Introduction to Electromagnetism
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Lecture - 17
Electrostatic potential due to a charge distribution – I a line charge of finite length

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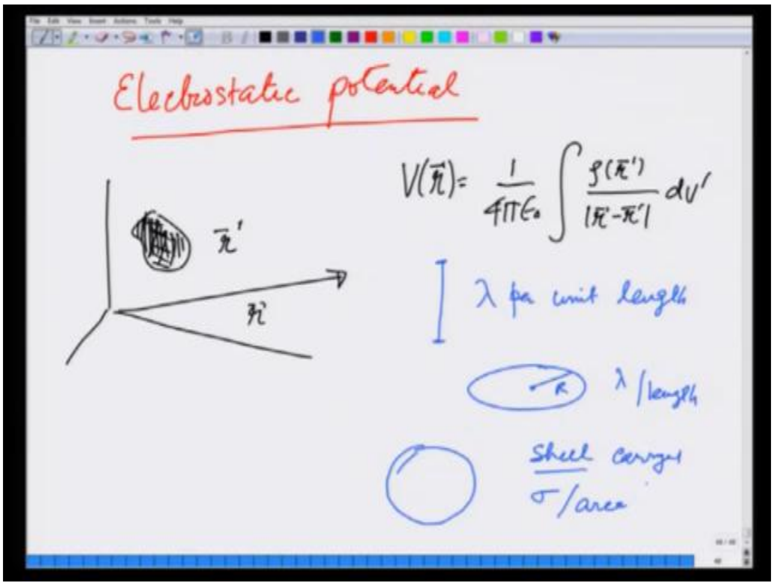

We have seen that the electrostatic potential due to a charge distribution is given as, if there is a charge distribution at r prime and I am calculating potential at vector r. Then the electrostatic potential V r is given as 1 over 4 pi Epsilon 0, integration rho r prime over r minus r prime d v prime, where d v prime indicate that, I am integrating over the r prime variable or over this volume, where the charge distribution is non-zero.

What we will do in this lecture is, use this to calculate potentials for two or three charge distributions. The examples I am going to take r going to be 1 a linear charge of charge density lambda per unit length. A ring of charge of radius r, again carrying charge lambda per unit length, this can be easily generalized to a disc of charge, which we did earlier in the case of electric field or force on a charge q. But, I am going to use it this time to calculate the potential due to a uniformly charge shell, a spherical shell carrying sigma charge per unit area.

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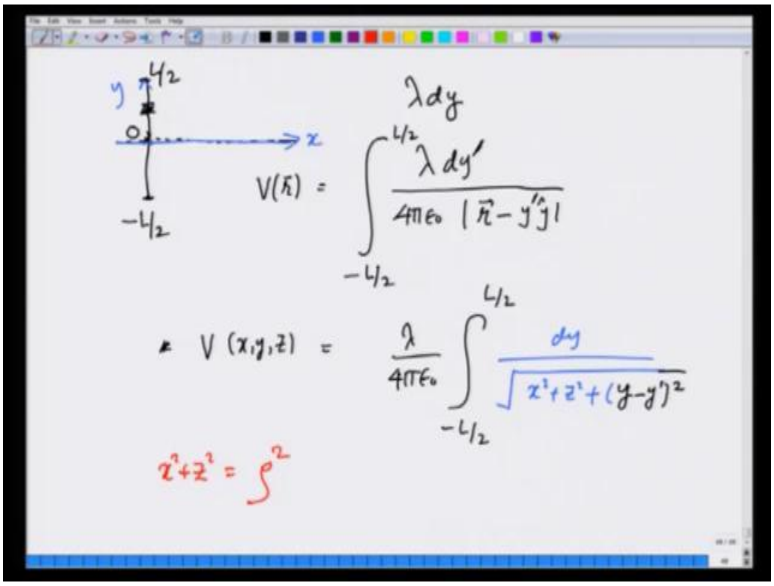

So, let us begin with the line charge and let us say, this is given at the origin with the center at the origin. The length is from minus L by 2 to L by 2. And now, you see the advantage of using potential to calculate electric field later, because in the potential, I do not have to worry about any components. All I have to do is, if I calculate the potential due to this small charge lambda d y, then all I have to do is, calculate lambda d y over 4 pi Epsilon 0 and the distance of that point from r is going to be r minus y, y unit vector.

And this integrate and y varying from minus L by 2 to L by 2, no components nothing just one calculation. So, let us just do that. So, this is going to be integral. So, this is V at r or I can write this as V x y and z is equal to lambda over 4 pi Epsilon 0 comes out and I have minus L by 2 to L by 2, the charge is sitting on the y axis x y. So, x and z components of the charges are 0, and therefore I have d y over square root of x square plus z square plus I am calculating it at some point y.

So, I should just change the notation a bit, I will put a prime to indicate as I have been doing to show that it is a y prime over which I am integrating. So, I am calculating field the potential at y, y minus y prime square that is it, that is the integral that we are going to do.

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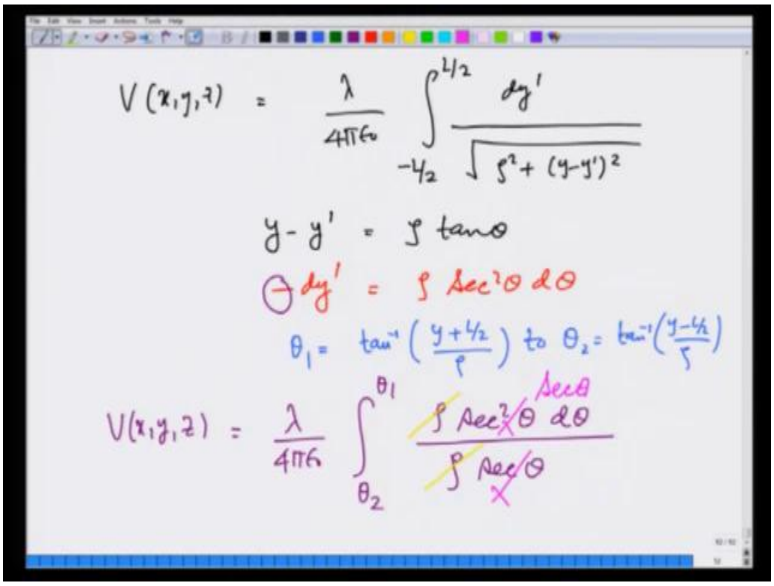

Let us now to do the integral, let us take x square plus z square to be some rho square. So, V x y z is given as lambda over 4 pi Epsilon 0, integration d y prime over square root of rho square plus y minus y minus y prime square y prime varies from minus L by 2 to L by 2. To perform the integral, let me write y minus y prime equals rho tangent of theta, so that I have minus d y prime equals rho secant square theta d theta and the limits are from theta 1 equals tan inverse y plus L by 2 over rho to theta 2 equals tan inverse y minus L by 2 over rho.

So, that the integral now can be written as V x y z equals lambda over 4 pi n Epsilon 0. This minus sign changes the limits, the other way, so I am going to have theta 2 to theta 1 rho secant square theta d theta over rho secant theta and this rho cancels and one of this secant theta is cancels and I am left with the secant theta here.

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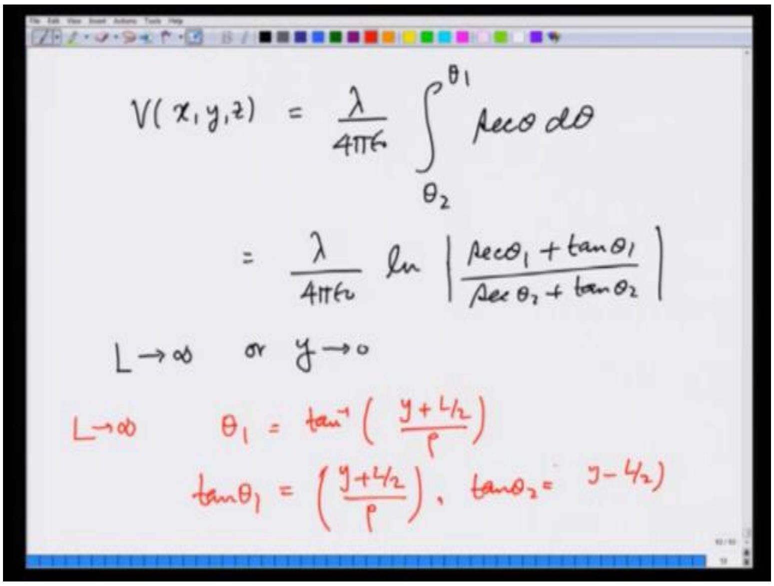

And therefore, the potential then becomes V x y z equals lambda over 4 pi Epsilon 0 theta 2 to theta 1 secant theta d theta, which is nothing but lambda over 4 pi Epsilon 0, log of secant theta 1 plus tan theta 1 divided by secant theta 2 plus tangent of theta 2; that is the answer. Now, our particular interests, either you take limit L will go to infinity or y going to 0. Let us take the limit L going to infinity. In that case, theta 1 which was equal to tan inverse y plus L by 2 over rho becomes tan theta 1 equals y plus L by 2 over rho and tan theta 2 becomes y minus L by 2 over rho.

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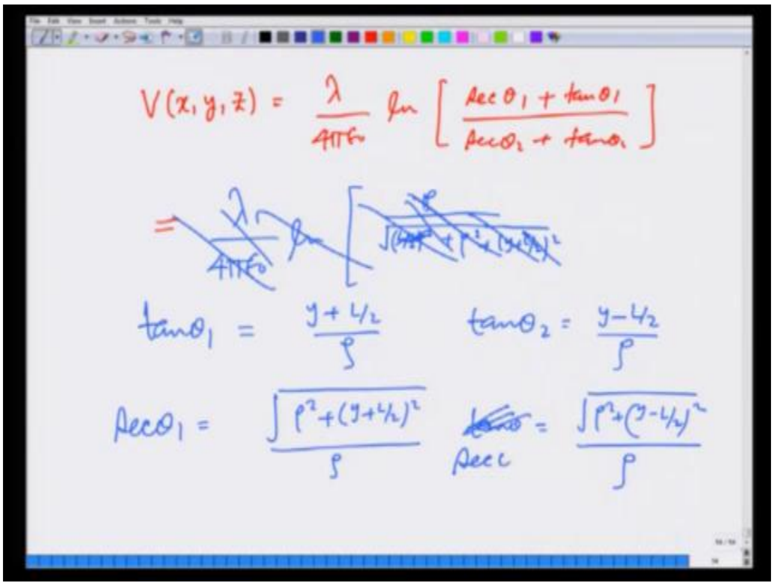

This gives V x y z as lambda over 4 pi Epsilon 0 log secant theta 1 plus tan theta 1 over secant theta 2 plus tan theta 2 equal to lambda over 4 pi Epsilon 0, log of secant theta 1 will be 1 over cosine theta 1 which becomes 1 over. Let us check that tangent of theta 1 is y plus L by 2 over rho, so this will become rho over squared root of L by 2 square plus rho square plus y plus L by 2 whole square.

We have tangent theta 1 is equal to y plus L by 2 over rho tangent of theta 2 equals y minus L by 2 over rho. And therefore, secant theta 1 is equal to squared of rho square plus y plus l over 2 square divided by rho tangent theta 2 is equal to square root of rho square plus y minus n by 2 square over rho, this is secant theta 2.

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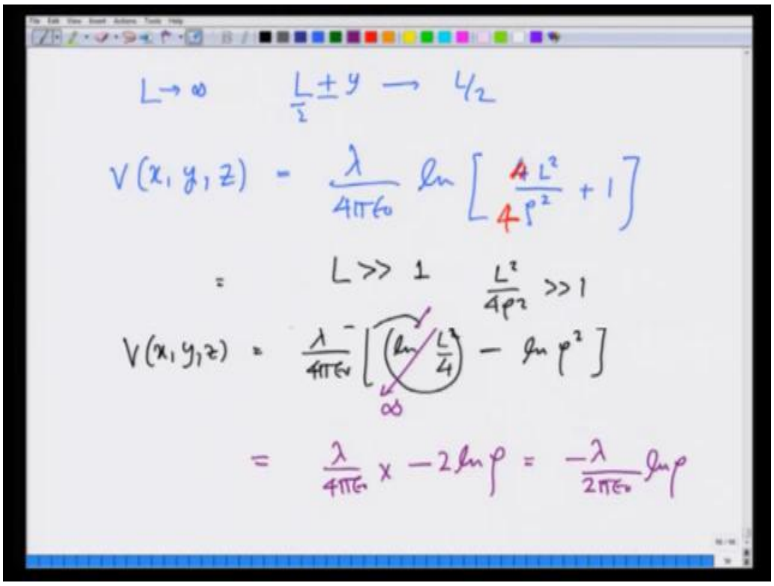

You work all this out and in the limit of L tending to infinity, so that l plus or minus L by 2 plus or minus y can be taken as L by 2, you get V x y z as equal to lambda over 4 pi Epsilon 0, log of 4 L square over rho square plus 1, 4 is not on top, 4 is a bottom. It is comes out to be this, which I can write as equals since L is much, much, much greater than 1, L square over 4 rho squared is going to be much, much, much greater than 1.

So, I can write all that as V x y z equals lambda over 4 pi Epsilon 0, log l square by 4 minus log rho square as I will tells to infinity this fellow goes to infinity. However, in calculating potential, I can always neglect that infinity part. So, I can write this as lambda over 4 pi Epsilon 0 minus times minus 2 log rho, which is minus lambda over 2 pi Epsilon 0 log rho. That is the answer for L tend to infinity, this match as well with congenital expression and that you know for an infinitely long wire.