Introduction to Electromagnetism
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Lecture - 15
Electric field as the gradient of Electrostatic Potential

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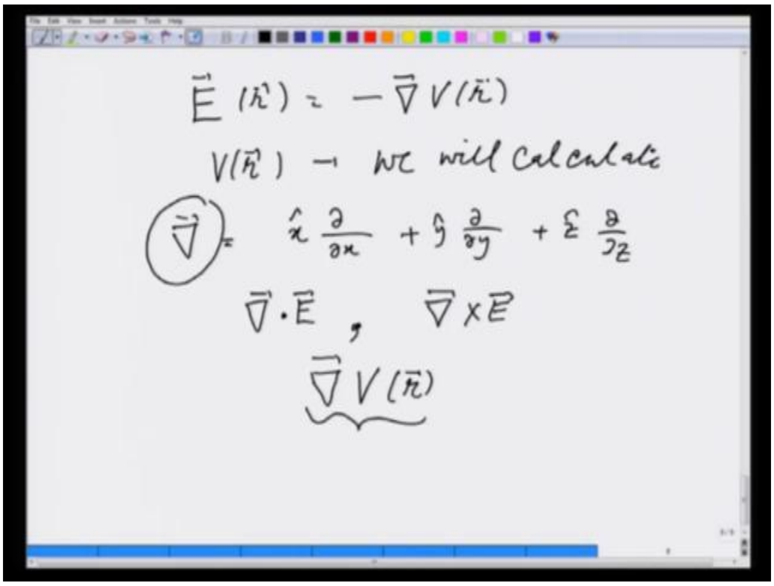

We saw in the previous lecture, that we can write E at any point as minus the gradient of potential, where V is we have to calculate in this lecture. But, before that let us understand, what this gradient operator means. Gradient we wrote as partial derivatives in the direction x with respect to x, plus y partial y plus z partial z. So, this operator, we use in three forms now, we have used it to calculate divergence of E, we have used it to calculate curl of E and now, we have used it to calculate gradient of a scalar field if you like. Let us understand the meaning of this gradient which will also give us insights into how E and V are related.

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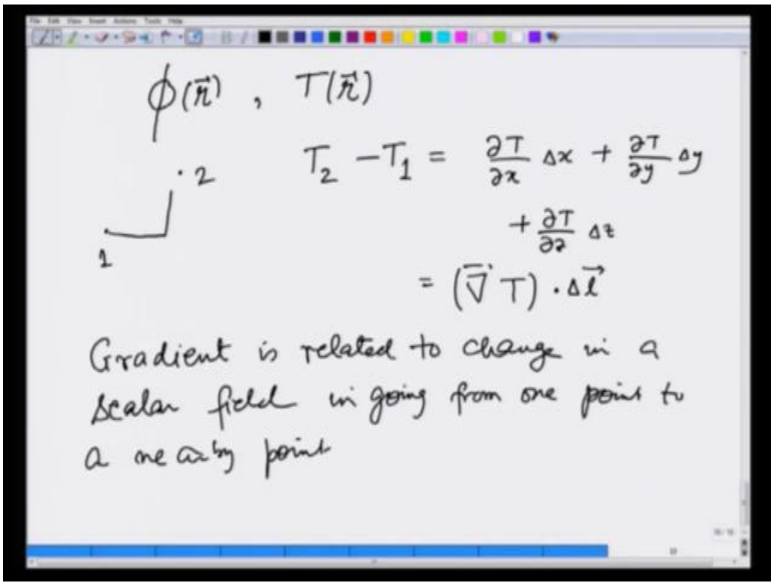

You see gradient was defined by calculating, suppose there is a scalar field, now let me write this as phi r not necessarily potential, but phi r or it could be say temperature distribution in a room. What we saw that, if I take two points and calculate the difference, let us say this is 2, this is 1, T at 2 minus T at 1. This is given as partial derivative of T with respect to x delta x plus partial T over partial y delta y plus partial T over partial z delta z, which we denoted as gradient of T dot delta l. So, gradient is related to change in a scalar field in going from one point to a nearby point. Remember in one dimension the d by d x times delta x is a change, this is the three dimension analogue of it.

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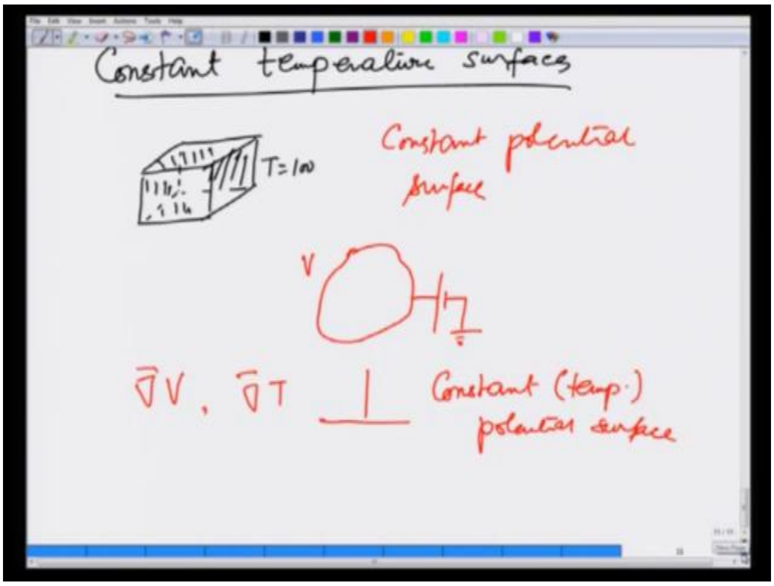

Let us understand how we can visualize this. So, suppose I take constant temperature surfaces, let us see, what do we mean by that, suppose I take a box and put all the surfaces at the same temperature. Unless, T equals 100 degrees or let us also look at constant potential surfaces, suppose I take metallic geometry and connect it to a battery, the other side the battery is grounded. So, that the potential all over is V or equal to the EMF, then this is also constant potential surface. Then the gradient of V or gradient of T is going to be perpendicular to the constant, property we are talking about temperature or potential surface, how do we see that.

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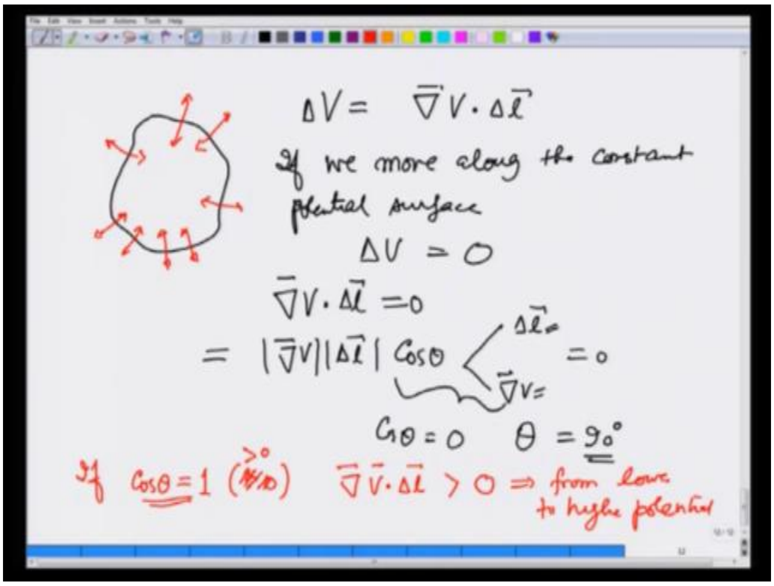

To understand that, let us make this constant potential surface, then I know from going from point 1 to point 2, the change in the potential is equal to grad of V dot delta l. If we move along the constant potential surface delta V is equal to 0 and grad of V dot delta l is 0, this is nothing but equal to modulus of grad of V, modulus of l, cosine of angel between delta l and grad of V and this is 0.

Always 0, if you moving along the constant potential surface; that means, cosine theta is equal to 0 or theta is equal to 90 degrees, what we have just seen is that, angle between delta l and gradient is 90 degrees. That means, gradient is perpendicular to the constant potential surface, but still we are not sure about this direction, would it go from higher potential to lower potential or lower potential to higher potential. That is also very easy to see.

If cosine theta is equal to 1 or greater than 0, then grad of V dot delta l is greater than 0 and I am moving, since this is greater than 0, this means we are moving from lower to higher potential. So, cos theta equal to 1 or greater than 0 positive, so that means, cos theta is positive, I am moving from lower to higher potential.

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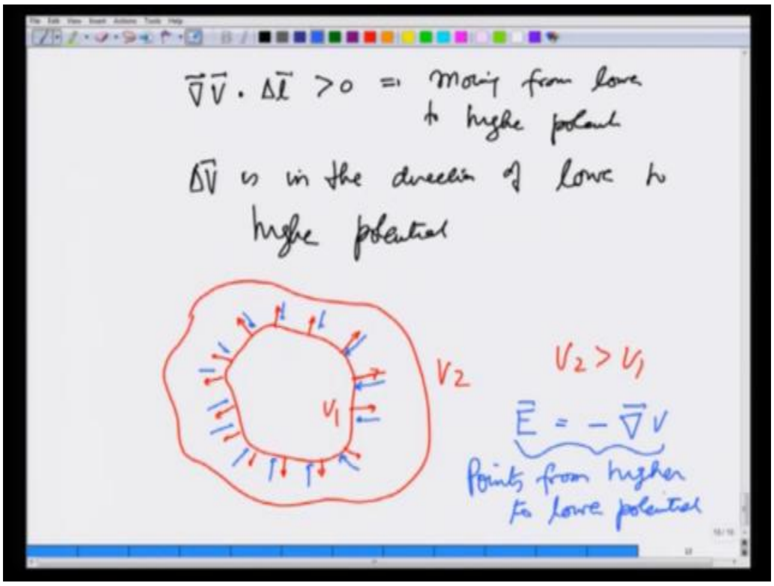

So, grad V dot delta l greater than 0, implies moving from lower to higher potential; that means, delta V is in the direction of lower to higher potential. So, let us see geometrically what does it mean, suppose I have two constant potential surfaces, one is this, the other one is this, this is V 2, this is V 1 and let us say V 2 is greater than V 1.

Then, gradient will be perpendicular to this at each point and pointing towards higher potential everywhere.

Now, remember that electric field is minus grad of V and that means, electric field would be from higher to lower potential, electric field would be pointing exactly the other way. So, electric field points from higher to lower potential. So, you understood the relationship between gradient and constant potential surfaces.

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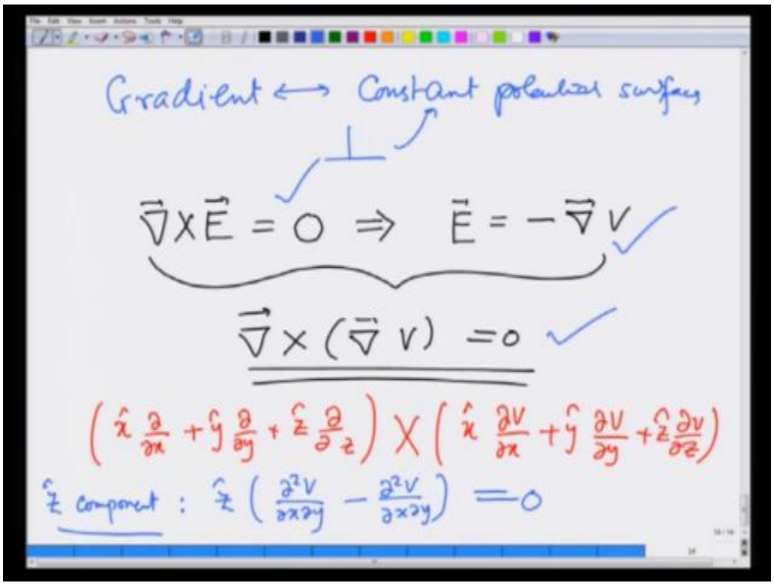

Gradient is related to constant potential surfaces, in fact, it is perpendicular to this and electric field points in the direction from lower to higher potential and electric field points from higher to lower potential, because minus the gradient. Further, you just seen that, why we can define a potential, because curl of E is 0 and this implies, that I can write electric field as minus grad of V. This also, if I combine these two together, this means curl of a gradient of any quantity will be 0, this is a mathematical property, is very easy to see.

So, we have just argued it from physical principles, but let us see, curl of gradient would be x partial x plus y partial y plus z partial z, cross product with x d v d x plus y d v d y plus z d v d z. Let us look at z component, x cross x gives me 0, x cross y gives me z, it will be d 2 v by d x d y, I am looking only exit component. So, x cross z gives me, why I am not worry about that, y cross x gives me z, but with the minus sign. So, it will become minus d 2 v by d x d y which is 0.

So, this curl of being 0, V being equal to minus grad of V and gradient of curl being 0, they are all related with each other.

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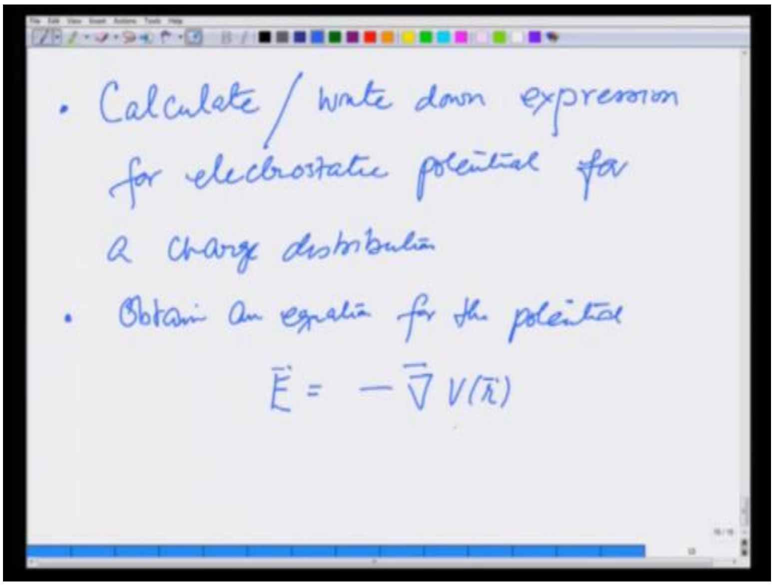

Next what we want to do is, calculate or right down expression for potential for a charge distribution obtained an equation, just like the electric field equations for the potential. We want to do so because, you may have noticed by now that E is a vector quantity therefore, whenever I wanted to calculate E, I have to calculate three quantities. On the other hand, if I now calculate the potential, which is a scalar quantity calculating electric field becomes quite easy, because I can just take the gradient of the scalar quantity and obtained the electric field on it, these two things will do in the next lecture.