Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 24
Energy of a Charge Distribution – II An example

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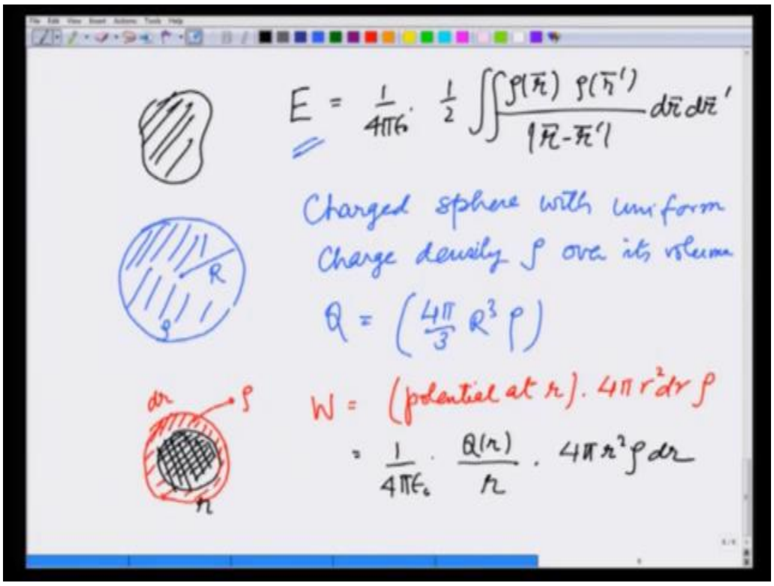

In the previous lecture we saw that if I have a distribution of charge, then the energy of this or the work done in assembly this charge is given by 1 over 4 pi Epsilon 0 1 half rho r rho r prime over r minus r prime d r d r prime. We are now going to solve an example for a given charge distribution, for this we will take a sphere of radius r which carries a uniform charge distribution rho inside.

So, charged sphere with uniform charge density rho over it is volume, so the net charge is going to be 4 pi by 3 R cube times rho. If I would calculate the energy, I will do it in two way, so that you see the equivalence of assembling the charges and this expression that I have written above. Let us first take that I assemble the charges, so that in between when I have a sphere of radius small r, I bring in the new charge and that would be I add a small shell over it, that also carries density rho and it is of thickness d r.

How much work have I done in doing so? The work done will be equal to potential at r times the net charges are brought in and that will be 4 pi r square d r is the volume times rho. Now, potential at r due to this black sphere that is of radius small r is going to be equal to 1 over 4 pi Epsilon 0 times a charge inside this r radius as sphere divided by r times 4 pi r square rho d r.

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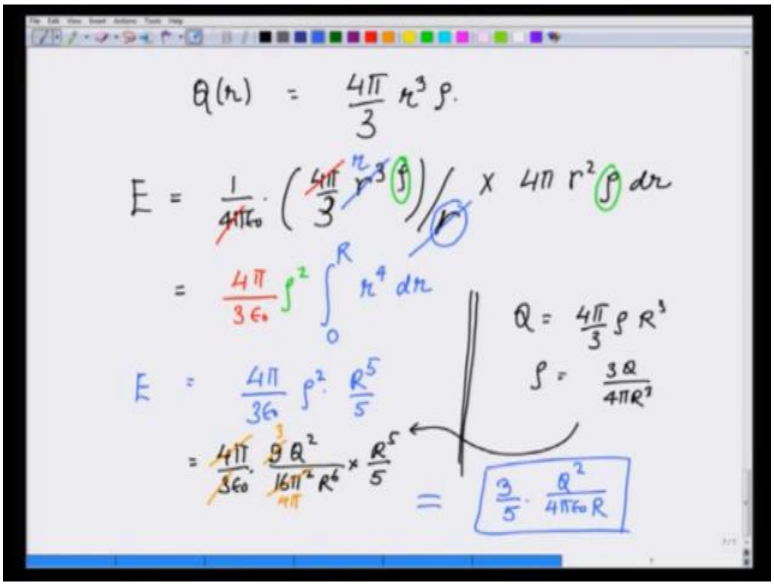

Charge inside, this sphere of radius r is 4 pi by 3 r cubed times rho and therefore, energy of this distribution is going to be 1 over 4 pi Epsilon 0, 4 pi by 3 r cube rho divided by r times 4 pi r square rho d r. Let us work this out, let me cancelled this 4 pi, so I get 4 pi over 3 Epsilon 0 this rho here and this rho here gives me rho square, this r cancels with r cube and gives me r. So, I am left with integration of 0 to capital R.

Because, that is the prove in which the charge has been assembled, r raise to 4 d r which gives me an answer 4 pi by 3 Epsilon 0 rho square times R raise to 5 that is the answer. Let us write this in terms of the charges, you already seen let me write down in this side, that Q the net charges 4 pi by 3 rho R cube and therefore, rho is equal to 3 cube over 4 pi R cubed. Substitute this here and you are going to get is equal to 4 pi by 3 Epsilon 0 rho square which is 9 Q square divided by 16 pi square R raise to 6 times R raise to 5 over 5, this 4 pi cancels with this gives you 4 pi, this 3 cancels with this gives you 3. So, you get the energy equal to 3 over 5 Q square divided by 4 pi Epsilon 0 R, that is the energy of a uniformly charged sphere with density rho.

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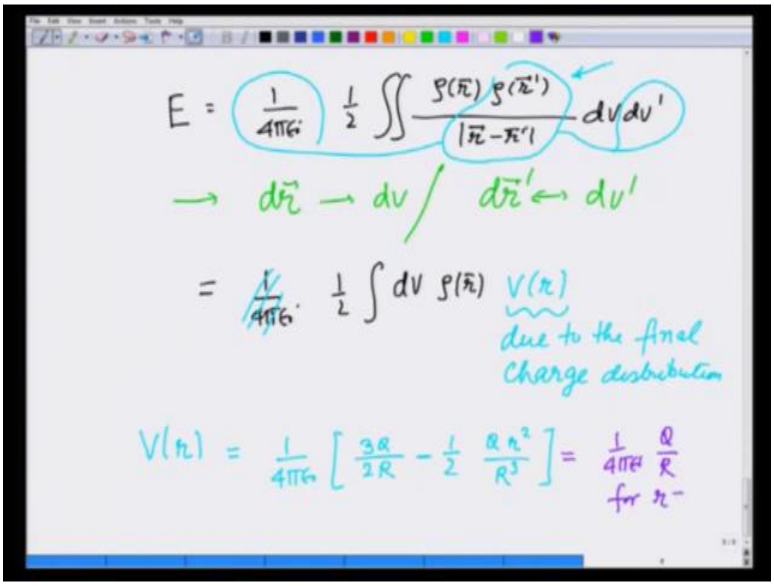

Let us calculate it using the other expression that we had derived, where we have said that energy is also given as 1 over 4 pi Epsilon 0 1 half integration rho r rho r prime over r minus r prime d v d v prime or one point I keep some time writing d r for d v and d r prime for d v prime, so do not get confused. Now, you can write this as 1 over 4 pi Epsilon 0 1 half integration d v rho r and this term rho r prime r minus r prime d r prime d v prime combined with 1 over 4 pi Epsilon 0 is nothing but I will remove this also it is nothing but V at r.

But, notice that this V of r is due to final charge distribution, just before this when we solve for the energy we first calculated V at r due to charge inside. Now, this V of r is a potential due to the final charge distribution, because what I am taking here this rho is a total charges density or charge density due to the full charge distribution. Now, in one of the assignments earlier, you solve for this V of r and this V of r is nothing but if you recall 1 over 4 pi Epsilon 0 3 Q over 2 R minus 1 half Q r square over R cubed, this is one of the assignments I had given earlier. So, that you can see that this is equal to 1 over 4 pi Epsilon 0 Q over R for r equals R.

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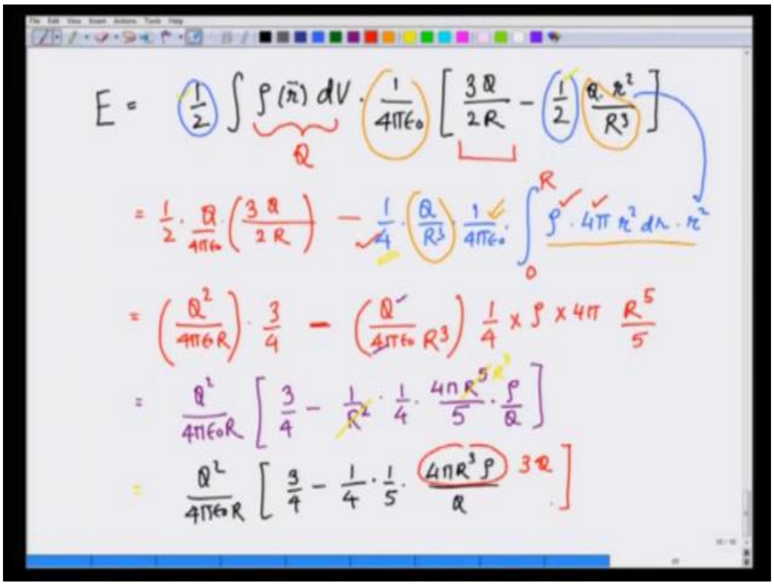

Therefore, the total energy is going to be given as 1 half integration rho r which is a constant d v, let me write the potential 1 over 4 pi Epsilon 0 3 Q over 2 R minus 1 half Q R square over R cubed, the first term is the constant. So, the integration comes out to be equal to this charge Q times the constant and therefore, I can write this as 1 half Q times 3 Q over 2 R and there is a 4 pi Epsilon 0, the denominator minus the next term is going to be this one half and this one half is 1 4th Q over R cubed there is a 1 over 4 pi Epsilon 0 and I am left with integration rho which is a constant times. Volume integral is going to be 4 pi r square d r and I already have a r square from here.

Let see if we have taken care of all the terms, 1 half and 1 half gives me 1 4th, this we have taken care of Q over R cubed, Q over R cubed, 1 over 4 pi Epsilon 0 which is taken care of here and then rho times 4 pi R squared even this is fine. So, I can now write this as equal to Q over 4 pi Epsilon 0 R, Q square over 4 pi Epsilon 0 R and I am left with 3 quarters minus Q over 4 pi Epsilon 0 R cubed 1 4th times rho times 4 pi. These are the constants coming from here and then I have R raise to 5 over 5, because this integral is from 0 to R.

So, this comes out to be Q square over 4 pi Epsilon 0 R, let us take this out, I am left with 3 quarters minus Q 4 pi Epsilon 0 R we have taken out. So, 1 over R square times 1 4th times 4 pi R raise to 5 over 5 times rho over Q, this again now cancel a few things this R square R pi gives me a R cubed here. So, I am getting equal to Q square over 4 pi

Epsilon 0 R 3 quarters minus 1 4th times 1 5th 4 pi R cubed rho over Q, 4 pi R cubed rho is 3 Q.

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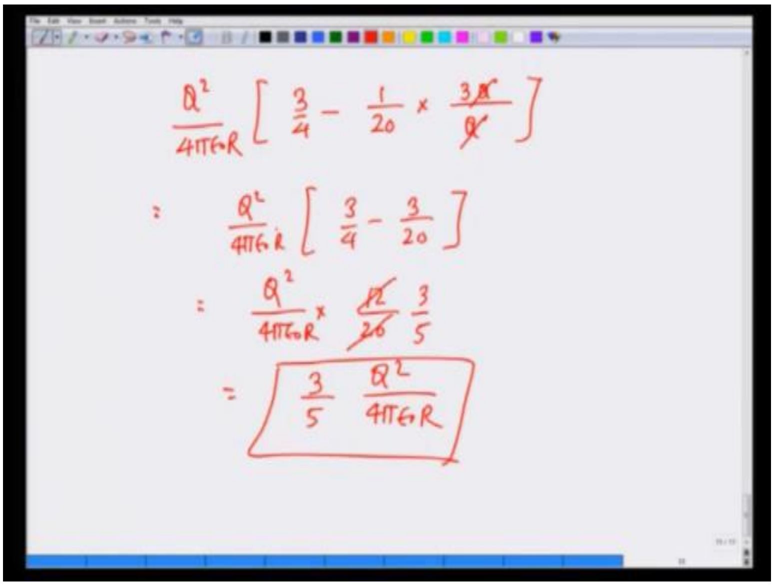

And therefore, the energy I get is Q square over 4 pi Epsilon 0 3 quarters minus 1 over 20 times 3 over Q over Q, this Q cancels and you get the R here Q square over 4 pi Epsilon 0 3 quarters minus 3 over 20, thus Q square over 4 pi Epsilon 0 times this will be 12 over 20 which is nothing but 3 5th which is 3 5th Q square over 4 pi Epsilon 0 R same answer as earlier. So, we have now solve for a distribution of charge energy of distribution of a charge with two different methods, one we thought of this being assembled by bringing charge from outside. And therefore, we did calculator the potential due to the existing charge and multiply by the boarding charge integrated and the other the directs expression which is 1 half v r rho r.

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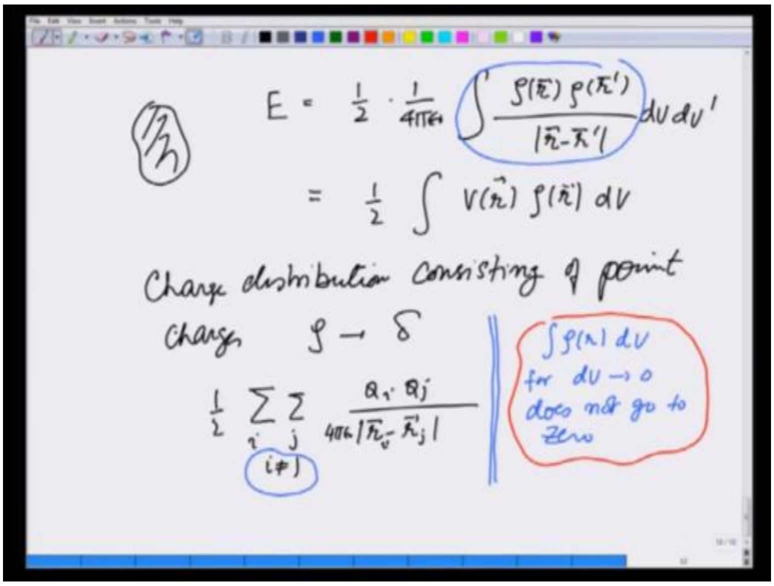

So, finally, to include this you seen that if there is a charges distribution or a collection of charges I can write the energy of this system as 1 half is 1 over 4 pi Epsilon 0 integration rho r rho r prime over r minus r prime d v d v prime which is also equivalent to 1 half integration v of r is the potential due to the full charged distribution times rho of r times d v. In the case of charge distribution consisting of point charges rho goes to nothing but delta function and I will get back by expression i j i not equal to j Q i Q j over r i minus r j summed over 1 half and 1 over 4 pi Epsilon 0.

In making this transition from continuous distribution of charges to charge distribution consisting of point charges, you have to be careful that in this case integration rho r d v for d v tending to 0 does not go to 0. Because, this point charges are like delta functions and therefore, in doing integration I have to explicitly avoid self interaction of charges.

Remember, earlier when I was discussing this formula I had said that in the continuous distribution this rho r d v integrated over smaller and smaller volume microscopically small very volume goes to 0, but in point charges does not. So, one has to be careful. I like you to make this connection after hearing this lecture.