Introduction to Electromagnetism

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Lecture - 18
Electrostatic potential due to a charge distribution - II;
a ring and a spherical shell of charge

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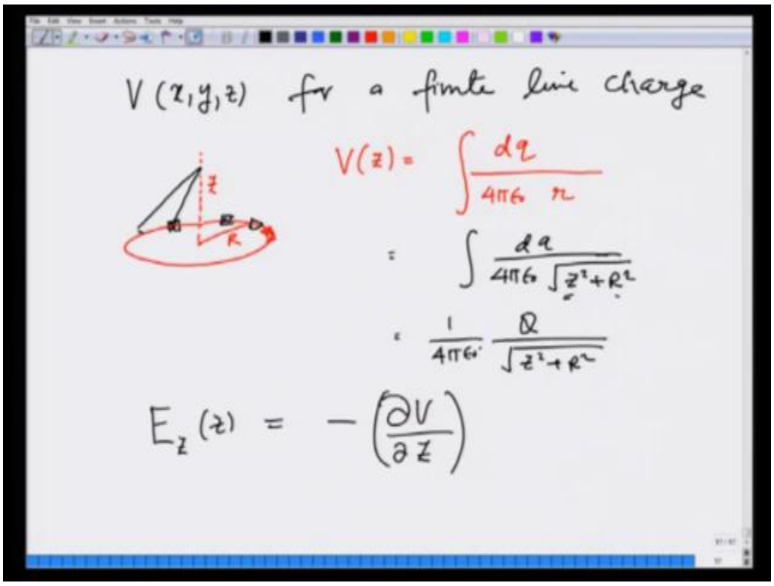

In the previous lecture, we calculated the potential for a finite line charge and then to the limit L going to infinity. In this lecture, we take a ring of radius r and calculate the potential on it is axis at a distance z. The calculation compared to electric field calculation that we did earlier in some earlier lecture is much easier. Because, all I have to do is calculate V at z is going to be d q, whatever that small d q charge I am taking on the ring divided by 4 pi Epsilon 0 times the distance from the small charge integrated over.

And in this particular case the calculation becomes very simple, because the distance from all these small charges that I take on the periphery, on the ring is the same for a given point on the axis. And that distance is integral d q over 4 pi Epsilon 0 square root of z square plus r square, where r is a radius of the ring. So, this integration does not really depend on z and r, it depends only on d q.

So, it can be written as 1 over 4 pi epsilon 0 Q, where Q is the total charge on the ring divided by square root of z square plus r square as simple as that. In your assignment, I will give you a problem to calculate the electric field in that z direction at z by taking gradient of this potential in this z direction or the z component of the gradient.

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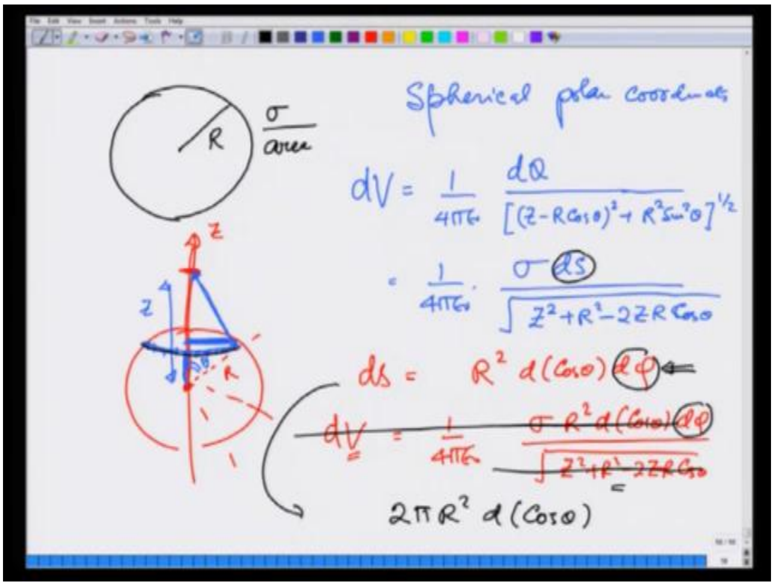

Using this, now I am going to calculate the potential due to a shell of charge. So, this is a shell of charge of radius R and carries sigma per unit area. Although, you may already know the answer, this will give you some practice into doing the calculations or integrations required in doing, you know potential calculations. So, for convenience, I take calculate the potential on the z axis, although because of the spherical symmetry, the potential is going to be same all around at the same distance.

So, later I can replace this z by small r, where r will indicate the distance from the center. So, this radius is R and I am calculating the potential at a distance z from the center, I am going to use spherical polar co ordinates. If I take a ring out here of charge at an angle theta, then I know this ring gives me a potential d v, which is equal to charge on this ring, we have just calculated this, 1 over 4 pi Epsilon 0 times the distance from the ring.

Distance from the ring periphery is going to be z minus this distance, which is z minus R, cosine of theta square plus this radius square, which is R square sin square theta raise to 1 half. So, this is going to be equal to 1 over 4 pi Epsilon 0, how much is the charge,

charge is sigma times d s, which is d s the area of this ring, divided by I can calculate this, it will come out to be z square plus R square minus 2 z R cos of theta.

Recall from one of the previous lecture that d s is R square d cosine of theta d phi. So, I can write d this potential is equal to 1 over 4 pi Epsilon 0, I just want to point out that do not confuse this d v out here with the volume integral. This is the potential, d v is equal to sigma R square d cosine theta d phi over z square plus R square minus 2, z R cos theta, just a word of caution.

The word of caution is that, I have written this d phi out here, but d q for d q I have written the charge on the ring. So, actually I should have carried out this phi integration already, because this indicates the total charge on the ring. I wrote this explicitly out here just to show that this can be integrated over, because in the integrant there is no phi. So, actually I should be writing, let me just cut this and correct.

This is just to tell you that this d phi is actually not required, d phi integration I should have already carried out to calculate the total charge on the ring. So, the total area of the ring I am going to write is 2 pi R square d cosine of theta.

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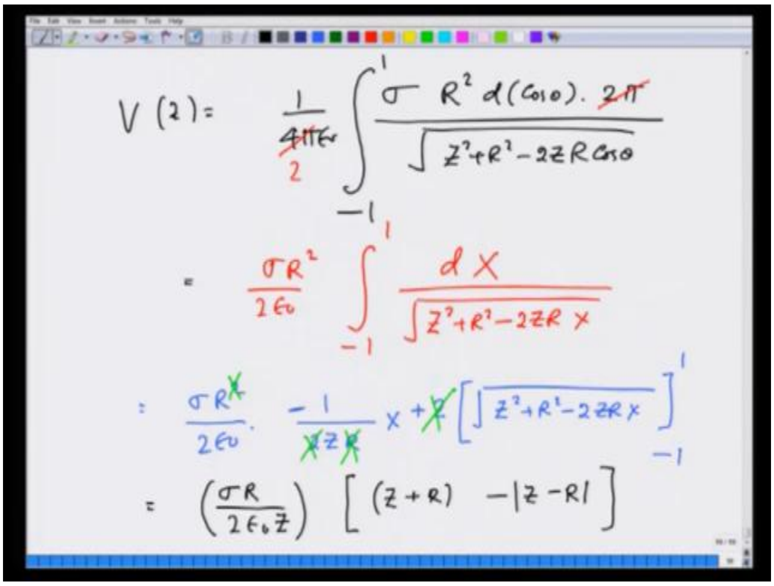

And therefore, the potential at distance z is actually V at z, it is 1 over 4 pi Epsilon 0 sigma R square d cosine of theta times 2 pi divided by a square root of z square plus R square minus 2 z R cos of theta and cosine of theta integrates from minus 1 to 1. So, this answer is this 2 pi cancels gives me 2. So, sigma over 2 Epsilon 0 R square integration minus 1 to 1, d cosine theta I can write it as d x divided by the square root of z square plus R square minus 2 z R x.

This integration is very easy carry out and I can write this as sigma R square over 2 Epsilon 0 times minus 1 over 2 z R times plus 2 square root of z square plus R square minus 2 z R x minus 1 to 1. This 2 cancels with this 2, this R cancels with one of the R and I get final expression as sigma R over 2 Epsilon 0 z. Inside I get, for 1 it becomes z minus R square with this minus sign that I will come later. So, modulus z minus R and here I get module z plus R, this is already a positive quality; that is my answer.

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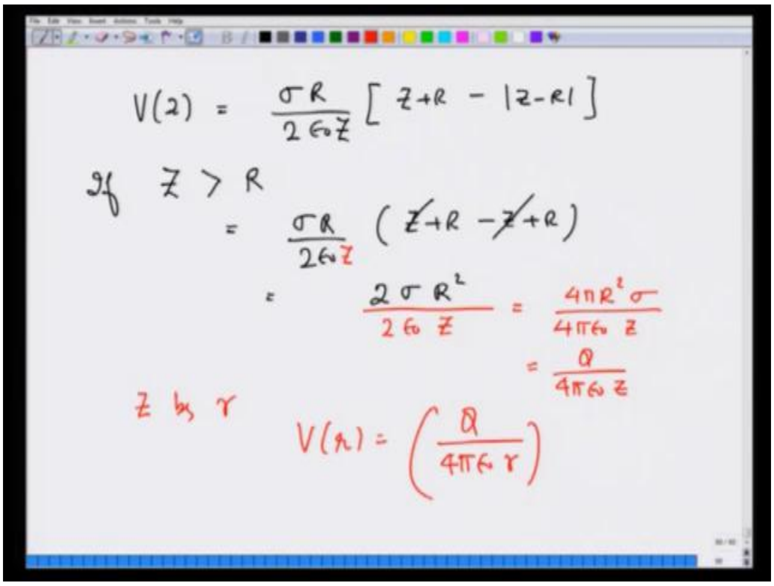

So, let us see what this answer is this answer is V z is equal to sigma R over 2 Epsilon 0 z plus R minus modulus z minus R. If z is greater than R then this goes to sigma R over 2 Epsilon 0 z plus R minuses z plus R. This z cancels and you get 2 sigma R square z here, z here, over 2 Epsilon 0 z, which can be written as 4 pi R square sigma over 4 pi Epsilon naught set, which is q over 4 pi Epsilon naught z. Now, I can replace z by R, because it does not really matter spherically system, and therefore V at R is nothing but q over 4 pi Epsilon 0 R.

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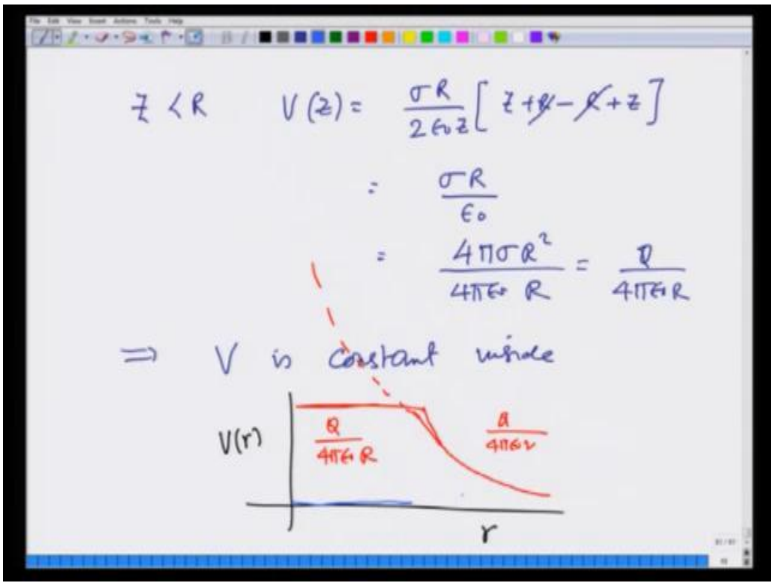

What about the other, inside this sphere, inside this shell, I am going to have said z less than R, and therefore V at z becomes equal to sigma R over 2 Epsilon 0, z plus R minus R plus z, is z here. This R cancels and I get sigma R over Epsilon 0, which is 4 pi sigma R square over 4 pi Epsilon 0 R, which is nothing but q over 4 pi Epsilon 0 R, implies V is constant inside.

So, how does V vary, V R with R is a constant inside, and then goes as q over 4 pi Epsilon 0 R. This should be little smoother, because this like this and this value is constant, which is this value is constant q over 4 pi Epsilon 0 R. You can see potential inside is constant and therefore, is gradient is going to be 0. The field inside therefore, field inside is 0, and then goes as 1 over r square outside.