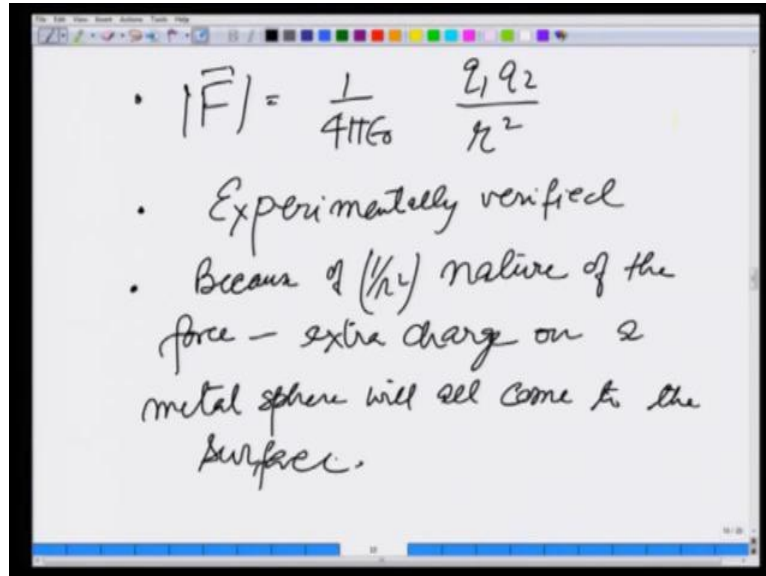


Introduction to Electromagnetism
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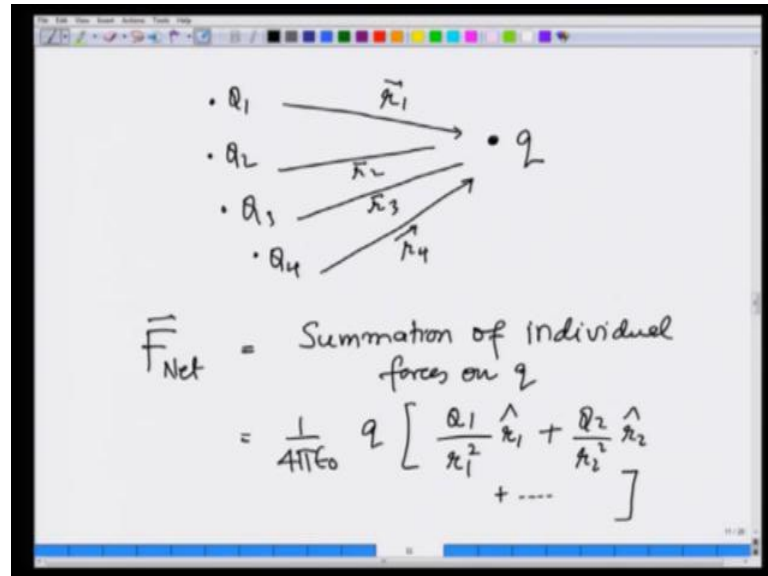
Lecture - 2
Coulomb's force due to several point charges

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The Coulomb force follows the principle of superposition; that is the force due to several charges is given as the sum of force due to individual charge. So, this is what we learnt so far.

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Now because of this nature; suppose I take now charge q_1, q_2, q_3, q_4 and so on, and try to find what is the force on a given charge q . Let this distance be r_1 . Let this distance be r_2 . Let the distance be r_3 . Let this distance be r_4 , and let me write this vectors. So, vector this is vector, this is vector, this is vector, this is vector. Then the net force is going to be summation of individual forces on q , and this has to be vector sums. So, this is going to be $\frac{1}{4\pi\epsilon_0}$, which is common; $q q_1$ over r_1 square r_1 unit vector plus $q q_2$ over r_2 square r_2 unit vector plus so on. Notice that vectors have been written in such a way, that whether they are positive or negative charges, all the directions of properly taken care of.

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$$\vec{F}_{\text{Net}} = \frac{1}{4\pi\epsilon_0} q \sum \frac{Q_i}{r_i^2} \hat{r}_i$$

$$= \frac{1}{4\pi\epsilon_0} q \sum \frac{Q_i}{r_i^3} \vec{r}_i$$


Principle of superposition

If charge Q_1 is doubled, force due to Q_1 also gets doubled.

So, I can write that the net force F_{net} , is equal to $\frac{1}{4\pi\epsilon_0}$ forces on q summation q_i over r_i square r_i unit vector. Sometimes this is also written as $\frac{1}{4\pi\epsilon_0}$ cube summation q_i over r_i cubed and make this a vector, is the same thing, because r_i vector divided by r_i is going to be r_i unit vector. This is also known as principle of superposition; that means, the net field can be calculated by adding fields in a linear manner. What it also means is, if charge q_1 is doubled force due to q_1 also gets doubled.

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Example 1 : Find the force on q



$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{(x^2 + \frac{a^2}{4})} \cdot \frac{x\hat{x} - \frac{a}{2}\hat{y}}{(x^2 + \frac{a^2}{4})^{1/2}}$$

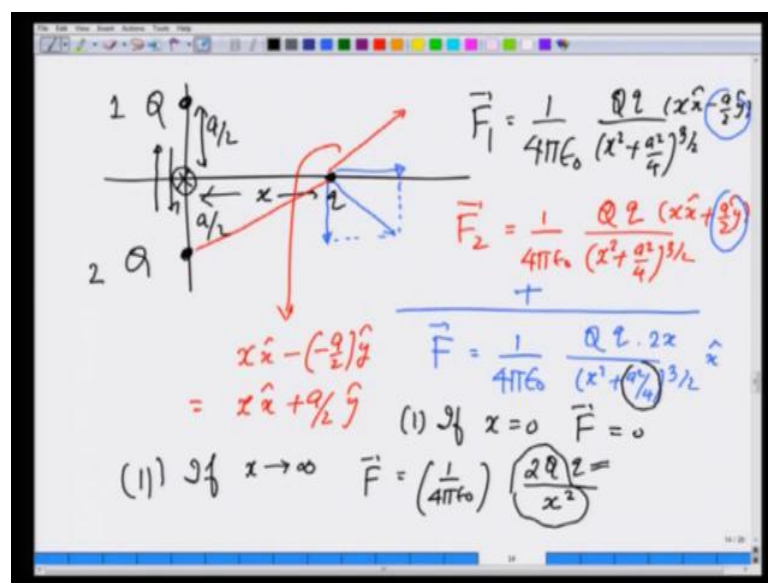
$$= \frac{1}{4\pi\epsilon_0} \frac{Qq}{(x^2 + \frac{a^2}{4})^{3/2}} (x\hat{x} - \frac{a}{2}\hat{y})$$

Unit vector = $\frac{x\hat{x} - \frac{a}{2}\hat{y}}{\sqrt{x^2 + \frac{a^2}{4}}}$

So, now let us use this to calculate forces on some configuration. Suppose I have two charges; example one, suppose I have two charges, separated by certain distance. Let us say this is in the x y plane, x y. Let this be charge q, let this be charge q, separated by a distance a. And I put a third charge q at a distance x from them, this is q. Find the force on q. Now force, as we said earlier on q is going to be force due to a upper charge. Let us call this 1, let us call this 2. So, this is going to be force due to 1 plus force due to 2. this is a vector force right. So, let us take the charge 1 and see what happens. Here is charge 1 q, here is charge q. This is here a distance x, this is here a distance a by 2. The force is going to be given by this vector.

So, F_1 is going to be $\frac{1}{4\pi\epsilon_0} \frac{q^2}{(x^2 + \frac{a^2}{4})^{3/2}}$. What is this distance, this is $x^2 + \frac{a^2}{4}$ square root. Square of that is going to be $x^2 + \frac{a^2}{4}$, times unit vector in this direction. And what is this vector. This vector is nothing but this vector minus this vector. So, this going to be $x\hat{x} - \frac{a}{2}\hat{y}$; that is a vector form charge 1 2 q, charge 1 2 q, and therefore unit vector is equal to this vector $x\hat{x} - \frac{a}{2}\hat{y}$ to y divide by its magnitude, which is going to be square root of $x^2 + \frac{a^2}{4}$ by 4. So, I put this here, I am going to get the unit vector $x\hat{x} - \frac{a}{2}\hat{y}$ divided by $\sqrt{x^2 + \frac{a^2}{4}}$. So, that my final answer in this case is, $\frac{1}{4\pi\epsilon_0} \frac{q^2}{(x^2 + \frac{a^2}{4})^{3/2}}$, which is nothing but the distance raise to three times a vector $x\hat{x} - \frac{a}{2}\hat{y}$.

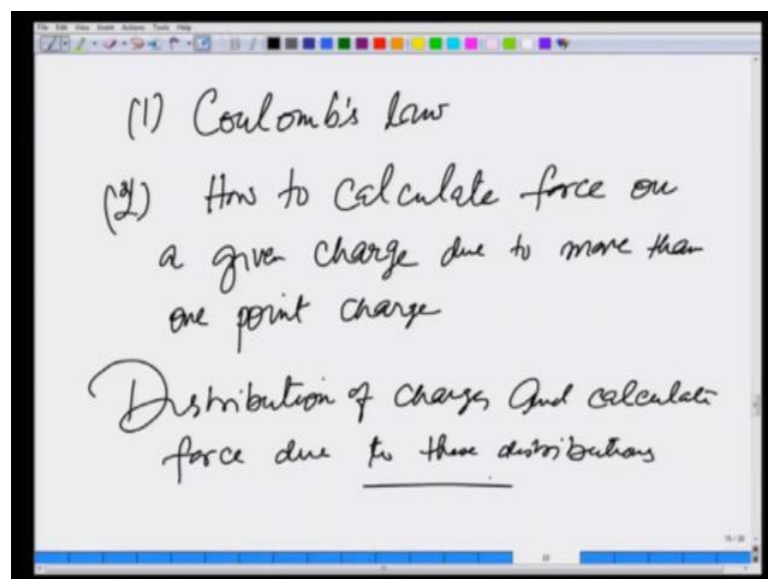
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Let me write it again. I have this configuration charge q here, charge q here, charge q here at a distance x a by 2 a by 2. I am calling this charge 1, I am calling this charge 2. Then force 1 F_1 if you got out of $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$. Let us see it makes sense. It tells me that force in the x direction is this way, and force in the y direction, is in the minus direction, which is perfectly because the net force is going to be like this, x component like this and y component like this. I can similarly now write F_2 ; F_2 has a same magnitude, except that, is along in this vector. And what is this vector. This vector is nothing but x x minus a by 2 y , which is x x plus a by 2 y , and therefore, F_2 I can write as, $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2 + a^2)^{3/2}}$, which is the same distance. This distance and upper distance is the same, a vector out here changes x x plus a by 2 y .

So, you now found both the forces, add them up and I get the net force, is equal to $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2 + a^2)^{3/2}}$. Notice that this part will cancel; 1 is positive sign, and 1 is negative sign for times 2 x x ; that is the answer. Note one, if x is 0 F is equal to 0; that make sense, because of this charge is here. This fellow pushes it this way, the other fellow pushes this way, and the net force is 0. 2, if x becomes very large, I can ignore a square by 4, and then the net force F is $\frac{1}{4\pi\epsilon_0} \frac{2q_1 q_2}{x^2}$. So, it is as if, at center of there is a there is a charge $2q$ sitting applying force on q , and the other certain things, we are noticing.

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So, to conclude, what we have learnt in this chapter one, is Coulomb's law. How calculate force on a given charge due to more than one point charge. What we are going to do in the next lecture is consider distributed charges, distribution of charges and calculate force due to this distribution.

Thank you.