Introduction to Electromagnetism
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Lecture - 14
Electrostatic Potential

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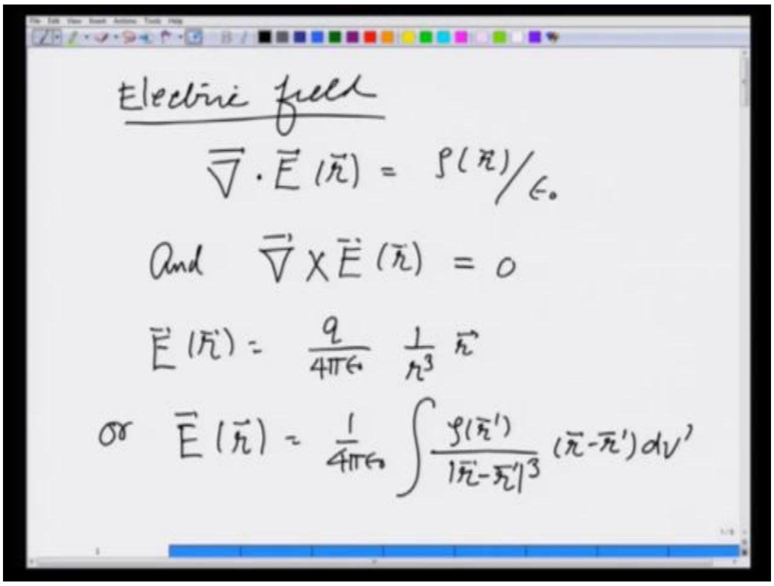

In the previous lecture on Electric Field, we have seen that, electric field satisfied the equations that it is divergence at any point was related to the charge density at that point and curl of E is always 0. We needed these differential equations, because there may be situations, where the electric field although the formula for electric field in free space. We had already written was q over 4 pi Epsilon 0, 1 over r cubed, r or for a charge distribution E r is given as 1 over 4 pi Epsilon 0 integration rho r over r minus r prime cubed r minus r prime d v prime. But, this is only in free space.

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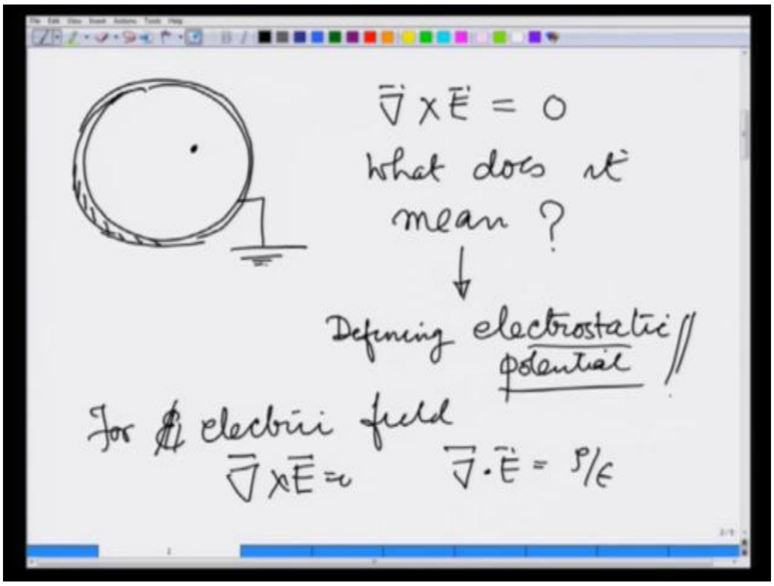

This need not be the case; for example if I take a charge say inside a grounded sphere. Then, this charge, since this is shielded by a metallic grounded sphere. It need not give a field which is given by the previous formula. And therefore, what one needs to do is solve those differential equations with the boundary condition, appropriate boundary condition, we will talk about it a little more little later.

For the time being, let us understand the meaning of curl equation. What does it mean? This will automatically lead us to defining, understanding this, we lead us to defining electrostatic potential, which is easier to deal with compare to electric field, because for electric field, we have two differential equations. We have equations for it is curl as well as it is divergence and we will see where electrostatic potential will have only equation, because this will be a scalar quantity.

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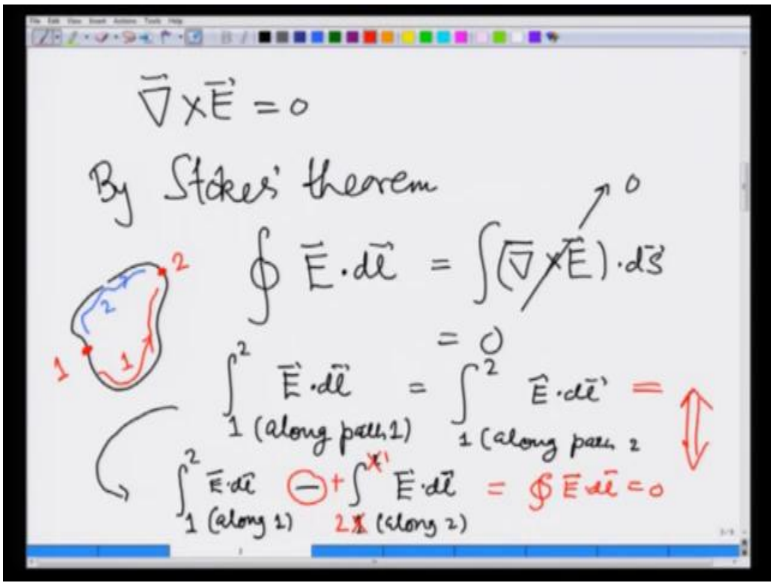

So, let us see, when I have curl of a quantity is equal to 0. By Stokes theorem, we have over a closed path the line integral of E, which is equal to the area integral of the curl of E, this we did last time is equal to 0, because this fellow is 0. And therefore, if I take two points 1 and 2 and let me choose one path along this call this path 1, second path along this call this path 2.

Then, integral 1 to 2, let us say along path 1, E dot d l is same as integral 1 to 2 along path 2. If the curl of E is 0, this is because from here, what I get is 1 to 2 along 1, E dot d l minus 1 to 2 E dot d l along 2. And now, I am going to change sign, I will make this plus and change this from 2 to 1, because going from 1 to 2 is same as minus going from 2 to 1. And therefore, it becomes 1 to 2 plus 2 to 1 and this becomes integral over the entire path E dot d l, which is 0. So, this condition implies this and vice versa.

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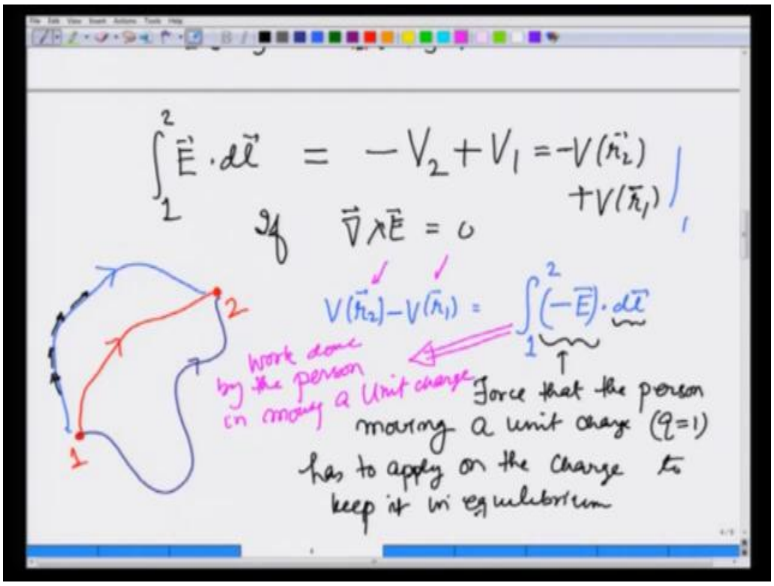

So, curl of 0, we have seen tells me that, if I travels between two points 1 and 2, the integral is independent of the path. And therefore, I can write E dot d l between two points 1 and 2 is independent of path and I am going to write this in a particular way, I will write it as minus v 2 plus v 1, if curl of E is 0. Let us understand, what these v's stand for and also write this in a better way, v at r 2 plus v at r 1.

So, I have taken these two points 1 and 2 and irrespective of what path I choose to go between these two E dot d l always turns out to be the same and that is why, I can write it just as the difference between quantities at these two points only. Let us write this slightly differently. So, I will write v at r 2 minus v at r 1 is equal to integral minus E dot d l from 1 to 2.

What is minus E? This is force that a person moving a unit charge q equal to 1 has to apply on the charge to keep it in equilibrium. So, minus E is the force that a person holding the charge has to apply to keep in equilibrium and d l is distance travelled. So, this is d l, this is d l, this is d l, this is d l and integral means, I am adding over all these d l's. This is the force applied by the person, and then the person is moving it, therefore, this represents the work done.

Let me write the different this represents the work done by the person in moving the charge, moving a unit charge and this v; I am going to call the potential. So, potential difference between points 2 and 1 is the work done by an external agency or a person in moving the charge keeping it in equilibrium, in moving it from 1 to 2, let us recap it.

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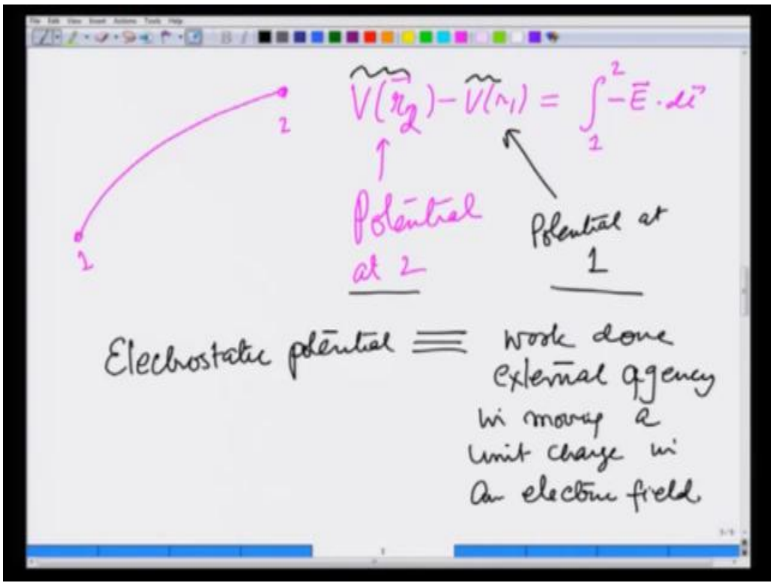

So, if I am going to points and a person moves a charge from point 1 to point 2, this is even as minus E dot d l from 1 to 2. So, the potential difference between point 1 and 2, this I am calling this potential at 2, this is potential at 1. So, electrostatic potential difference between two points is the work done in moving a unit charge by a person taking it from 1 or 2 and it is independent of the path and that is why I can define this potential.

So, now what I would be understood is that electrostatic potential is equivalent to work done not by the electric field by an external agency or an external. Let us write it agency in moving a unit charge in an electric field from point 1 to 2. Notice that, even if I add a constant to both these the difference remains the same and therefore, potential is defined only up to a constant. Because, after all is the work done or the potential difference which is the physical quantity.

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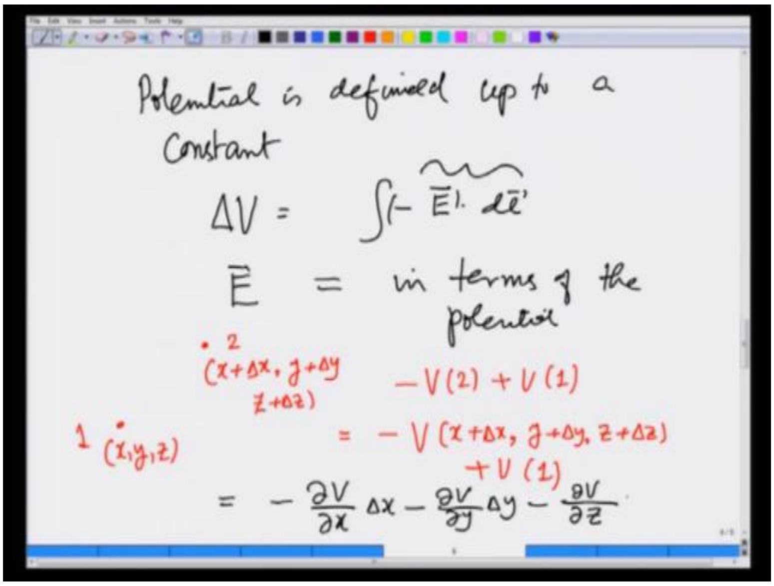

So, adding a constant to a potential does not make a difference and therefore electrostatic potential is defined up to that is means a constant. That means, I can add or subtract any constant in it and nothing no physics changes. What we have seen is that, we have related potential difference to the electric field work done, how about the inverse relationship, how do I express E in terms of the potential. You can already see that sense delta v is the integral, it has to be some sort of derivative and this introduces the idea of gradient.

So, for that, let us take two points nearby point 1, which is at x, y and z and point 2, which is at x plus delta x, y plus delta y and z plus z. Then, minus v at 2 plus v at 1 is going to be minus v at x plus delta x, y plus delta y z plus delta z plus v 1 and this by definition of partial derivatives is equal to minus partial v with respect to partial x delta x minus partially v or partial v over partial y delta y minus partial derivative of v with respect to z delta z.

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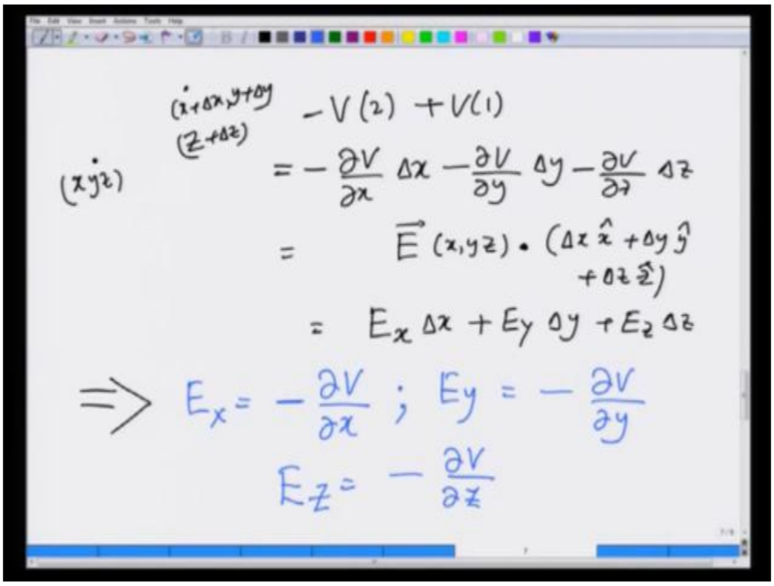

So, between points 1 and 2, x, y, z, x plus delta x, y plus delta y, z plus delta z; we have seen that v at 2 plus v at 1 is minus partial v minus partial x delta x minus partial v partial y delta y minus partial v partial z delta z and this is equal to E. Let us say at x, y, z dot d l, which is going to be delta x in x direction. It was delta y in y direction plus delta z in z direction, which is equal to E x delta x plus E y plus delta y plus E z delta z, I can take delta x is equal to 0, delta y equal to 0, delta z equal to 0 or any other combination.

And since, so there arbitrary and this immediately therefore, implies that E x is nothing but minus partial derivatives of v with respect to x, E y is nothing but partial derivative of v with respect y and E z, z component of electric field is minus partial derivative of v this with respect to z.

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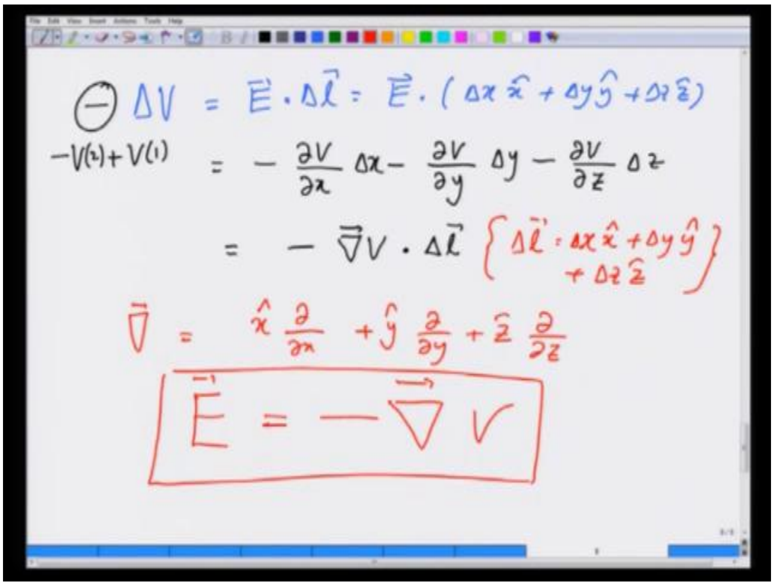

And delta v, which is E dot delta l or E dot delta x x plus delta y y plus delta z z is nothing but equal to minus partial v over partial x, delta x minus partial y, delta y minus partial v over partial delta z, I should write the minus sign here, because this is minus v 2 plus v 1. I am going to write this as symbolically as minus gradient of v dot delta l, where delta l is equal to delta x in x direction plus delta y in y direction plus delta z in z direction. A.

And gradient, I am going to write this as x partial x plus y partial y plus z partial z and this gives me that E electric field is nothing but minus gradient of v. This is the inverse relationship of what we learnt earlier. In the next lecture, we are going to learn a little few things about the gradient operator and how v and E are related geometrically.