Introduction to Electromagnetism
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Lecture - 26
Electric field and Potential in a Conductor

We have been talking about the electrostatic field and electrostatic potential, and now we want to apply it in different situations.

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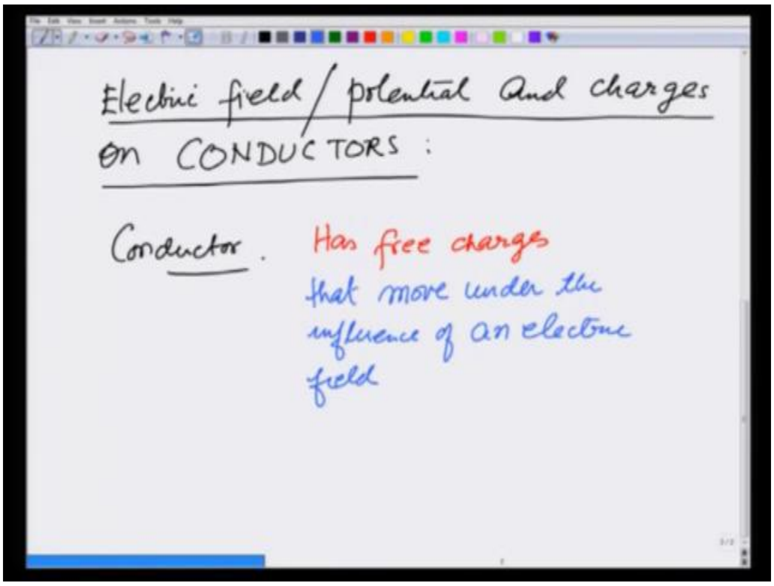

In today's lecture we are going to talk about Electric field Potential and Charges on conductors, this is a very special class of materials. And therefore, we talk about them in this lecture, later we will also talk about polarizable materials or dielectrics and we start with conductors today. A conductor by definition has free charges that move under the influence of an electric field, and therefore it behaves in a particular way and I am going to take it is properties one by one and explain them.

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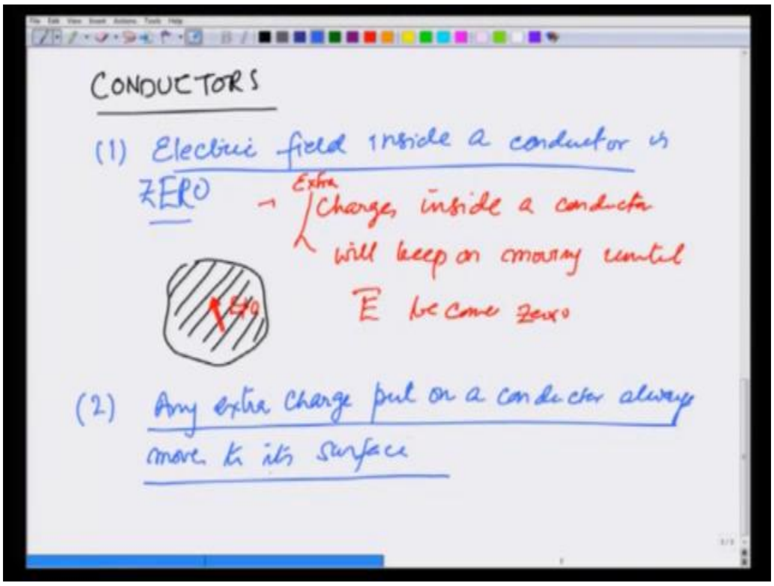

We are working on properties of conductors, so let us take property 1, electric field inside a conductor is zero that is very easy to understand, if I have a conducting material and if there is a field inside, E not equal to zero. What this field will do is, make the charges move and these charges will keep on moving until the field becomes zero. So, charges inside a conductor I should say extra charges, because otherwise everything is neutral, conductor will keep on moving until E becomes zero.

Because the charger mobile and therefore, E inside a conductor will always be zero, as a consequence number 2 any extra charge put on a conductor always moves to it is surface. This is something we have already talked about in first few lectures, but now we will explain it further.

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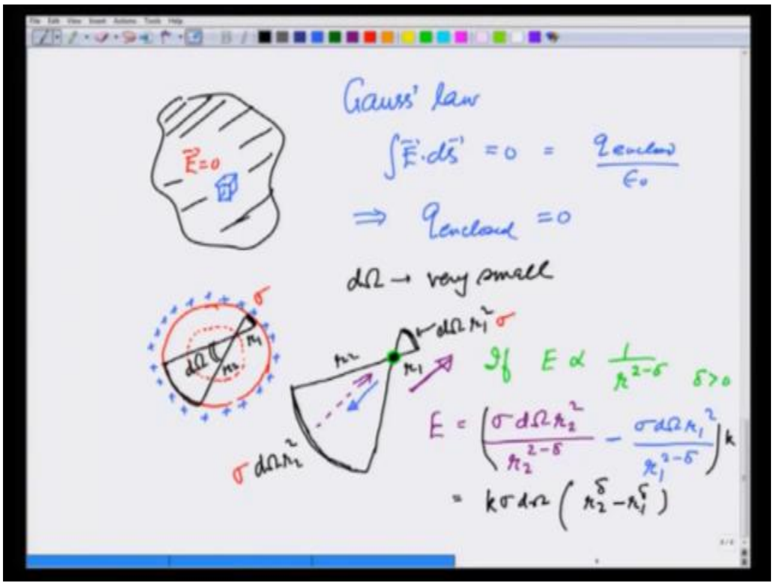

So, to understand this let us again take this conductor which is supposed to be solid and E is 0 inside. If I take any small volume of any size, any shape, any things then by Gauss's law, integral E dot d s since E is 0 and this is nothing but q enclosed by Epsilon 0. This immediately implies inside q enclosed in any volume, any shape, any size always 0 and therefore, charge there is no extra charge inside a conductor. This also we have talked about earlier in the context of, how do we test that the Coulomb force 1 over r square.

Let me elaborate it little more on that now. Suppose I take a spherical conductor, so that there is spherical symmetry, there is no distinction between one direction and the other and put some extra charge on it, this extra charge will be distributed is spherically symmetric manner. Since the charges which are like repel, so I would expect that these discharges were moved, let us see this are positive charges moved to the surface.

Let us see what happens after that if the field is not 1 over r square or Gauss's law is not satisfied. Keep in mind that Gauss's law is equivalent to field being as varying as 1 over r square. So, for that what I will do is, I will make, take a very small surface here and take an on the opposite side, the surface making equal solid angle d omega, let the distance here be r 1, let the larger distance be r 2, d omega is very small.

Now, what we learnt earlier in a spherical co ordinate? The area of this... So, let me just make this picture again what I am taking is, I am taking this small surface on one side r

1, larger surface on the other side r 2 and this area is d omega r 2 square, this area is d omega r 1 square. And let us calculate the field at this point, the middle point where the two lines are meeting due to the charged on two sides. The charge on this surface is going to be sigma, if the charge density outer is sigma times d omega r 2 square, on this side is going to be d omega r 1 square sigma.

Now, let us take if E varies as 1 over r raise to 2 minus delta, where delta is a positive number, then the field at this point in between is going to be E equals sigma d omega r 2 square divided by r 2 raise to 2 minus delta and that is going to be in this direction. This is a field direction and minus sigma d omega r 1 square divided by r 1 square minus delta and that is going to be in this direction.

So, that the net filled comes out to be and there is a constant k which would be 1 over 4 pi Epsilon 0 which we are not concerned about right now, I am more concerned about the r dependence. So, this is k sigma d omega, they are all the same r 2 raise to delta minus r 1 raise to delta.

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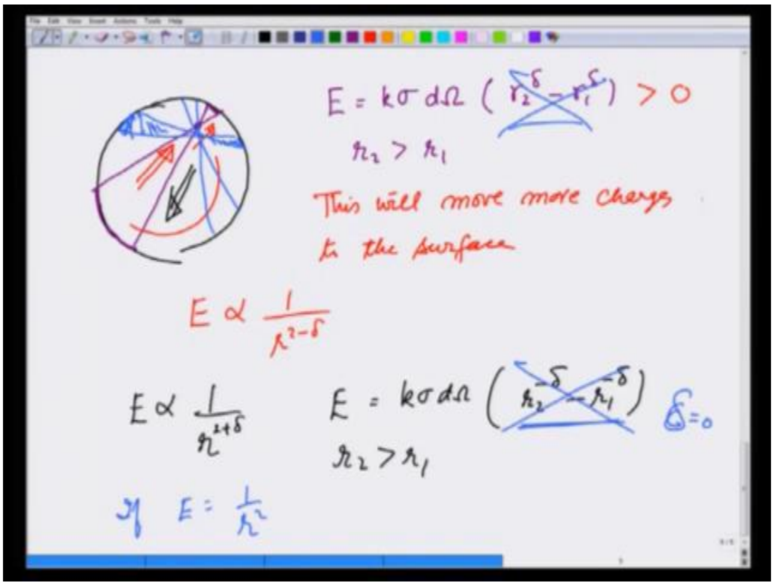

Let me recap, I am taking this sphere and I am focusing on field at this point due to these two small areas and I found that this field E is sigma k times d omega r 2 to raise to delta minus r 1 raise to delta. If r 2 is greater than r 1, then this is field in this direction, so there is a net field and in this direction. What would this do? This... So, this is let us take this red direction to be positive, this will move more charges with the surface, with the result that opposite charge is going to be left inside.

So, what we have seen is, if a field is proportional to 1 over r raise to 2 minus delta, the extra charge comes from the surface and then it pulls more charge. And therefore, there is going to be opposite charge left inside, charge inside is not going to be 0. Let us take the other situation; the other situation let me take E proportional to 1 over r raise to 2 plus delta. In that case is net fields E will be equal to k sigma d omega r 2 raise to minus delta minus r 1 raise to minus delta.

And therefore, if r 2 is greater than r 1 the field will be in the opposite direction, I am showing that black here. So, now, these charge extra charge in the surface will start pushing in the same charge inside pushing towards center. So, final distribution there will be some charge in the same sign at the center and opposite charge and the rest of the volume. In these two cases what we are, now I can generalize the argument, because I can make this kind of appositively located sections everywhere.

And therefore, what you see if the field deviates from 1 over r square, then charge inside is not zero. And therefore, if E is 1 over r square, then you will see these two terms cancel exactly; that means, if delta is 0, these terms cancel, these term cancel and the charges inside becomes 0. So, what we have seen as a consequence of Gauss's law or has a 1 over r square behavior, the charge any extra charge on a conductor moves to the surface. Thus this is a very physical way of looking at you that I talked about.

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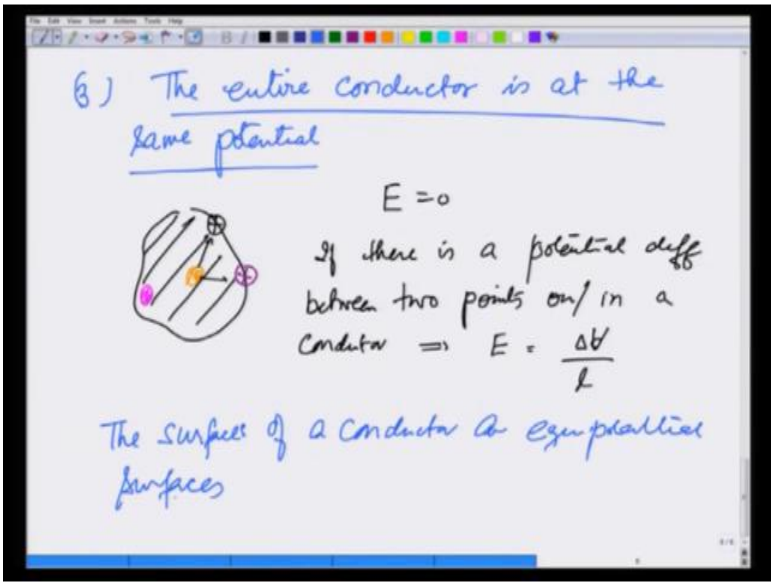

Next point number 3, the entire conductor is at the same potential; that means, if I have this conductor potential at any two points, take this point here, this point here, a point inside, a point on the other side, they will all be at the same potential, reason is very simple. Since, E is 0 there is inside a conductor, there is no work done in moving from one point to the other inside a conductor, if there is no work done there is no potential difference, it has to be the same.

Look at other way, if there is a potential difference between two points on or in a conductor, this implies there will be a field. Because, field is nothing but delta potential divided by the distance, if there is field is going to move charges, until the field becomes 0 and the move in the field becomes 0, the potential is the same. So, all the surface of the conductor are equipotential surfaces, the surfaces of a conductor are equipotential surfaces, because the potential is the same everywhere.

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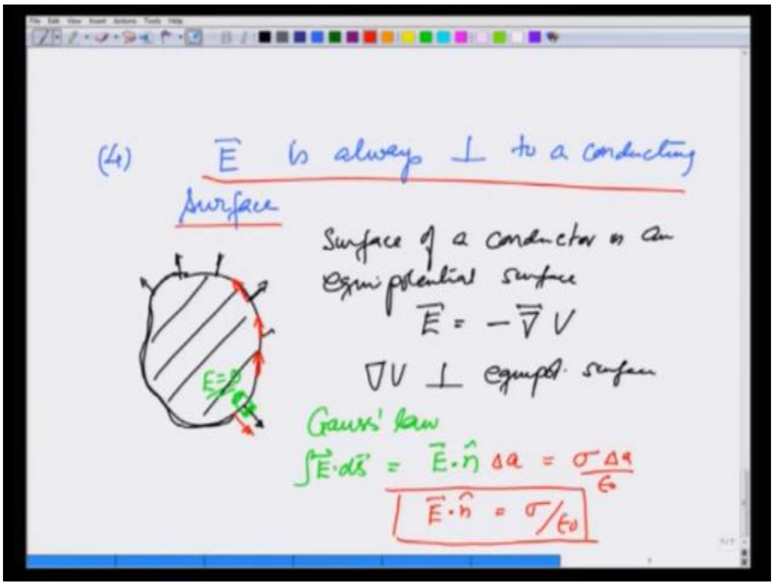

Next, E electric field is always perpendicular to a conducting surface, this is also very easy to understand, again let us look at a contractor, we have already said that the surface of a conductor. So, first I will give you mathematical argument is an equipotential surface and E is nothing but minus radiant of V. Since, the potential is the same on this entire surface and gradient is always gradient of V is perpendicular to equipotential surfaces.

So, it is automatic that field will be perpendicular to this, physically suppose the field was not perpendicular physically, suppose there was a component I am showing by red along the conducting surface. Then, this will make the charges available, mobile chargers available on the conductor move and they keep moving until the field parallel to the surface become 0. So, finally you interpreting E becoming perpendicular to the surface.

As a consequence, I can also write what this surface charge density will be on a conductor. So, that what I will do is, I will choose a small box here cylindrical or whatever to make a cylinder. Since, the field is perpendicular that makes the field perpendicular here, it could be coming out or it could be going in. Then, if I apply the Gauss's law E dot d s we contribute only from the two base or the upper surfaces of the cylinder and field inside E inside is 0.

Therefore, E dot d s is nothing but E dot n, where n is the vector coming up out of the surface, coming out of the volume through that surface E dot n times that small area d a and that should be equal to charge density times delta a divided by Epsilon 0 by Gauss's law and therefore, E dot n is equal to sigma over Epsilon 0. This is how surface charge density and the electric field on the surface of conductor are related. If the field is going in, then you can see E dot n is negative, charge density must be negative, if the field is coming out charges density is positive.

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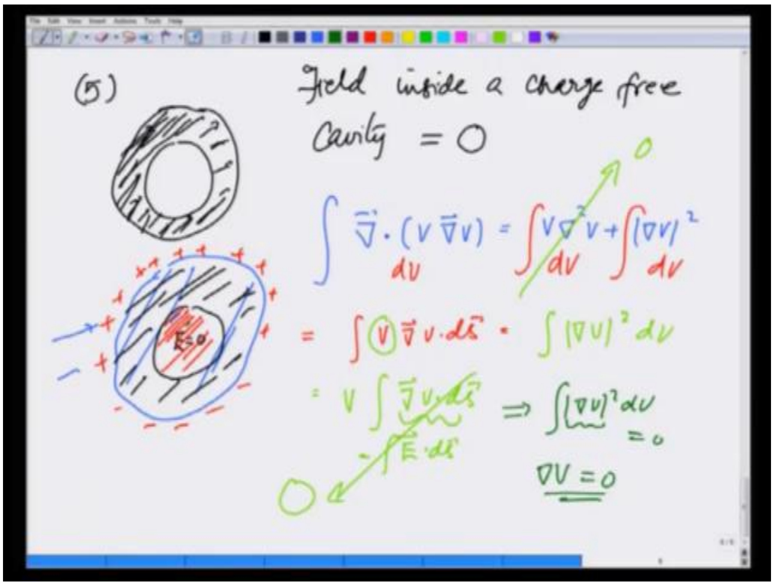

Next, if I make a close cavity of a conductor with no charge inside q there is nothing inside, then field inside a charge free cavity is also 0 that is again very easy to see first let us we have physical argument. Let me assume that this whole thing was a solid conductor. So, whatever charges I gave or whatever field I applied from outside, no matter what I did all the charges plus or minus decide on the outer surface. Therefore, no charge inside if I now removed is section of this to make a gravity inside.

So, let us make this gravity and erase this electrically I am not disturbed anything, I am not moved charge I have not done anything. And therefore, the situation inside remains the same as earlier, and therefore E becomes 0, the other way mathematical way of looking at it is, what we had already done few lecture ago and that is if I take divergence of v grad v which is equal to v Laplace in v plus grade of v square and integrate over this volume inside. So, I am integrating over this volume inside the gravity d v integration d v integration d v.

This comes out to be equal to integration v grad of v dot d s which is equal to since there is no charge inside this term goes to 0. So, I am left with grade v square d v, again this v is the constant over the surface of a conductor. So, I can pull it out grade v dot d s this is nothing but E dot d s if you like with the minus sign, but there is no charge inside and therefore, this term vanishes. What I am finally, left with therefore, is grad v square d v is 0, this is a positive definite quantity and therefore, grad v must be 0 if this has to vanish and therefore, there is no field inside.

So, if I take a conductor, no matter how thin the shell is field inside will always be 0 whatever I do from outside and this is used to make shields to protect instruments from outside disturbance. People also say sometimes when if you are caught on a thunder storm go inside a car which is made of metal, then you will be shelled it in any distributors outside and this is also known as faraday's cage.

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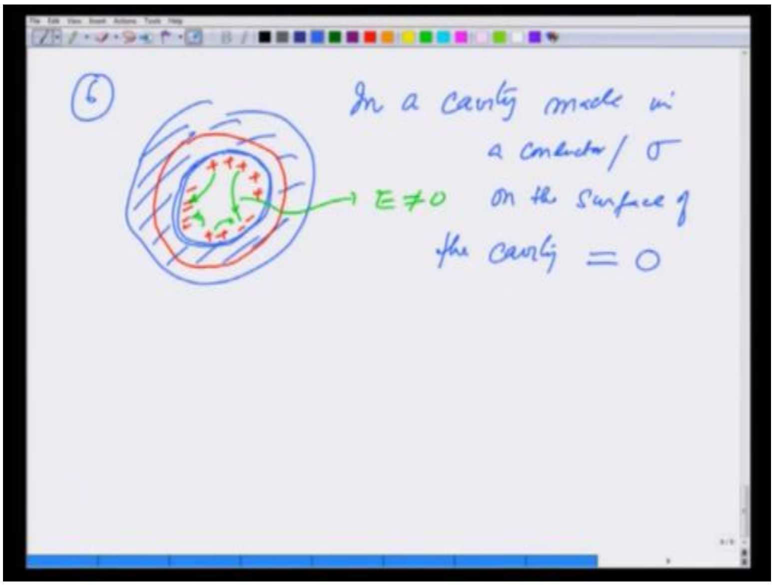

Further, final point, if I take this gravity although the net charge is 0 on this enough surface by Gauss law, because field inside a conductor 0. If I take this Gauss's surface and integrate give me net charges 0. Let me ask question can there be charge distribution inside may be positive somewhere, negative somewhere, positive again, negative some on the other surface and the answer is no, this can never happen.

It is, because if the charges distributed then there will be some electric field lines from positive to negative and; that means, E is not zero. But, we have just seen in the previous point that E 0 inside the gravity and therefore, they cannot be any distribution of charges. So, in a gravity made in a conductor sigma on the surface of the gravity sigma means surface charge is always 0. So, these are some properties of conductors we have seen and you can see it is because a mobile charge at charges there are charger available in a conductor it has these specific properties.