Introduction to Electromagnetism
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Lecture - 11
Line surface area and volume elements in Cartesian and Cylindrical Coordinates

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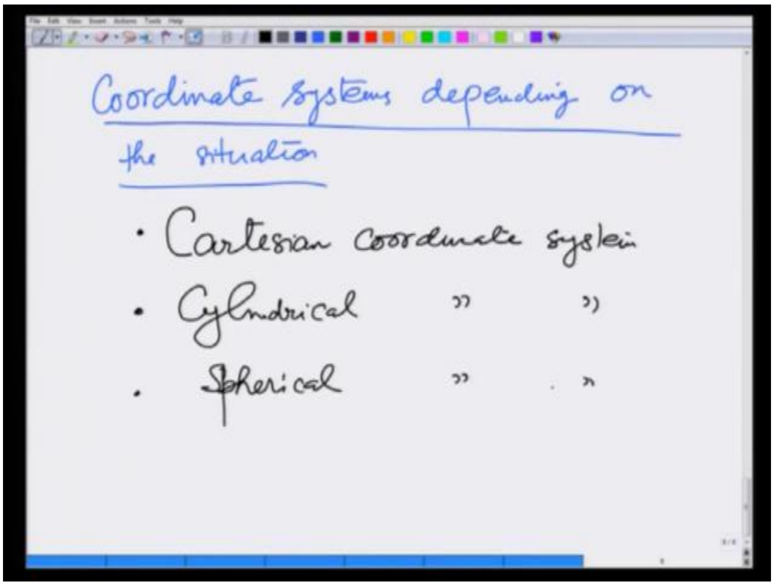

As you notice so far, we have been dealing with the calculating electric field or force on charges and for this, what we need to do is, use different coordinate systems depending on the situation. So, what I will do in this lecture is just review three coordinate systems for you and their line elements and surface elements and volume elements, so that you can use them easily. I am going to review for you Cartesian coordinate system, I am going to review for you the cylindrical, coordinate system, and finally the spherical, coordinate system. These are the other ones that we use most often in a many problems.

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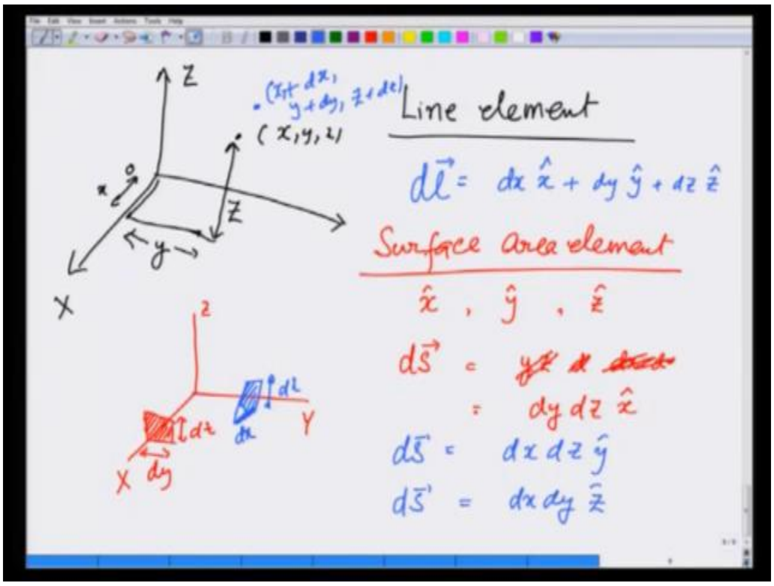

So, let us start with Cartesian coordinate system in which from the origin a point is given by it is x y and z coordinates. So, if I am giving a point x, y and z, I am moving in direction by x, y direction by y and then z direction by z. So, this distance will be y, this distance will be x and this height will be z. Line element in this coordinate system is, when I go from point x, y, z to a next point nearby, which could be x plus d x, y plus d y and z plus d z.

So, line element d l vector is given as d x unit vector in x direction plus d y unit vector in y direction plus d z unit vector in z direction. Next, I am going to deal with surface area element and I want you to get used to the idea of surface area having a vector nature. Particularly, when we saw that we are calculating flux of electric field, then we used surface area as a vector quantity.

Similarly, when we were calculating divergence and things like those and when you use Divergence theorem, we took surface area as a vector quantity. So, in Cartesian coordinates, I want to identify surface area perpendicular, surface area in x direction, surface area in y direction and surface area in z direction. Surface area in x direction by convention is going to be perpendicular to the x axis.

So, it is parallel to the y z plane and this distance being d y, this distance being d z, I am going to write surface area element d s as y z, d x, d y, no it is not d x is perpendicular to x. So, therefore, I should write it as d y, d z in the x direction. Similarly, the area element in the y direction is going to be perpendicular to the y axis and it is going to have the x element x width d x, height d z.

And therefore, in that direction d s is going to be d x, d z in the y direction and in the z direction, similarly I can have this as d x, d y in the z direction. So, I defined surface area element in the Cartesian coordinates.

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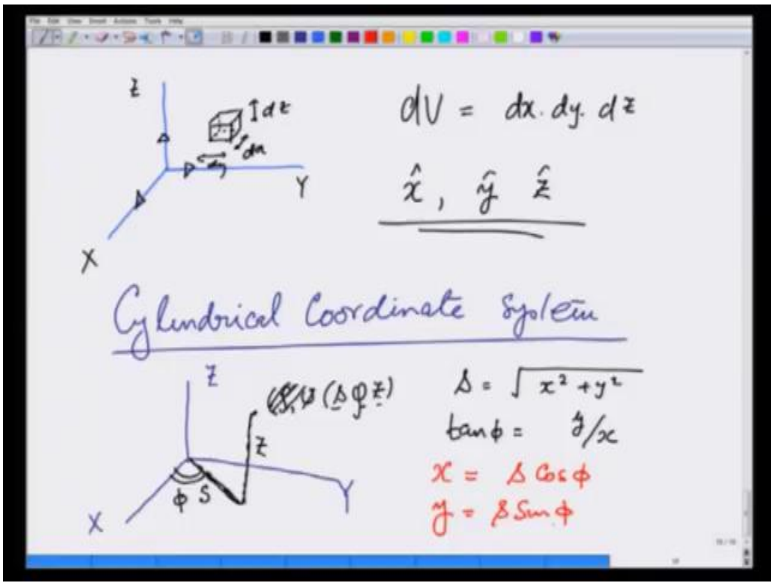

Third, I am going to find for you the volume element. Naturally, volume element is in Cartesian coordinate going to be a box, a small box of the size, this is x, y, z of the size d x, d z and in this direction d y. So, volume element is going to be d x, d y, d z. Unit vectors, we already know are x, y and z and these are fixed in space, these are fixed unit vectors, they do not change direction and magnitude obviously is 1.

Next, let us consider cylindrical coordinate system and this is useful, when we are doing do dealing with situations with it is cylindrical symmetry, that means, there is symmetry about the z axis, nothing changes with respect to when you go around the z axis. So, let me make a coordinate system first, this is my original x, y and z coordinate system. In cylindrical coordinates, I specify a point by measuring it is distance in the x y plane from the origin.

And we are going to call this S, I am using the notation used in the book by Griffiths. Then we measure the angle from the x axis, which I am going to call phi, and then height from the x y plane or the z coordinate, which I am going to call z. So, this is specified by rho, not rho, I said S, S, phi and Z, I need to three coordinates to specify the system and we have given this name S, phi and Z.

You can immediately see the relationship that S is going to be square root of relationship between this and the Cartesian coordinates, x square plus y square, tangent phi is going to be y over x and z is z. By inverting these equations I can also write x as equal to S cosine of phi and y is sine S sine of phi.

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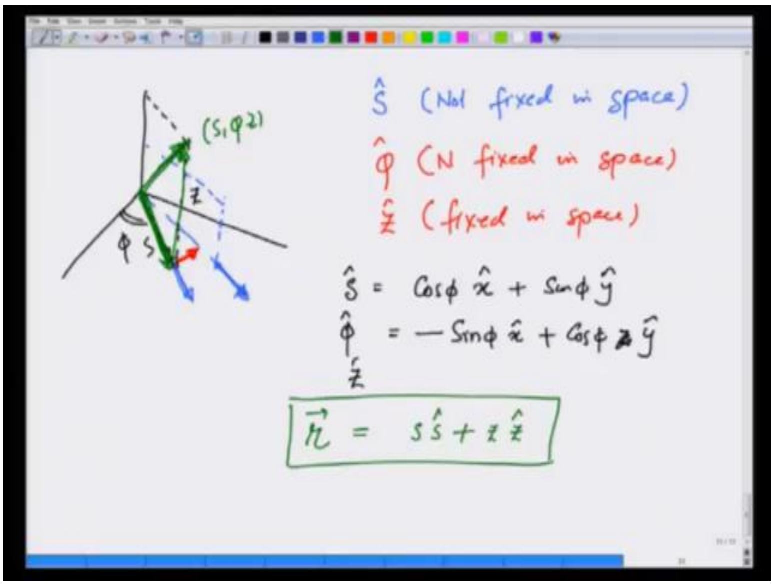

Let us look at the unit vectors. If I look at the unit vectors in cylindrical coordinates, this angle is phi, this distance is S and this height is Z. The unit vector in the S direction is going to be in the direction pointing away from the origin. Naturally, you see, if I go to a different point, which is say here, you are going to see that, S unit vector has changed direction.

So, this S unit vector pointing away from the origin and it is not fixed in space, it depends on what point you are at. Then, I define unit vector phi, which is perpendicular to S unit vector in the x y plane. So, phi is this red unit vector that I am showing and this is also not fixed in space and 3rd is the Z unit vector, which is like Cartesian coordinate system is fixed in space.

From the diagram, you can also see that S unit vector is nothing but cosine of phi, S unit vector plus sine of phi, y unit vector, phi unit vector is minus sine of phi x unit vector plus cosine of phi, y unit vector and z is obviously z. If I want to show the vector from vector position of this point, the vector position of point S, phi and Z, this is going to be sum of the vector in S direction here plus Z. So, the vector r is given as S in S direction plus Z in Z direction; that is a position of the point, the vector from origin to the point. Let us look at the line elements, area elements, and volume elements in cylindrical coordinates.

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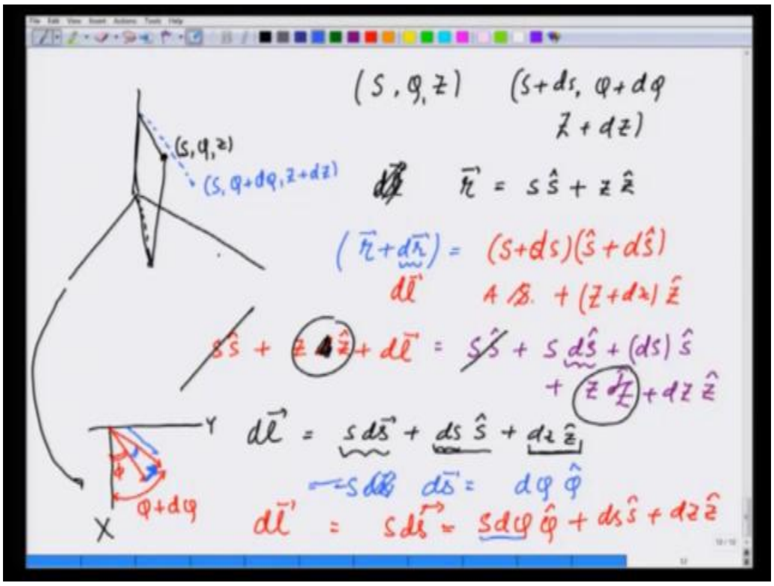

So, if I am going from point S, phi and Z to another point which is S plus d S phi plus d phi and z plus d z, then the line element d l let us look at geometrically, how it looks. So, I am made a point here, which is S, phi and Z and it is position is given by vector r, which is S S plus Z Z. Next point is at some nearby, which is at S phi plus d phi z plus d z.

So, r plus d r is obtained or which I can write as d l is obtained by going from S to increasing distance by d s, S plus S, I can write it right here. As I said earlier, S is not fix, so I am also changing the direction of S plus z plus d z, which is fixed. So, by keeping only the first order, I can write this as S S plus S d s unit vector, this is the change in the unit vector plus d s; that is change in the distance in the direction of S plus Z in the unit vector Z direction plus d z, Z and this is equal to on the left hand side r, which is S unit vector S plus Z d z plus d l.

Let us cancel certain terms S S cancels, this was Z Z plus d l, this term cancels with Z Z on the right hand side and therefore, I get d l is equal to S, d s plus d s, S plus d z, Z. Notice, if I am moving in S direction by d s, I obviously, collect a vector d s S, I am moving in Z direction of d z, I collect a vector d z, Z unit vector, this is the additional term.

And let us see, what is d s, d s unit vector has changed. So, let me look at the x y plane, this is my X, this was my Y and what we have done is d s was like this, it has changed slightly. This was pointing in the direction phi and now it is pointing in phi plus d phi direction and the change, I will show by blue is this unit vector, this vector not unit vector, this vector.

Notice first that this change is in the direction phi and this magnitude is going to be d phi times this length which is one. So, this is going to be S d s or d s vector is going to be magnitude is one times d phi in the direction phi. And therefore, I can write the line element d l as S d s, which is equal to S d phi in the phi direction plus d s S plus d z Z that is a line element in cylindrical coordinates.

It makes perfect sense, because if you look at term by term, this is the distance moved in phi direction, this is the distance moved in S direction, this is the distance moved in Z direction.

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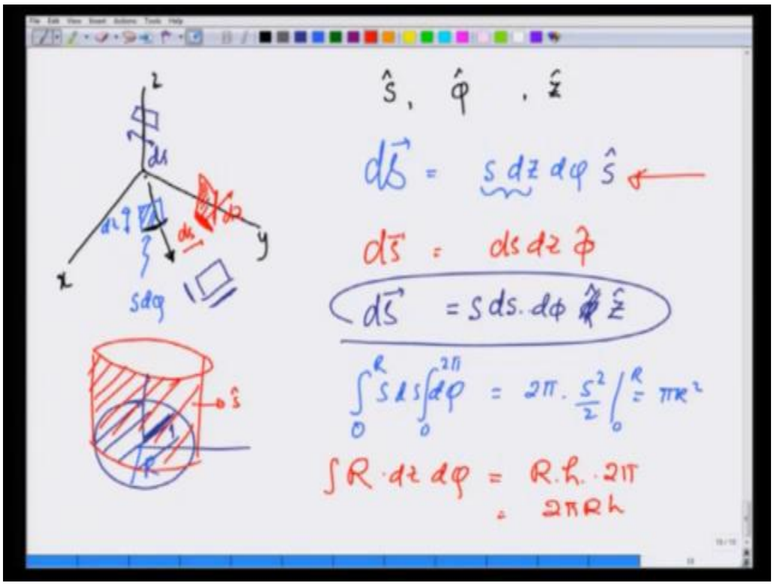

Next, let us look at the area element, if I look at the area element in cylindrical coordinates, I am looking at x, y, z at area perpendicular to S, area perpendicular to phi and area perpendicular to Z. This is the S unit vector, if I am looking at area perpendicular to S, you can see one element is going to be in the x y plane like this and the second element of this area is going to be in the Z direction. This distance is going to be as we just saw S d phi and the height is d z.

So, the area element d s in phi direction is going to be S, d z, d phi; you can see this as unit is of distance square. Now, next how about in direction phi, I am going to take an element which is in the Z direction and perpendicular to the phi direction. This is going to the area in the direction phi. This distance perpendicular to phi is going to be d s and the height is d z and therefore, in the direction phi d s is going to be d s, d z in the direction phi, I should have written direction S here.

And I if I am looking at area element perpendicular to Z, it is going to be d s and I will be moving like this d s and S d phi. So, d s the area element in the Z direction is going to be d s times S times d phi, it is in the Z direction. You can confirm this quickly, if I were to look at the area of a unit circle in the x y plane, let us say this is a unit circle; that means, the radius is 1 or you can also take the radius to be r.

Then, the area would be integration S d s d phi, S varying from 0 to r, d phi varying from 0 to 2 pi and you get 2 pi times S square by 2 0 to r, which gives you pi r square. Or, if I were looking area of this side of a cylinder surface area of that, you can see that, this area is in the direction S. And if I were to calculate the magnitude and look at this, then the area would be integration S which for this surface is fixed radius R, d z d phi which is nothing but R, d z is going to be the height of the cylinder times 2 pi which is 2 pi R H, a familiar result.

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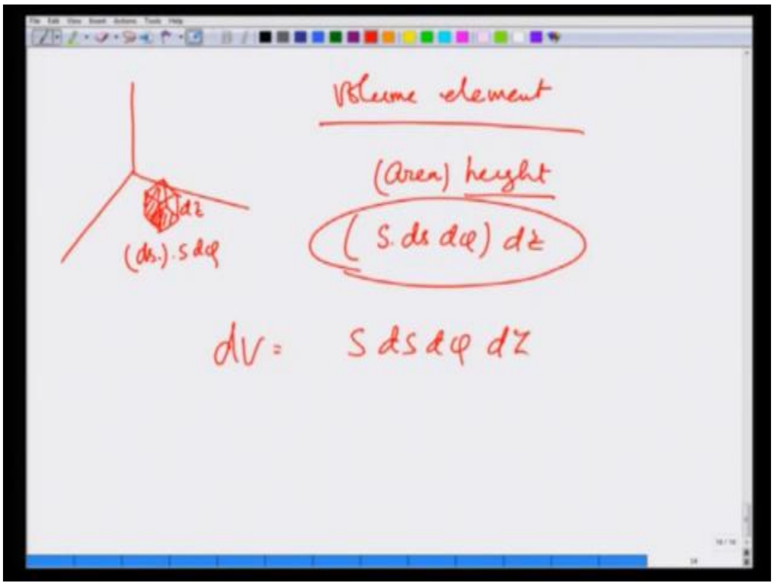

Third, let us look at the volume element in the cylindrical coordinates; I can look at it in many different ways. A volume element is nothing but some area times the height. So, I can choose any area I like, suppose I take the area in the Z direction, this area we just saw is nothing but d s times S d phi. So, S, d s, d phi and height for this volume element is going to be d z. So, I can just multiply by d z; that is the volume element, you can make other areas, hence find that this is correct. So, volume element d v is nothing but S, d s, d phi, d z.