Introduction to Electromagnetism
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Lecture - 23
Energy of a Charge Distribution – I

We have looked at how charges or charge distributions give rise to electric field and potential, and how to calculate them. Now on, for 2 or 3 lectures, we are going to focus on what happens to the energy to system, when we assemble a set of charges or assemble a distribution of charge.

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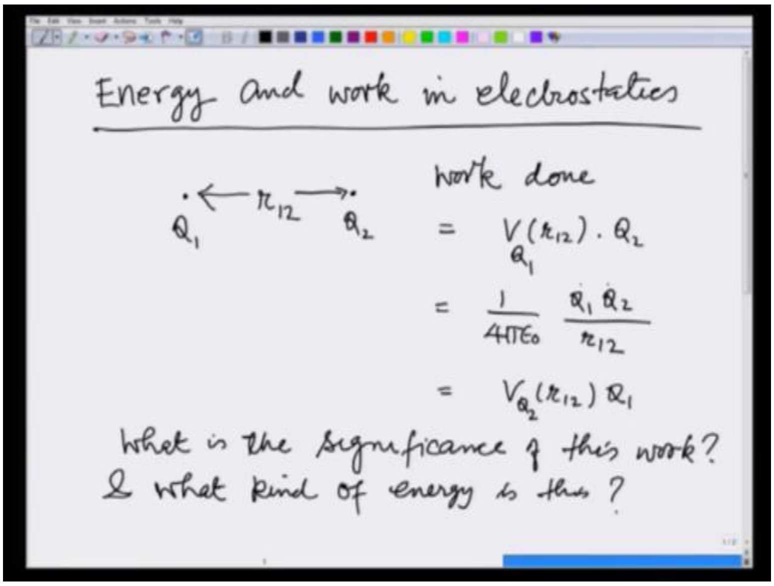

So, what we are going to work on is the energy and work in electrostatics. Let us start with the simplest example, if I have a point charge Q 1 let us say, and I bring another point charge Q 2 over distance, let us say r 1 2 from it. Then by the definition of the potential the work done in doing so will be equal to the potential at r 1 2 due to charge Q 1 times Q 2 and this is equal to 1 over 4 pi Epsilon 0 Q 1 Q 2 over r 1 2, you see this is symmetry in Q 1 and Q 2.

So, I could also have a sort of this work as the word of bringing charge Q 1 over distance r 1 2 from Q 2 or the work in bringing Q 1 in the potential of Q 2 at a distance r 1 2 from Q 2. What we are interested in now is, what happens when I bring in assembly of charges many, many charges. And finally, we go to a charge distribution. So, that it is described by charge density rho. Now, first what is the significance of this work and what kind of energy is this? Well, since this is energy due to position of charges, it is potential energy and this significance is that this is the amount of work that we have done in an assembly in this charges.

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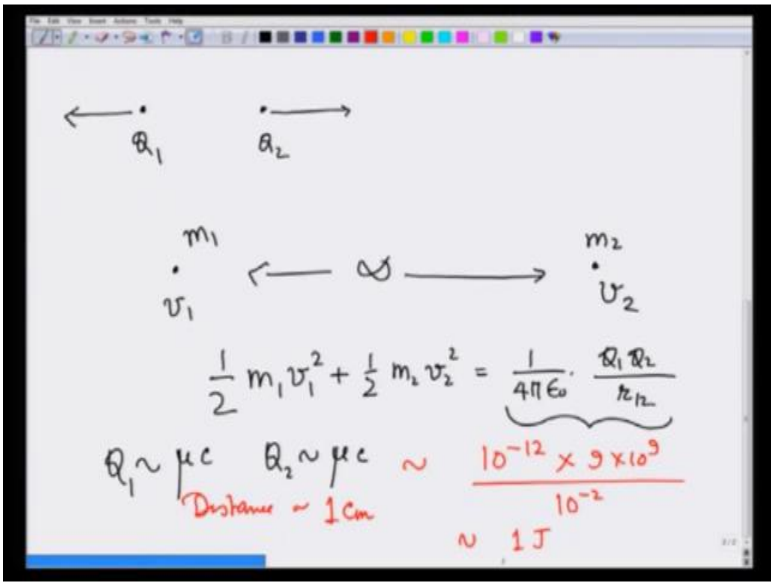

And therefore, if I leave Q 1 and Q 2 by themselves, if they are same charges they repel from each other and therefore, it start rushing away. Finally; reaching infinite distance let us say this is infinite with speed v 2 and v 1. So, that their energy have let us say the mass of first charge is m 1 and mass of the second charge is m 2, then the total kinetic energy m 1 v 1 square plus 1 half m 2 v 2 square will be equal to this initial energy that we said is in the charge distribution and therefore, this will be equal to 1 over 4 pi Epsilon 0 Q 1 Q 2 over r 1 2.

What kind of magnitude of energy are we talking about? Let us say Q 1 is of the order of micro coulomb, Q 2 is of the order of micro coulomb, then this potential energy we are talking about and let me will be of the order of 10 raise to minus 12 times 9 times 10 raise to 9 divided by the distance is say 1 centimeter between them. So, this will be 10 raise to minus 2 of the order of 1 2, this is a kind of energy we are talking about. If this is micro coulombs, then the energy is of the order of 1 joule if they at distance of 1 centimeter. Imagine, what would happen if they were 1 coulomb, then the energy would be 10 raise to 9. We can already see that 1 coulomb is a lot and lot of charges.

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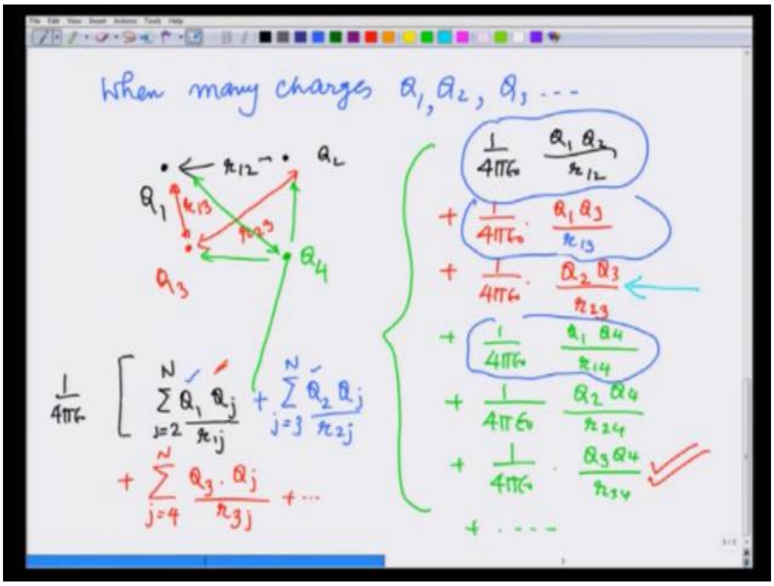

Next, what happens when many charges Q 1, Q 2, Q 3 and so on are there, let us assemble them one by one. So, I have charge Q 1 let us bring Q 2 to a distance r 1 2 and I am doing, so I have done work which is 1 over 4 pi Epsilon 0 Q 1 Q 2 over r 1 2. Let me now bring in charge Q 3. Let us say this r distance, r 1 3 from charge Q 1 and r 2 3 from charge Q 2, in bringing this charge I am doing work against a forced provided by Q 1 and force due to Q 2.

And therefore, the energy is going to be sum of the potential energy due to these two charges. So, I am going to get the additional terms which are 1 over 4 pi Epsilon 0, potential energy of Q 3 due to charge Q 1, and notice that this is all using principle of super position, we just adding it up. So, this is going to be Q 1 Q 3 over r 1 3 plus 1 over 4 pi Epsilon 0 Q 2 Q 3 over r 2 3.

Next let us bring in charge Q 4, this will now be coming in the potential of these three charges. So, now, I will add three contributions, so this is going to be 1 over 4 pi Epsilon 0 Q 1 Q 4 over r 1 4 plus 1 over 4 pi Epsilon 0 Q 2 Q 4 over r 2 4 plus 1 over 4 pi Epsilon 0 Q 3 Q 4 over r 3 4, and then I can keep adding these terms. If you look at the pattern and all these, you will notice that I can write this as, if I have charge Q 1 if I bring next n minus 1 charges, then I have Q j summation j equals 2 through N. Of course, there is 1 over 4 pi Epsilon 0 outside.

So, these are the terms let me point them out here and presence of one I have this term, I have this term, so Q 2, Q 3, Q 4 and so on. Next, in bringing these charges I also done work against potential due to Q 2. So, it will be Q 2 Q j I have already accounted for interaction between 1 and 2, so j will be equal to 3 to N. So, this r 1 j, this is next term is divided by r 2 j, next I will be counting the potential energy due to bringing all these charges in presence of charge 3, so I will have q 3 times q j.

However, notice that I have already accounted for Q 1 Q 3 in this term and Q 2 Q 3 in the second term, and therefore I have j equals 4 through N over r 3 j and so on. The last term the red one accounts for this term, this term was already accounted for the second term and so on.

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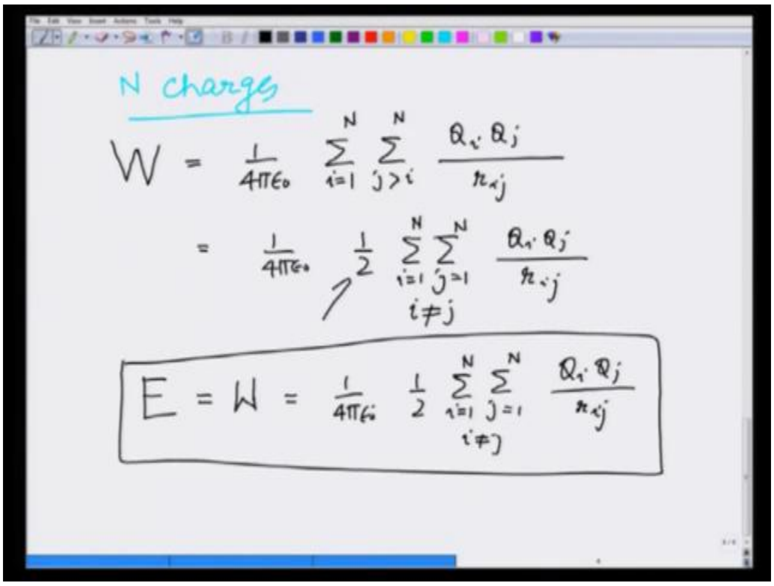

So, in general when I have an assembly of charges, when I have N charges I can write the potential energy is equal to 1 over 4 pi Epsilon 0 summation i equal to 1 through N and presence of each other charge I am doing to work from j, which is greater than i up to N, Q i Q j divided by r i j. I would like you to expand this and see that this sets well you got derived earlier. I can also write this as 1 over 4 pi Epsilon 0 1 half of i equals 1 through N j equals 1 through N; that means, I am now allowing j to very not just from greater than i to N, but all the j's.

So, I will be double counting each Q i Q j, so I divide this by a factor of half here. So, you keeping i not equal to j Q i Q j over r i j. So, that is the expression for the energy of an assembly of charges. So, let us write this again I should be careful, let me write not this not as V, but as W that is work done. So, energy of this assembly of charges is equal to the work done bringing all these charges together, which is going to be 1 over 4 pi Epsilon 0 1 half summation i equals 1 through N summation j equals 1 through N i not equal to j Q i Q j over r i j, this is the energy that is there in assembly of two charges.

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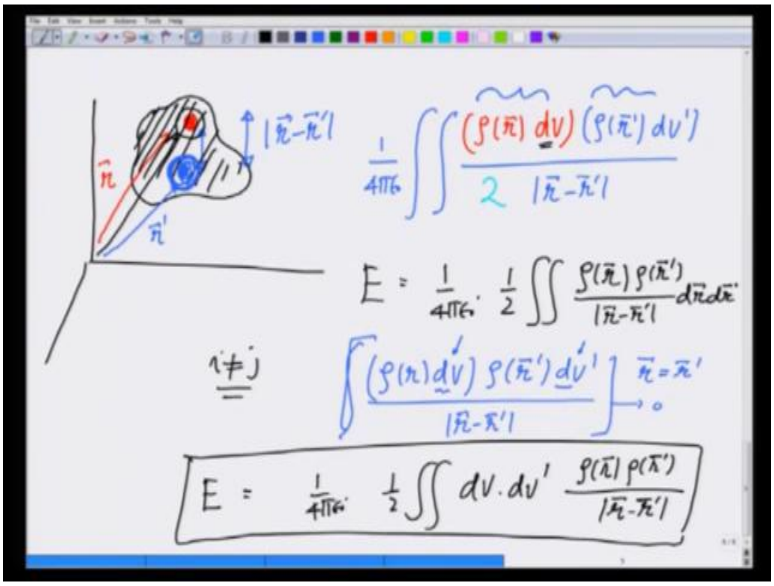

Let us now look at what happens when I go to a distribution in what; that means, is if I have a charge distribution. I can think of this charge distribution as if there is a charge somewhere in this volume shown by red they where charge somewhere in this volume shown by blue, and there is an interaction between let us write the position of charge and red to be let me show this by red r and the one by blue by r prime, then the distance between them is going to be modulus of r minus r prime.

Then the interaction energy between these two is going to be density at r times volume small volume d v that is the charge there, then density at r prime times of volume at r prime divided by the distance between them. And then I integrate over v and v prime and to 1 over 4 pi Epsilon 0 when we do this integration; that means, I am taking this charge multiplying and adding all these charges, and therefore I should also be dividing by a factor of two here, just like we did in counting a charges Q i and Q j.

So, if final expression for the energy then comes out to be 1 over 4 pi Epsilon 0 1 half integration row or row r prime over r minus r prime d r d r prime. You may ask what happened to i not equal j term, vol we are taking these volumes here all this red volume here or the blue volume here infinitesimal small. And therefore, when I multiply row r d v and row r prime d v prime with r equal to r prime this becomes infinitesimally very, very, very small.

In the sense that this divided by r minus r prime close to 0 and therefore, there is no problem of self energy like Q i, i equal to j term in the previous expression that we derive earlier here where we put i not equal to j. So, when you have a continuous distribution, so that these volumes of microscopic or macroscopically very small. So, that the product goes to of 0 then there is no problems this remain 0 and there is no problem of self interaction. So, finally, we write E of distribution of charge which is 1 over 4 pi Epsilon 0 1 half double integration d v d v prime row r row r prime row r minus r prime. In the next lecture, we are going to solve some examples of energy of some distribution of charge.