surface, infinitely large conducting surface grounded it and put a charge Q in front of it. We solved this early using the image method and found out that the charge induced on the surface becomes exactly minus Q and the problem can be solved by putting a charge minus Q in the back side. So, that the potential on the conductor becomes 0.

Now, we ask the question, explain the problem and say, what if I put another conducting surface, infinitely large in front of it, does it also induce minus Q and this is also grounded or there is something else. Problem is little difficult, because what happens now if I make a situation again, let see this, I have this conductor, I have a charge here Q, I have this conducting plate again here. This charge will form minus Q on the backside and minus Q here, these minus Q is again will form image plus Q plus Q and so on.

And therefore, this becomes a non-terminating series, I cannot possibly submit or use it the method of images to get the charge induced on the two surfaces and this is where method Reciprocity theorem will coming very handy. So, we create two situations, one which is the real situation, the one which for which I want to get the answer and the other, where I know the answer.

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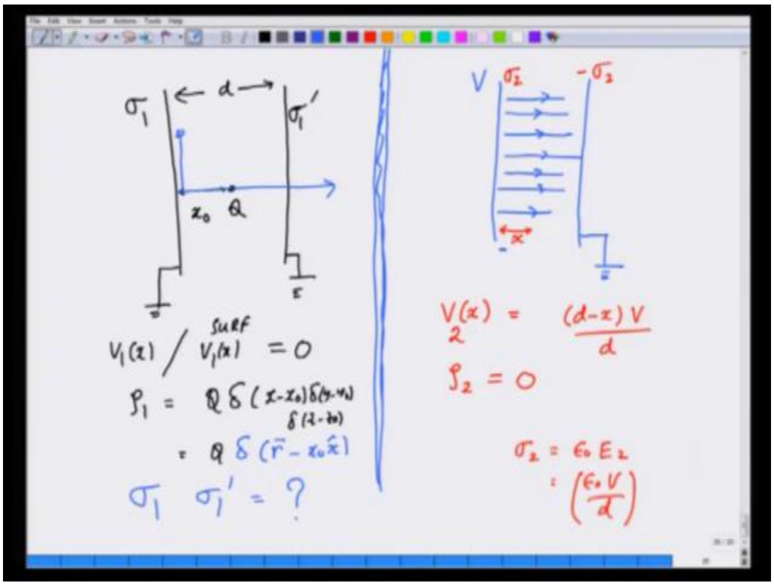

So, let us take this conducting surface one, which is grounded conducting surface two, which is also grounded, take the distance between the two plates to be d. And let us take the distance, the charge form one of the plates that let us say the left plate to be x 0 and this is charge Q and I want to know, what are the induced charges on the two plates. Let us take second situation in which I know the answer in that situation will be like this, I will take one plate is to be grounded and give potential V to the other plate.

If I give potential V to the other plate, gone down to 0 and in the field, I know in this case is uniform. Therefore, at a distance x from the surface, the potential at x is going to be d minus x times V divided by d. You can see 1 x is 0 this goes to V, when x is equal to d this goes to 0 and in between it varies linearly.

This is going to be my known situation V 2 x is this. What about rho 2? Rho 2 is 0. What about sigma 2? There is some sigma 2; that means, there is some positive sigma 2 here, negative sigma 2 here. Such that, sigma 2 is equal to Epsilon 0 times field in this situation, which is going to be Epsilon 0, V divided by d. You know all these from the capacitor problems you solved in your 12th field. Situation one is where I want to know sigma 1, this is going to be some other sigma whatever, sigma 1 on the other plate.

Let us call it sigma 1 prime, because the two surface charges need not be the same, V 1 x is something, but I certainly know that V 1 x on the surface is 0. Because, the potential is or the plates are grounded, rho 1 is equal to Q delta, if you like x minus x naught delta y minus y naught delta z minus z naught, if this position is that. Let us also take, so this I can write as Q, let us also take origin to be write here, so that things becomes simple.

So, y naught, z naught is 0, I can write this as a delta r minus x naught x, sigma 1 or sigma 1 prime is what I am interested in. Is the problem statement clear? So, we have situation one, which is our real, situation two which we have created and we know the answers in situation two, using that we will find the answer in situation one.

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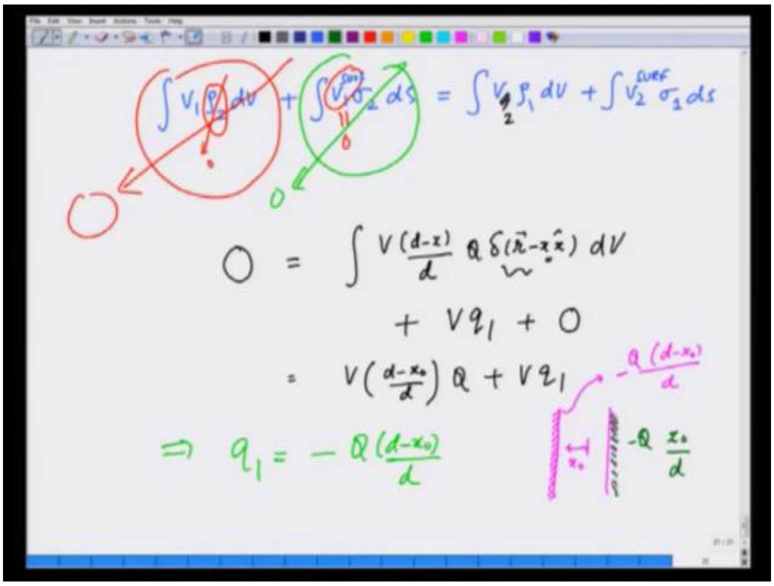

So, to do this, I am again going to write V 1 rho 2 d V plus V 1 sigma 2 on the surface d s is going to be equal to V 2 rho 1 d V plus V 2 on the surface sigma 1 d s, V 1 I do not know, but rho 2 certainly is 0. So, this term drops out, V 1 on the surface is 0, this term is 0 and therefore, this entire term also drops out. On the left hand side, I get 0, which is equal to this should have been V 2, this is equal to V 2. V 2 is V d minus x divided by d, rho 1 which is Q delta of r minus x naught x d V plus V 2 surface.

V 2 on the left surface is V, sigma 1 V is constant, sigma 1 d s give me charge induced on the left surface, on the right surface V 0, so that gives me 0. So, I have written all terms, this is equal to V d minus x over d, x will now become x naught, because of this delta function times Q plus V Q 1 and this indefinitely gives you Q 1 equals minus Q, d minus x 0 over d.

So, if I look at this capacitor again, this distance was x 0 and the charge density or Q 1 on this is equal to nothing but Q the distance from the other plate divide by d, the minus sign. You can similarly show that on this side, on the right side, let me show on the right side with different color, on right side the charge is going to be Q again with the minus sign, x naught divided by d. So, this gives you another application of Reciprocity theorem, how can be used effectively to find induced charges in different situations.