Introduction to Electromagnetism
Prof. Manoj K. Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 22
Laplace’s equation in some other physical phenomena

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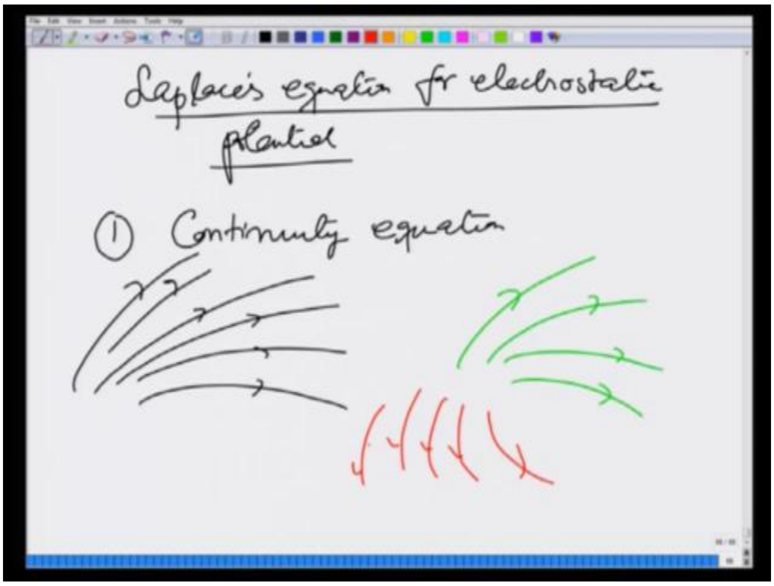

We have been looking at Laplace’s equation for electrostatic potential. Let us now look at some other situations where this equation is valid, that also helps us in connecting the electrostatic potential with those situations and may give us some insight. The first thing I do in this case is the continuity equation and deriving all these relationships, I am going to make use of Divergence theorem. So, imagine a fluid or heat or some particles flowing, so let us make different situations, maybe there is heat I will make it with color or there are some particles going around diffusing, then what happens what is continuity equation tell us.

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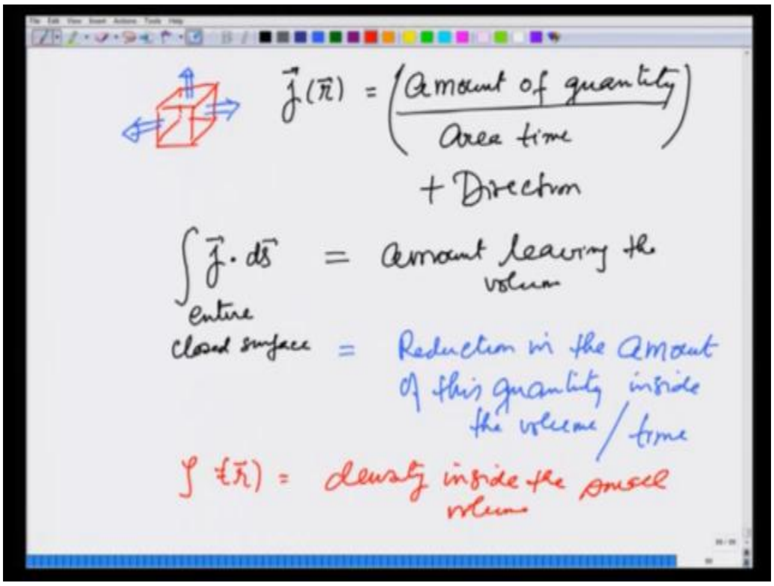

So, in this flow if I look at a particular volume, this current passing through this, we define this earlier. So, let us define current j r as equal to the amount of quantity, I am writing quantity because it could be fluid, it could be some particles, it could be some heat per unit area per unit time and it is directional. So, it also plus the direction of it, so it flows in a certain directions. So, that if this is a volume, the net amount leaving the volume would be j dot d s from each surface and if I integrate over the entire surface, entire closed surface, this is going to be equal to the amount leaving the volume.

Why am I saying leaving and not entering, this is because I am taking the conventional sense s is pointing out of the volume. So, if j dot d s is positive it means things are leaving, if j dot d s is negative this means things are entering. So, j and s are at the same direction that things are leaving, but they should also equal to the reduction in the amount of this quantity inside the volume per unit time. So, let us define the density rho, as the density rho at r has the density inside the small volume, this is a very small volume that I am taking.

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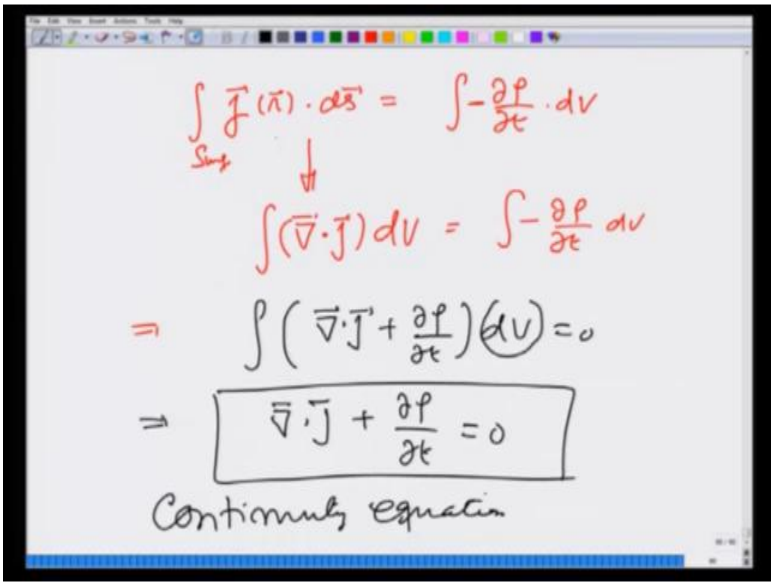

Therefore, I am going to have j at r dot d s is equal to j at r is being calculated in this surface is equal to integral d rho by d t with a minus sign by it is reduction times d, the volume. By Divergence theorem, this is divergence of j in the volume is equal to minus d rho by d t in the volume and this gives me integration of divergence of j plus d rho d t d v is equal to 0. Now, this is small volume can be taken in any different shape, any different size.

So, this is really arbitrary and this implies del divergence of j plus d rho d t is 0 and this really comes from conservation of mass, conservation of that quantity, this is known as continuity equation. If rho is not changing, the density is not changing; that means, everything that is coming in it also leaving, divergence must be zero as we learnt in the case of defining diverges. In that case, divergence of the current would be zero. Let us now apply this in two different situations and that gives you very beautiful feeling for electrostatic cases also.

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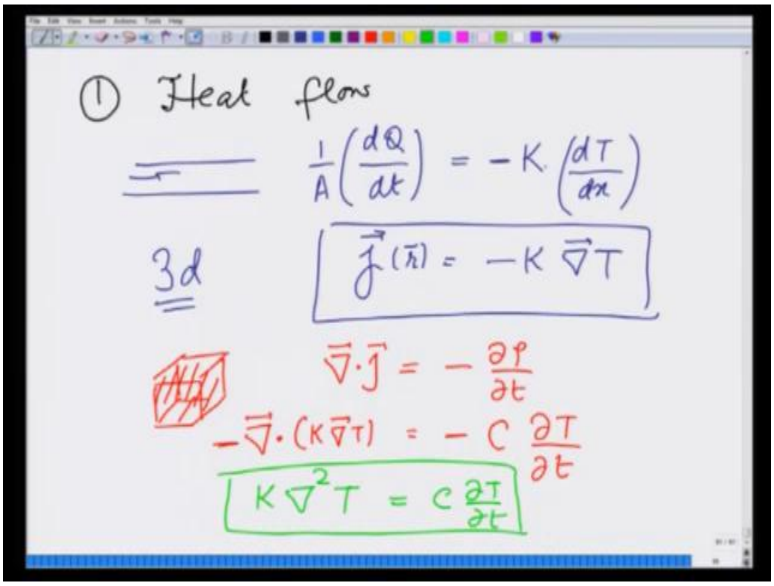

One is heat flow, you have learnt in your 12th rate, that heat flow in one dimension, the amount of heat d Q d t per unit time per unit area is actually equal to the thermal conductivity and the differential derivative of T x with respect to x. I am going to put a minus sign here, because that tells you that heat flows in the direction of lowering temperature.

In 3D, this generalizes to the per unit area, the heat current this was 1D is flowing one direction, but in the three dimension case heat current is, that is the heat flowing per unit area per unit time is equal to minus K gradient of T. So, let us again consider a small volume, whatever current is flowing out because of that the quantity of heat, a quantity of heat energy inside is lost. So, if I apply del dot j is equal to minus d rho d t, this would come out to be minus, the temperature is reducing and that means, heat flows on some specific heat times d T over change of temperature with time.

And this should be equal to divergence of by continuity equation K grad of T with the minus sign outside. This becomes therefore, K del is square T is equal to C d T d t. This is what will determine, how temperature changes with time given some boundary conditions.

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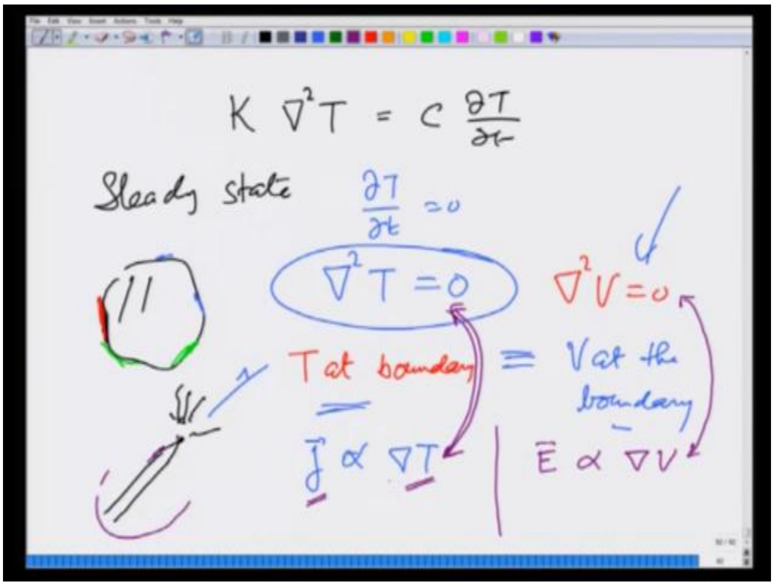

So, now let us, so you derive the equation K, Laplace on T is equal to C d T by d t, let us see this steady state situation; that means, the temperature does not change your time. This could be for example, if I take a volume and maintain the temperature on the sites, maybe here is one temperature, here is another temperature, here is third temperature, maintain different temperatures. Then, temperature inside the time does not change with time, because it has a stabilize, then d T d t will equal to 0 and del square T is equal to 0, that is a steady state temperature equation.

Compare this with the del square V equal to 0, this is the same equation, all I have done is replace V by T. Maintaining a potential on the surface is equivalent to T at boundary is equivalent to V at the boundary. That means, if I either solve this I know the solution for this, if I solve this I know the solution for this. So, this is the, I can call it heat diffusion equation. Now, I am going to connect it something you know.

You have learnt and you have been told that, when lightning conductors that are put on building have a very sharp edge and that is because near the sharp edges, the electric field becomes very large. If the electric field becomes very large, most of the charges or most of the current in the atmosphere comes here or lightning strikes at that point and then goes to the ground directly. Now, let us analyze that situation here, the current or heat is proportional to grad of T.

The electric field is proportional to grad of V. So, whatever the relationship is between V and E is the same relationship between T and the heat flow. Now, if E is very, very strong near the edges and there is no other charge giving rise to, this is Laplace equation, so there is no external charge, same must be true here. If there is no source are sink, then in steady state the heat flow should be proportional to grad of T. What happens to grad of free electric field in the case of lightning conductor? It is very strong near the edges. Similarly, the heat flow should be very strong near the edges, because it is proportional to grad of T, do we see that situation in life and you do.

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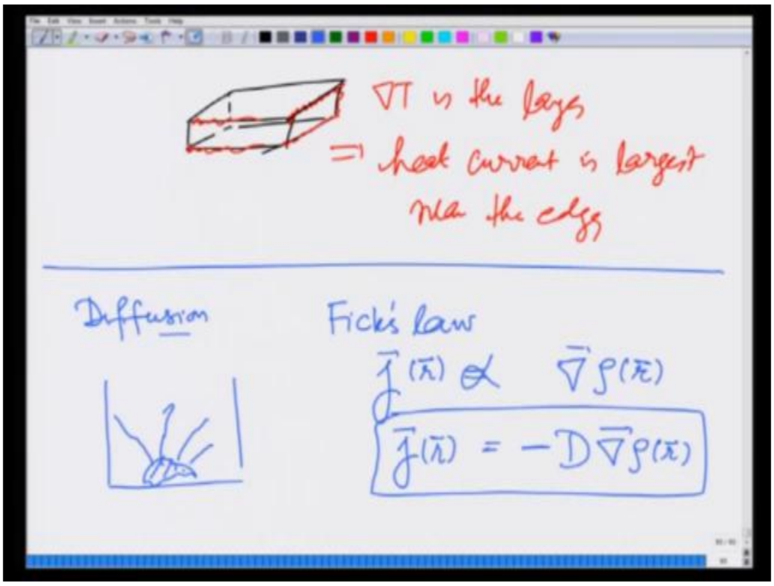

Next time you buy an ice cream or you have a piece of ice cream, notice how it melts from the edges. It starts melting from the edges, so that you start seeing a round shape there and that is, because at the edges grad T is the largest and this means the heat current is largest near the edges. So, this gives you a connection between the electric field and the heat flow or the potential and the temperature. Another example of this I am going to take relates to diffusion.

Diffusion is where if you put some material in a fluid, it starts diffusing slowly or evaporation is also a kind of diffusion if you like loosely. Then there is something called Fick's law that tells you that the diffusion current at any point is proportional to gradient of the density of the particles diffusing or this is equal to minus some diffusion co efficient gradient of rho. So, if the gradient is large, then there will be more current of diffusion, if gradient is small if gradient is zero, there will be no diffusion.

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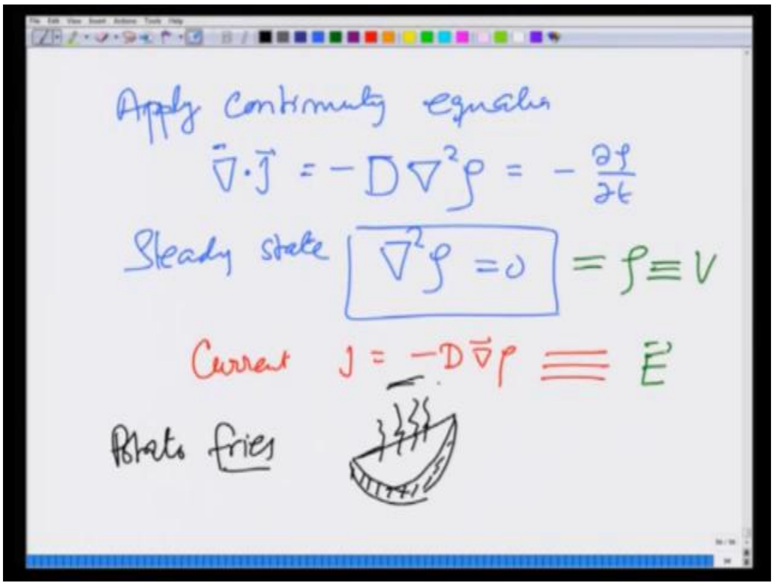

Now, apply continuity equation; that means, del dot j; that means, del dot j which is minus D del square rho is equal to minus d rho d t that is a reduction in the particles. And in steady state, when there is a constant gradient being maintained, so that there is no need inflow or outflow, you should have del square rho is equal to 0. Again a Laplace equation and the current j which is equal to minus D grad of rho is like equivalent to the electric field and this rho is equivalent to the potential.

So, again this is similar relationship, so again from the edges I should see a large diffusion. Do I see that? Next time, you take some potato fries or something else which is fried with sharp edges, so potato fries have sharp edges for example. Now, there have these edges out here. What happens in frying? Frying water evaporates or water diffuses out because of the heat, from these equations suppose we have assume kind of a steady state where do you expect the maximum diffusion to take place from.

It will be from the edges, because that is where electric field is maximum and that is where the current of diffusion would be maximum and you should see the edges getting ground faster and that is indeed what happens? So, what we try to do in this lecture is, connect the Laplace equation with other known situations, so that next time you see, you get the feel for the electric field.