Introduction to Electromagnetism

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Lecture - 21
Method of images II:
Point charge in front of a grounded metallic plane and grounded metal sphere

In the previous lecture, we used the uniqueness of solution to find the potential for a charge kept in front of a metallic surface or a metal or a surface, which was grounded or which had v equal to 0.

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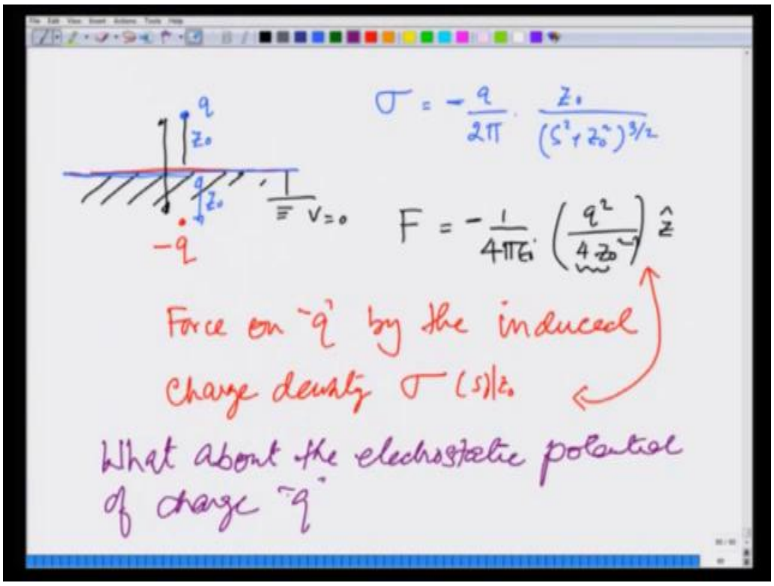

And we found that the potential can be obtained by putting a minus charge minus q charge for a charge q in front of the surface, which is at a distance Z at the same distance from the surface. We also calculated the surface charge sigma which came out to be q over 2 pi Z naught over s square plus Z naught square raise to 3 by 2. Now, what about the force on this charge, if it is brought in front of a metal surface? Because, it will obviously be attracted by these negative charges that is a minus sign here, negative charges that have been induced on the surface.

All can be calculated the force directly from this minus q. It is so happens that the force on charge q can be calculated as the force between these two charges, the charge itself and it is may charge. And therefore, force on this is given as 1 over 4 pi Epsilon 0, q square over 4 Z 0 square and it is attractive. So, let us put Z unit vector with the minus sign here. Notice that this goes as 1 over 4, Z naught square, I will leave this as an exercise for you to calculate force on q by the induced charge density sigma s, Z naught and show, that this is equal to this.

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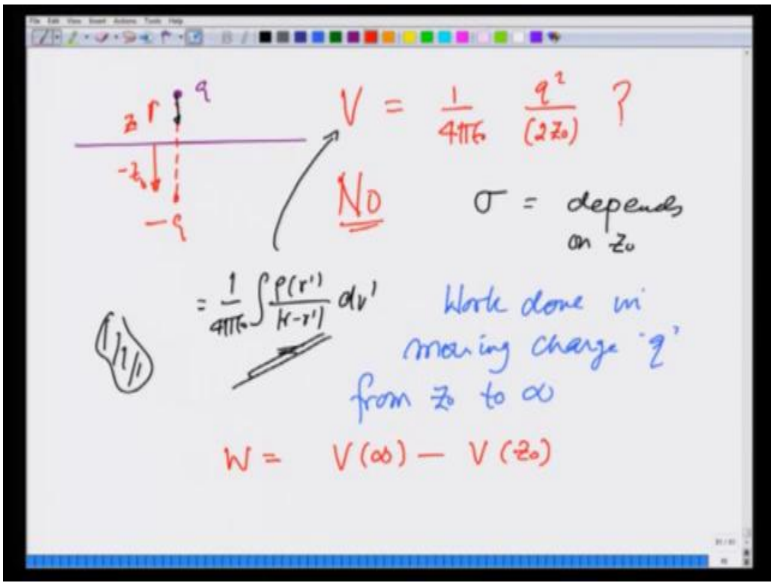

What about the electrostatic potential of charge q? Is it that, we have this image charge q and image charge minus q, add a distance Z 0, Z 0. Can I write the potential V as 1 over 4 pi Epsilon 0, q square over 2 Z 0? Because, this is the potential energy between two charges and the answer is no, there is a very simple reason for that. The reason for that is that, remember when we calculate the potential due to a given charge distribution, we can write this as integration rho r prime over r minus r prime d v prime 1 over 4 pi Epsilon 0.

And it is the work done in bringing a charge from infinity to that point, provided the charge density remains fixed, it does not change. However, notice in this case the image charge problem has charge comes in or goes out, the charge density sigma depends on Z 0 and therefore, I cannot use this formula directly. This formula, use of this formula directly gives me this answer with rho being the delta function. In this case, I explicitly must calculate the work done.

So, let us calculate the work done in moving charge q from Z 0 to infinity and that work done, W is going to be the potential energy difference of the charge particle at infinity minus V at Z 0. So, let us calculate this.

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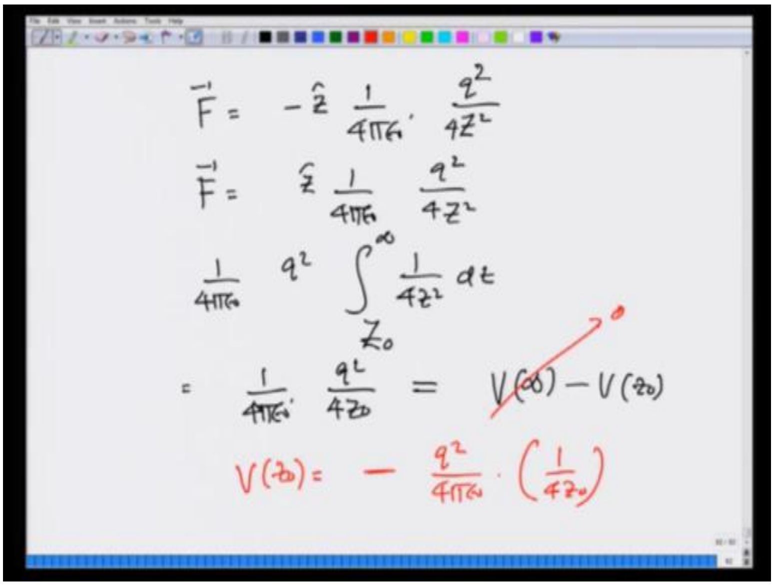

The force on the charge is minus Z, 1 over 4 pi Epsilon 0, q over 4 Z square, if it is at a distance Z, the force applied by us is going to be 1 over 4 pi Epsilon 0 in the Z direction, q over 4 Z square. So, work done by us is going to be 1 over 4 pi Epsilon 0 and so this is q square q, q square integration 1 over 4 Z square d Z moving from Z 0 to infinity and that gives me 1 over 4 pi Epsilon 0, q square over 4 Z naught.

This is equal to V potential at infinity minus potential at Z 0, taking at infinity potential to be 0, I get V Z 0 equals minus q square over 4 pi Epsilon 0, 1 over 4 Z naught. So, it is not 1 over 2 Z naught, but 1 over 4 Z naught.

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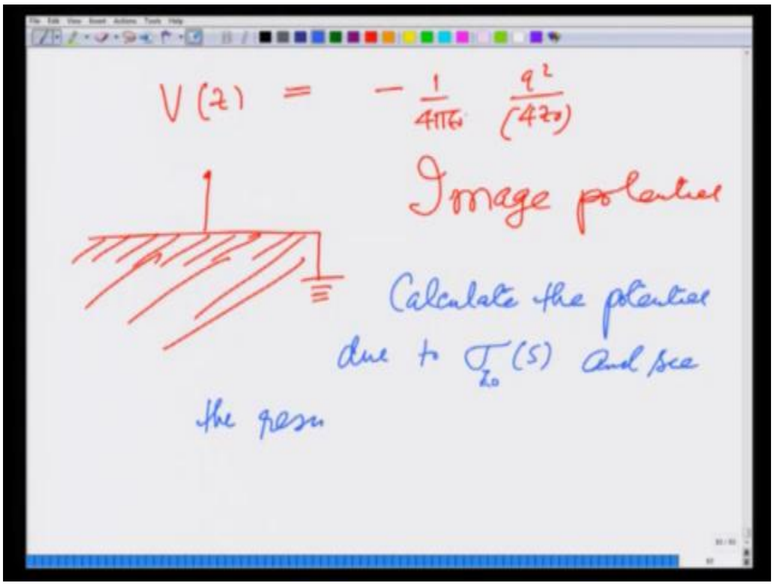

These are known as this is known as the image potential. So, V Z, add a distance Z in front of a grounded surface or in front of a metal, which is at 0 potential. The potential energy of a charge q is given as minus 1 over 4 pi Epsilon 0 q square over 4 Z naught and this is known as the image potential. I again emphasize, this is not equal to like the electrostatic potential that we calculate between two charges, add a distance 2 Z naught. Because, in this case as you move the charge, the test charge, which is q in this case the charge distribution itself changes.

So, we have calculated the potential and the force on the charge due to the image charge. We have already calculated the surface charge density, what I would like you to do is calculate the potential due to sigma S, because of the charge at this Z 0, and see what the result is.

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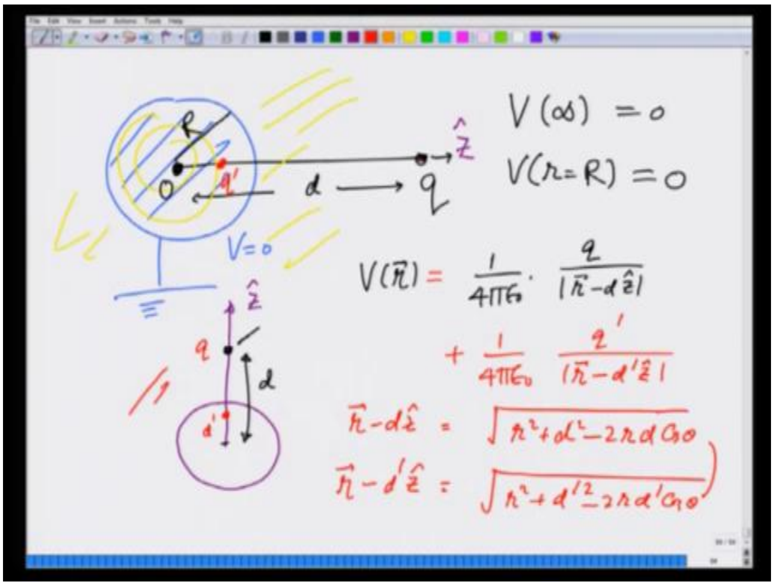

A related problem that can be solved using image method is, if I take a sphere. Say a metallic sphere and ground it that means the potential on this is 0 and take a charge q in front of this at a distance, let us say d from the center of this sphere. It so happens that in this case also I can find another charge, outside the region of interest; that means, my region of interest here is this outside volume, I can find charge inside this metal. So, that the boundary condition is satisfied.

What is the boundary condition here? V at infinity is 0. If I take this as the center or the origin, then V at r equal to R, if R is a radius of this sphere is also 0. If I can find another charge that satisfies, that combine with this potential of q satisfies both the boundary conditions, then I have found my solution, because that is a unique answer. So, let us try and see if I put a charge inside, along the same line as the center line, which joins this center of this sphere and the charge q, a charge q prime and see if I can satisfy both the boundary conditions.

So, let us write the potential V at r vector. Now, I am going to take this direction to be the Z direction, you imagine a bit that I have taken this direction to be Z direction, if you cannot then I will give you also. Another picture, here I have taken this direction to be the Z direction and taken this charge q out here, add a distance d from the center. Put another charge q prime at a distance d prime from the center along the line joining the center and the charge q or along the Z direction.

Then, V r I can write that as 1 over 4 pi Epsilon 0 q over mod r minus d at Z; that is a potential due to this charge q and the image charge which I am going to write in a different color plus 1 over 4 pi Epsilon 0, q prime over r minus d prime Z. So, I have put it in a different distance, but inside the metallic sphere. So, that outside the Poisson's equation remains the same, r minus d Z is equal to square root of r square plus d square minus 2 r d cos theta r minus d prime Z. Similarly, square root of r square plus d prime square minus 2 r d prime cosine theta; theta remains the same because both the charges are on the Z axis.

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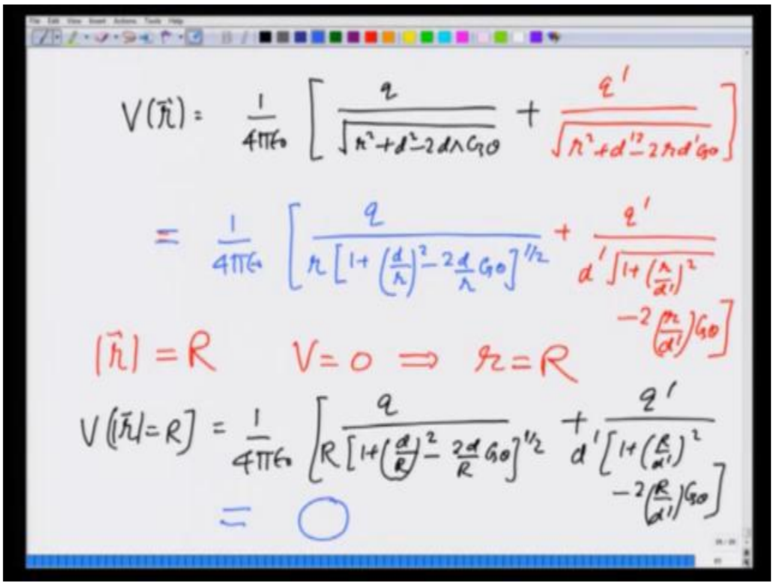

So, let us now write the potential V at r to be 1 over 4 pi Epsilon 0; that is common, q over square root of r square plus d square minus 2 d r cosine of theta plus q prime over square root of r square plus d prime square minus 2 r d prime, cosine of theta. To generate these charges, I am going to do some manipulations here. Although, they can also be generated by initially looking at two different charges and finding a surface, where the potential is 0, but here I am going to just do a little manipulation.

I am going to write this as equal to 1 over 4 pi Epsilon 0, q, I will take r common and inside I am left with this 1 plus d over r square minus 2 d over r cosine theta raise to 1 half plus q prime, I will take d prime common out here, d prime square root of 1 plus r over d prime square minus 2 r over d prime cosine of theta, close it. What do I want, I want at r modulus equal to R, V be 0.

So, let us take modulus r equal to R or this also implies r equals R, let us write the potential. So, V at mod r equal to R is equal to 1 over 4 pi Epsilon 0, q over r, 1 plus d over r square minus 2 d over r cos theta, raise to one half plus q prime over d prime 1 plus R over d prime square minus 2 R over d prime cos theta and I want this whole thing to be 0.

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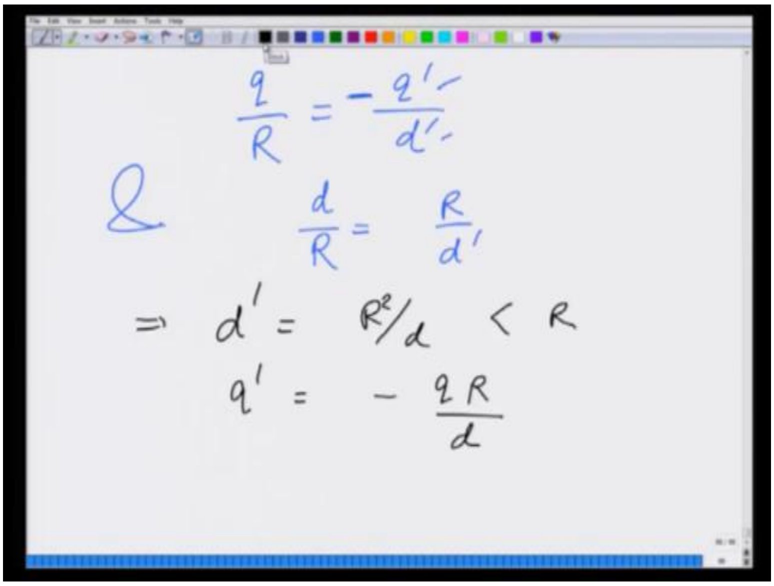

I can ensure that, If I take q over R to be equal to q prime over d prime with the minus sign in front and d over R to be equal to R over d prime, I have got two unknowns q prime and d prime and I have got two equations. So, this gives me d prime equals R square over d, which is certainly less than R because d is greater than R and q prime equals minus q R over d.

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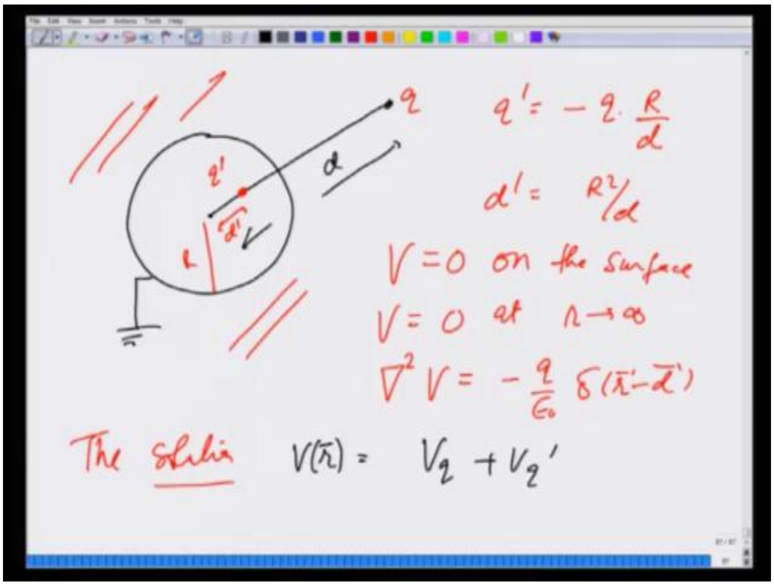

So, what I have found is that, if I have this sphere which is grounded and I put a charge at a distance d from it is centre, then if I put another charge here q prime, this is q, q prime equals minus q, R over d, radius is R. Add a distance d prime which is R square over d, then V is 0 on the surface and V is 0 at r equal to infinity. So, we have found this combination of charges that satisfies the boundary condition and in this region, outside this sphere, del square V is also equal to minus q over Epsilon 0 delta r minus this d vector, if it is on the Z axis it will become d Z.

So, we have found a combination of charges that satisfies the equation Poisson equation in the outer region, also satisfies all the boundary conditions, we have found these solution. That means, potential in this case V r is going to be V due to this q plus V q prime. So, what I have tried to convey to you is that, using the uniqueness of the solution, we can calculate at in certain situations, for example, we have done two examples where charge in front of a plane grounded surface and charge in front of a grounded sphere or grounded metallic sphere.

That we can find charge combinations that satisfy the boundary condition and the correct Poisson equation, and therefore that gives me the correct solution for that situation. I will leave the problems that I did in the case of planer surface charge in front of a planer surface for you to try and also give it in the assignment.