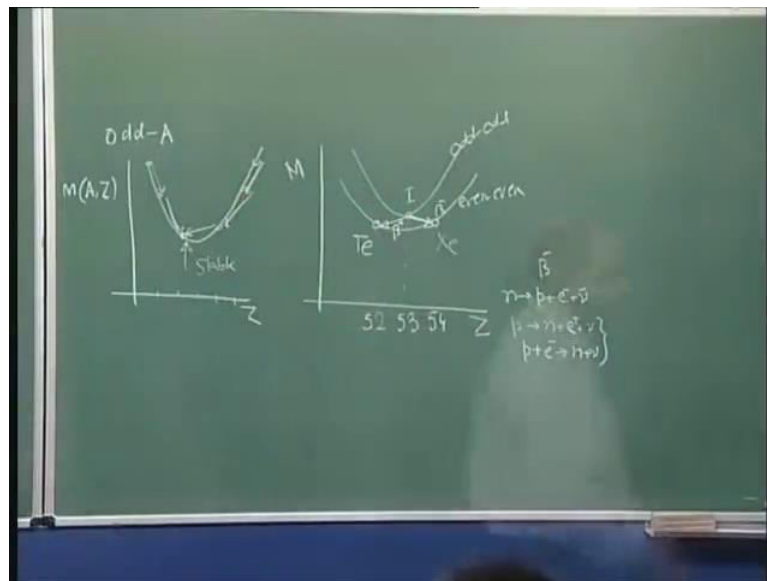


**Nuclear Physics Fundamentals and Application**  
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**Lecture - 9**  
**Semi empirical Mass Formula Cont...**

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So, we were talking of mass parabola. And that is mass as a function of  $Z$  for a fixed  $A$ . And we had talked about odd  $A$ , mass parabolas, where the masses for a particular  $A$  as a function of  $Z$ , they are on a parabola with nuclei like this. And then you have some stable nuclei and all other nuclei will be either beta plus decay or beta minus decay electron capture, and finally you will get a stable nucleus.

But then we talked about cases where this capital  $A$  is even. And when this capital  $A$  is even, you have two categories or two sets of nuclei; one corresponding to even  $Z$  and even  $N$  and the other corresponding to odd  $Z$  and odd  $N$ . Both of them will give you capital  $A$ . So, if you fix that capital  $A$  and then plot this  $N$  versus  $Z$ , mass versus  $Z$ , you will have two parabola. One corresponding to odd odd, and the other corresponding to even even. And these two parabola are shifted vertically with respect to each other, which one is up? For odd odd it is up or for even even it is up?

Student: odd odd it is up.

So, you will get two parabola; one for the odd, odd nucleus; another for even even nucleus. Remember  $A$  is fixed. Now, how the nuclei are distributed on these two parabola, that will decide what are the stable nuclei. The example that I gave  $A$  equal to 128, you had one nucleus here tellurium. Remember? Another here?

Student: Xenon

Xenon, Right? Here it was xenon, here it was tellurium. And there was some nucleus here, that was iodine, Right? What was the  $Z$  number of tellurium and Iodine and xenon? This was 52, this was 53, this was 54 and then you had other nuclei. Both of these were stable. This is also stable and this is also stable and iodine nucleus can decay either to this tellurium or this xenon. Both ways it can decay. So, this is beta plus active, beta minus active. It can decay through beta minus.

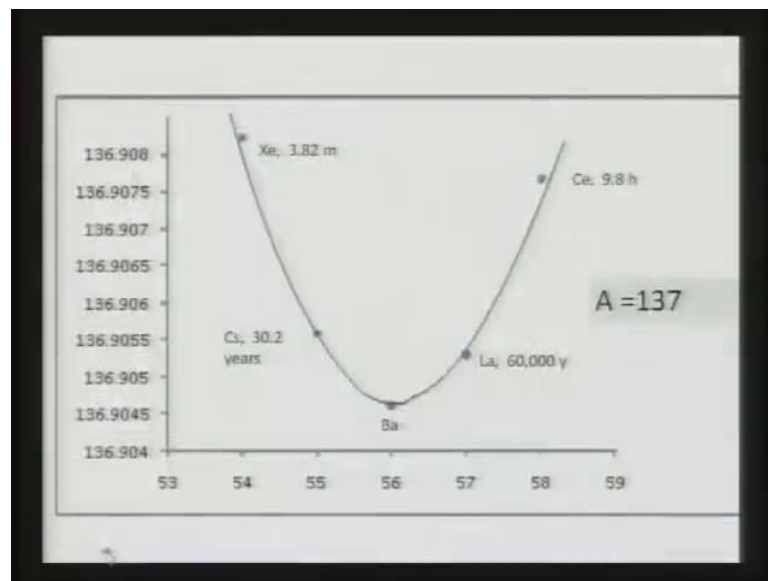
If it decays through beta minus. Iodine where will it go? Tellurium or xenon? Beta minus, if iodine decays through beta minus, neutron converting to proton and an electron, right? That is beta minus, so neutron converting to proton. So, proton number will increase. So, this is beta minus and this is beta plus, Right? This is beta plus, this is beta minus. Where  $Z$  is increasing and goes to  $p$  and plus electron this you need to know. That is beta minus and this is beta plus. Remember beta plus you have.

So, in beta plus you have proton, one proton converts into neutron. So, proton number decreases. So, from 53 it has become 52 and beta plus is also competed by this electron capture. So, it can be beta plus or it can be electron capture both way it can come from here to here. Now depending on how these nuclei are distributed for a particular given  $A$ , we can have other situations also. We can have other situations in which we have only one stable nucleus, with even  $A$ .

That is also possible or in some cases three stable nuclei that is also possible. I will show you now some slides where the real masses, atomic masses are plotted against  $Z$ . For even  $A$  I will show all three cases. Two stable, one stable and three stable. And this time it is it will be real mass data. The thing which we did in the previous lecture, we calculated mass using semi empirical mass formula. So, the idea was to show that the semi empirical mass formula brings out the features which are actually seen in the nuclei.

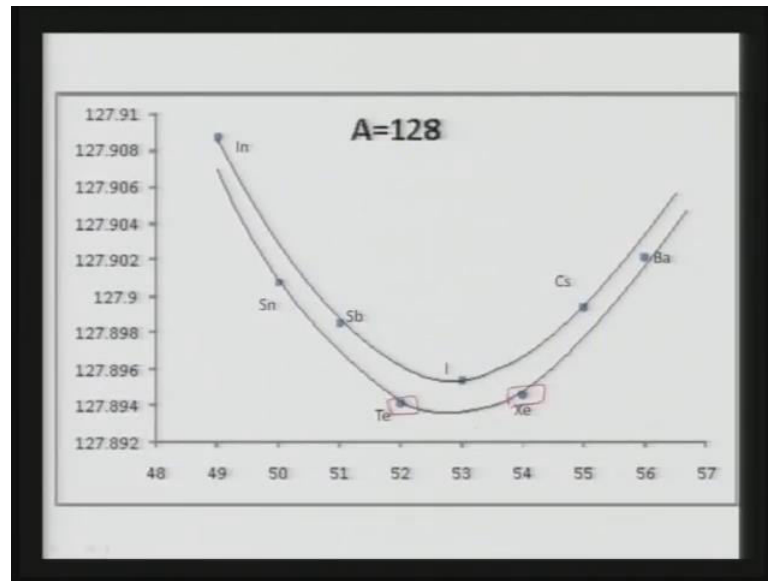
But then this is semi empirical and it gives some kind of an average behavior. It is not very accurate, Right? Accurate things if you need, you have to go to the experiment and measure it much accurately and then look at it. But then to understand what is going on at that in that real laboratory you need to have some kinds of formulas, some kinds of equations, some kind of models, some kind of theory which can tell that this is how why the nuclei are behaving this way. The semi empirical mass formula was that. Now, let me show you some real mass versus  $Z$  distributions.

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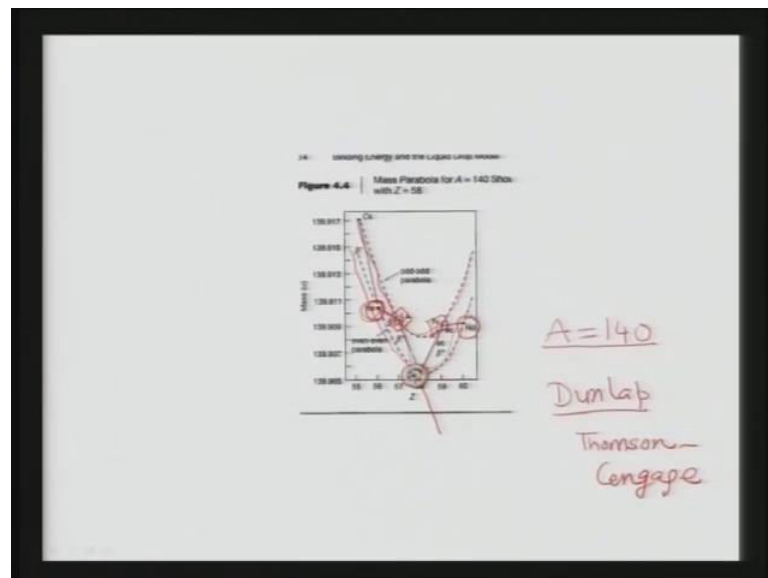
This is the slide that we had seen earlier, this is from semi empirical mass formula that we calculated. But then these numbers are accurate. Xenon is radioactive with 3.82 minutes, cesium 30.2 years and barium is stable, lanthanum 60,000 years, these are real data. Although this masses that I have drawn here. They are from this semi empirical mass formula. This is again from the semi empirical mass formula.

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And this is the thing which is on your black board also. This is iodine and then tellurium and then xenon.

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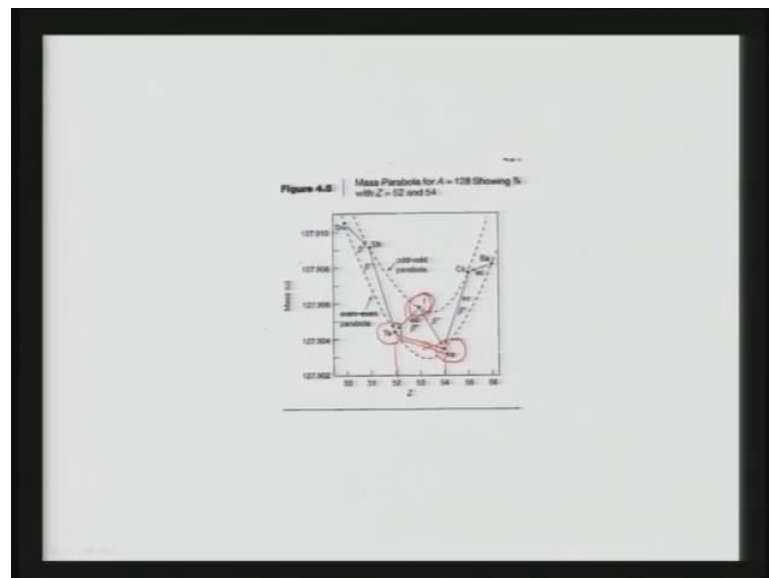


Now look at this one. This is for capital A equal to 140. This is for capital A equal to 140. This is real data and the data taken from the book Dunlap. In the first lecture I have given other details, publishers and all that. So, it is from this book. This is from Thomson the publisher is Thomson. Now, this Thomson company has become Cengage, but when the book was published it was from that Thomson publications.

So, this is for  $A$  equal to 140, you have two parabola and the situation is such that you have a see this CE sitting here on that even even parabola, then N D sitting here on that even even parabola. And then you have barium here. So, this is even even parabola. And odd, odd parabola where are the things? This is lanthanum, odd, odd parabola. This is lanthanum la and this is pr, this is an odd odd parabola. Now you can see you have only one stable nucleus and that is this see. This pr will beta plus decay to ce like this. And this la will beta minus decay to ce.

This is only stable nucleus on even even parabola. You have next nucleus here this ND which is at slightly higher mass than this pr. So, this will not be stable. This will go to this pr. This is not stable. On the other side of the even even parabola you have Barium which is at an higher mass than Lanthanum. So, Barium will decay to Lanthanum ba will go to la. So, you have only one stable nucleus here, Right? It is even case  $A$  equal to 140. You have two mass parabola, this odd odd mass parabola is shifted up with respect to that even even parabola. But then on even even parabola we have only one stable nucleus here.

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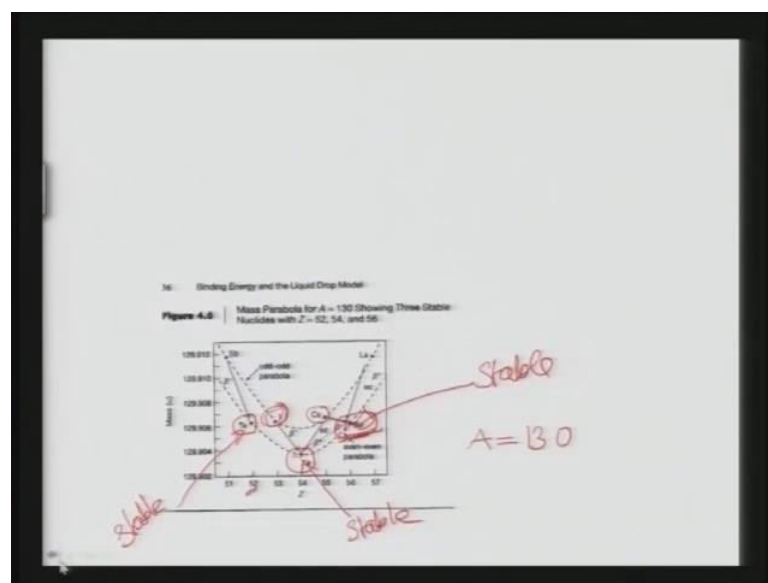


Now, this is that 128 once again I am showing. Not once again because these are the real data that I am showing now. Here is the Iodine. And on the even even parabola you have tellurium here and xenon here and both are stable, Right? Tellurium is at higher energy

than xenon. So, you can think that tellurium can go to xenon and then the energy will be reduced and that is not wrong to say.

But then  $z$  increases by two units. Tellurium is  $Z$  is equal to 52 and xenon is 54. And in between  $Z$  equal to 53 is at a higher mass energy. So, through beta decays it cannot go from tellurium to xenon. This path is not allowed. But then there is a process called double beta decay and so on. Much much less probable that may happen theoretically but has not been observed at least for this isotope. So, both of them are stable.

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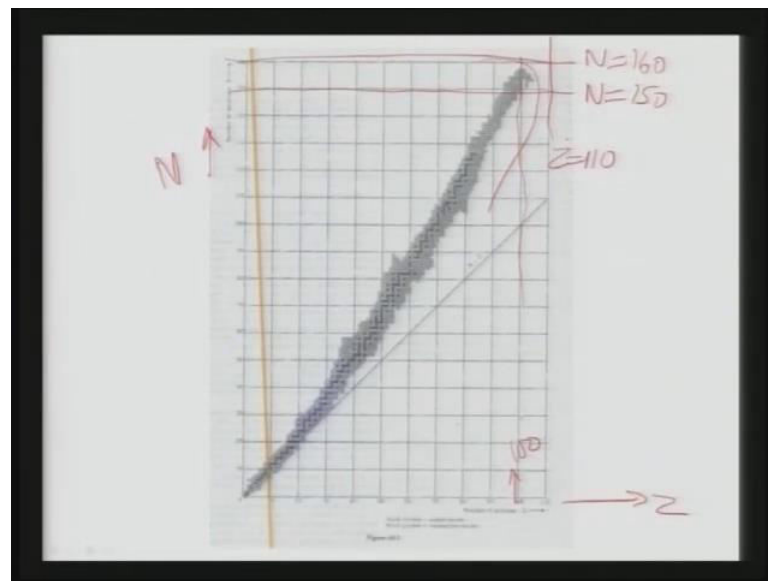


And then you have this case. In this case what you have is, this is  $A$  equal to 130 and you can see you have tellurium sitting here,  $Z$  is equal to 52, which is stable because it can go to iodine. But Iodine is a slightly higher energy. This is iodine, this is iodine slight. So, tellurium will not go to iodine. Tellurium then will not go to xenon we have already discussed. So, tellurium is a stable, xenon is a stable.

Just like  $A$  equal to 128 that we saw last slide, but then here you have this barium sitting here and cesium here on the odd odd. Now, look at this barium where will it go? This barium, where will it go? It will not go to cesium because cesium is at higher energy. Cesium will go to barium this transition. So, barium will not go to cesium. Barium will not go to xenon.

It can only go to xenon through double beta decay. That is very rarely seen in some isotope, some cases. So, barium is stable this is a stable, xenon is a stable and terillium is stable. So, three stables, Right? So, you can have depending on the situation you can have one stable nucleus, or two stable nuclei or three stable nuclei with a particular value of capital A which is even. If it is odd then you have just one stable nucleus. Now before I leave this stage let me also show you something else.

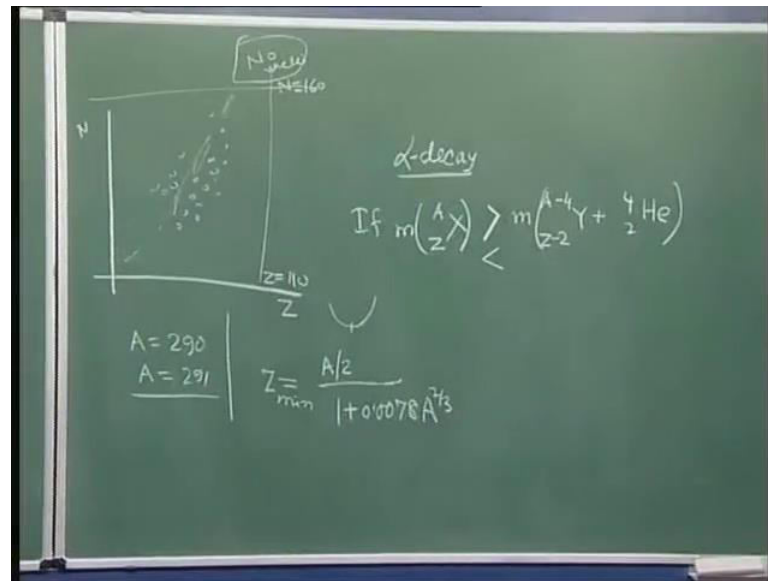
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This diagram you remember? This is Z and this is N. Now these black things in the middle they are the stable ones and the open circles, they are unstable nuclei. But which are seen, which are either exist in nature or we can easily make in laboratories, but then after somewhere here see this is Z equal to 100. So, if you look at this point here, here, here and this is Z equal to 100 and from this side it is 150. This is 150 and N equal to 150. N is equal to 160 and Z equal to 110.

Now, this diagram is drawn only up to here and not beyond. And there is a reason for it. If you go even for higher values of Z and N, you will find that nuclei do not exist. So, that is why this diagram is drawn only up to this point, say Z equal to around 110 and N is equal to 160. So, you do not have nuclei with Z say greater than 110, 115, 120 and so on and corresponding N 170 and 180. Those nuclei do not exist. Just keep this in mind I am coming to the board to discuss that.

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So, on this N Z diagram. On this N Z diagram what you show? All the stable nuclei which are observed and also unstable nuclei, which are observed, but will decay either beta decay, alpha decay or some kind of decay. So, those are shown on the N Z diagram. So, there are some stable things which are somewhere like this here on this type of curve and then you have some open things. These open things are correspond to those unstable nuclei, but then after somewhere around say Z equal to 110 and N equal to say 160.

Beyond this you do not have anything, nothing, no nuclei, but why? If I add these two it will be somewhere around A equal to 270 or so. Why do not I have a nucleus A equal to 290 or 291? Because if I go with that semi empherical mass formula. Let us take a easy case odd 291. So, A equal to 291, you will again get a mass, mass parabola. In the semi empherical mass formula you can put A equal to 291 and then put some Z values, or you know if that if that minimum of the parabola is first for that the minimum will be at A by 2, 1 plus 0 point how much?

So, I can always get a Z from this. If I put A is equal to 290 or 291 or like that and the closest integer value of Z. So, I can always get stable nuclei in principle. For odd A there is one stable nucleus. If it is even still you will have either one or two. Most of the time two stable nuclei, but may be one stable nucleus or three stable nucleus, but there is no stable nucleus at all.



So, what is going on there? All this mass parabola that I had discussed talks of a fixed  $A$ . And  $Z$  and  $N$  can change. If we change  $Z$  correspondingly  $N$  also changes because  $Z + N$  is same. So, what we talking is about beta decays, one Proton changing into one neutron or one neutron changing to one proton. So, this mass parabola, this mass parabola is essentially for beta decay processes. So, that stability that you had seen in all these diagrams it is against beta decay, beta minus decay or beta plus decay.

So, if you think of only in terms of beta decays a Proton converting into Neutron or a neutron converting into Proton. Yes there will be a nucleus with minimum mass. But then there are other decay modes also possible and one of the common decay mode is alpha decay. We will talk about length of alpha decay. But some preliminary only. In alpha decay what happens if you have nucleus with  $A$  mass number and  $Z$  protons and then  $(( ))$ , on the other side you have  $A - 4$   $Y$   $Z - 2$  and plus  $4 \text{ He } 2$ , this is alpha particle.

Now, one has to look at the masses, mass of this side and mass of this side.  $A$  is the same number of nucleons on the two sides. You have capital  $A$  nucleons here and you have capital  $A$  nucleons here. But then here they are separated into two parts. Here it is all in one intact. Now look at the masses if this mass is smaller than this mass? If this is smaller than this. Will it go for this without doing something externally? No, a smaller mass will not convert into this larger mass unless you supply energy.

So, it will not alpha decay. It will be stable against alpha decay. But then if the opposite is true, if opposite is true. Then if this nucleus is there? This nucleus can decrease its total mass energy by emitting this alpha particle. If these nucleus breaks into two parts, one alpha particle and one remaining nucleus which we call daughter nucleus. So, if it breaks into two parts like this it will lower its energy and therefore it will be unstable against alpha decay, and larger the mass difference, Remember?

Larger the mass difference, smaller will be the life time. And the mass difference is such large that the life time is not miserable, then you will won not find those nuclei. So, with heavy nuclei that happens. As you look for this equation, whether it is greater than this or it is less than this. If you verify for what values of  $A$   $Z$  this mass will be greater than this mass and it will become alpha active. And for what values of  $A$ , this mass is less than

this mass and it will not alpha decay. We can do that once again using our semi empirical mass formula because masses we can calculate using the formula.

For a given A, given Z, I can calculate using that semi empirical mass formula. How many mass atomic mass unit is this? How many mass atomic mass unit is this? And how many atomic mass unit is this? Then I can check whether this is greater than this or it is smaller than this. So, let me again show this on an excel sheet, this calculation. Now, excel sheet is there on your screen.

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alpha-stability								
Z	A	1H1	mn	av	as	ac	asym	m(A,Z)
47	110	1.007825	1.008665	15.5	16.8	0.72	23	102479.2
45	106	44122.04	59191.6	1705	385.6894	431.3425	53.52727	98746.64
Mass (daughter)+m(alpha)								109.8819
Mass parent								109.8814
Neutron drip line								
Z	A	1H1	mn	av	as	ac	asym	m(A,Z)
18	58	1.007825	1.008665	15.5	16.8	0.72	23	54108.5
16	54	16897.8	37581.97	899	251.7259	84.06781	191.931	54108.5

And look at this alpha stability thing. What I have done here? I have put in this cell. This is the value of capital A. Can you read that? How much it is? 110, very good. Then this is capital A. Then in this cell I have capital Z which is 47 here. Now this 47, I have not put. I have put a formula here. With this capital A it is calculating Z to be 47. And how is that calculated? Using the same formula for mass parabola, minimum Z min, that is yeah. That is on your board, in your green board. It is there? Z min is equal to A by 2 divided 1 plus 0.0078 capital A 2 by 3.

So, that at least from beta part you are safe. So, we are putting appropriate Z so that if it is it alpha stable, surely it is beta stable also. That is how this is calculated. Now below that you have 45. That is too less 47 minus 2 is 45. So, this is for you parent nucleus, and this is for you daughter nucleus. This capital A is 110 for parent nucleus and after alpha decay it will be 106. So, this is 4 less. And then we have the same things. This is at

hydrogen atomic mass, the neutron mass, this is semi empirical mass formula. You have  $m_p$ , you have  $m_n$ , you have  $m_e$ , you have  $m_{\alpha}$  and so on. So, all those things are there and then I have also put.

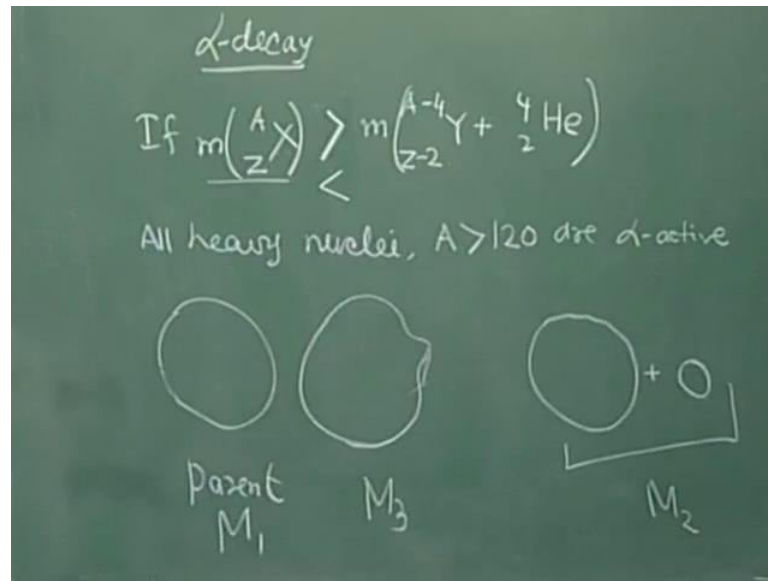
So, the mass will be calculated and then I have also put somewhere helium. This is helium mass, we need that also to calculate the right Hand side of this inequality. So, what will happen? It will calculate using all these data. It will calculate mass of this daughter plus alpha particle here. How much it is? 109.882 U, Right? That is mass of what? Mass of the daughter nucleus plus the mass of the alpha particle. And the mass of the parent nucleus is calculated here.

So, this is parent mass and this is daughter mass, which one is smaller and which one is greater? So, will the alpha decay happen in this place? If the original mass is less than after decay you are creating larger mass it will not go by itself, Right? Now let me change, let me change this 110 to something else. Let us make it 150. Now what happens? Mass of the parent nucleus is here 149.923 and mass of the daughter plus mass of the alpha is. So, which one is greater? Mass of the parent is greater. And mass of the daughter plus alpha is smaller.

So, the alpha decay can take place without supplying any energy, Right?? It is energetically favorable. So, go further. This is 150. Let us put something else 200. What happens? Which one is greater? Mass of the parent nucleus is greater. Anything you put larger number for capital A and you will find that parent is larger and all these nuclei are alpha active. All heavy nuclei are alpha active, Right?? I am giving appropriate Z so that the minimum mass can be ensured. This is the Z for which the mass is minimum.

If you change Z and N keeping capital A fixed. This is the Z which ensures minimum mass. I am working there, but alpha separation by itself, alpha separation by itself makes it radioactive. All heavy nuclei are alpha active. You want to test some lighter nucleus nuclei. Let us say you have this, let us put 80. A is equal to 80, see what happens. Mass of parent is smaller and mass of the products is larger, it will not take place. So, alpha decay in general will not take place for lighter nuclei. Only for capital A equal to 120 or 150 you will find that they become alpha active. So, let me come to the board.

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So, all these heavy nuclei heavy nuclei, in this case is something like 120 or so are alpha active. They can decay by alpha decay process, but you do have lots and lots of nuclei with  $A$  greater than 120 up to say 220 or 230 like that, which are stable or which have very large alpha decay lifetimes. Why is that? That is another story which we will deal in detail, but I will just give you the glimpses, product is parent nucleus is something like this let us. This is the parent nucleus.

Let us say capital  $A$  is equal to 160. And then the daughter nucleus will be a little more smaller and then plus this alpha particle. This mass is say  $M_1$  and this mass is  $M_2$ , this combined mass. So, if  $M_1$  is greater than  $M_2$  you will expect an alpha decay, Right? But what happens that going from here to here there are intermediate deformations, Right? This nucleus, this alpha particle is getting out of this parent nucleus.

Alpha particle is getting out means if you think in terms of classically you will have something like this. Alpha particle is getting out. It is in the process of getting out. Finally it will be like this. But in between things like this will happen. Several stages several stages it slightly it has slightly peeping out and then more and more of it will come out. And finally, everything will come out. Now, one can calculate masses here, calculate masses here and also calculate masses in this intermediate states. Because in this shape also when it is slightly here all Coulomb interaction this and that are different. So, the masses will be different.

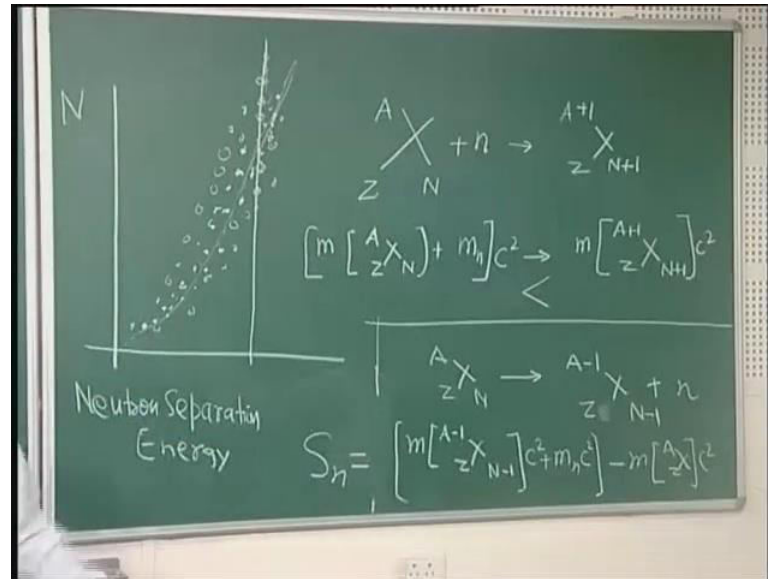
So, this mass let me for just this one say  $M_3$ . So, most of the time what you will see is although  $M_1$  is greater than  $M_2$  and you will expect that alpha decay is energetically favorable and it should decay in that way. But when  $M_3$  is greater than  $M_1$ , Right? So, this is  $M_3$  is greater than  $M_1$ . Although this  $M_2$ , once it decays this energy is less than this energy, energetically favorable, but how does it go from here to here?

If I have not supplied any energy from outside, because this mass is larger, and this mass is smaller and things do not move from smaller energy to larger energy. Things move from larger energy to smaller energy. This is larger and this is smaller, that is fine. Then this is stopping in between. It is a kind of barrier you have to first clear this barrier, you have to go up and you can come down. When you come down you really come down. This energy will be less than this energy. But then in between you have to surmount this barrier of extra energy. So, in Contour Mechanics we have these barrier penetration phenomena which finally makes this alpha decay possible in many of the cases.

But then depending on how high is that barrier that decay probability can be very, very small and the lifetime process can be very, very larger about  $10$  to the power  $20$  years. If the lifetime is about  $10$  to the power of  $20$  years it is as good as stable. So, you do find lots of stable nuclei with  $A$  value with capital  $A$  value  $150, 160, 170, 180$  or  $200$  or so or you have nuclei in that range which are alpha active. But then the life time is quite large, ten to the power  $8$  years,  $10$  to the power  $9$  years, thousand years.

So, all those exist because of this intermediate thing. We will talk much more about this alpha decay process later when we go into that decay chapters. So, that explains why you do not have nuclei on that side, Right? Fission is similar, fission where the difference between fission and alpha decay is that, in this the alpha particle is coming out and the remaining nucleus stands there. In fission almost two equal mass nuclei come out. So, this breaks into nearly equal masses, that is fission. So, this is about alpha decay there are other modes neutron emission directly proton emission directly. So, that we will be talking next. So, once again recall that  $N$  versus  $Z$  diagram where the stable and unstable nuclei are shown.

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So, you have this Z this side, N this side and then the stable nuclei they are somewhere around such a line. You have stable nuclei and then you also have the unstable nuclei shown generally in the books by open circles. So, up to Z 110 and N 160 normally these diagrams are given in the books. Now consider a particular Z. So, suppose take this Z. So, from here if you draw a line you have this nuclei on this line. Some of them are stable some of them are unstable. So, if these are the stable ones.

So, you have some unstable here and so on. Now to the left of this line the stability region somewhere here, if this is the line going, the stability line going. If you are left of it these are neutron rich. And therefore, the neutron will try to convert itself into proton that will give you beta minus decay. So, this unstable nuclei will be beta active, beta minus active, but then if you keep on increasing this neutron on this Z. That means you are going up and up and up and up. At a certain stage neutron emission itself becomes energetically favorable.

So, first if you are somewhere close to this value of stability, if neutron rich, but then the neutron emission is not energetically favorable. Neutron conversion to proton is energetically favorable. That is how it will reduce its neutron number, but then at a certain stage if you pump in too many neutrons. Then at a certain stage even the neutron emission itself will become energetically favorable. And then we say that this particular

nucleus will not accept anymore neutron, because by emitting neutron it reduces its energy.

So, the reaction I am looking for is you have a nucleus, say  $A$  total mass number,  $Z$  protons and neutrons. And if I try to put one more neutron in it, the product will be  $A + 1$   $X$   $Z$   $N + 1$ . So, if one more neutron is forced to go in. It will give this type of nucleus.  $X$  is same because  $Z$  is same. So, the name of the element is same. Energetically what is the situation? Beside you have mass of this nucleus here and plus mass of neutron and time  $C$  square. That is the total mass energy when the neutron is separate and this nucleus is separate. And once it gets into it and makes one single nucleus, then the mass this side is the mass of  $A + 1$   $X$   $Z$   $N + 1$  and into  $C$  square is the energy.

So, one have to compare this mass and this mass, if it so happens that mass of this nucleus plus mass of this neutron and time  $C$  square, total mass energy. If this side is smaller and this side is greater, this reaction will not take place. This mass is smaller than this mass. Things go towards the lower total energy. So, if this mass is larger and this mass is smaller you cannot pump that neutron in. You can come from the other side also. If you have a bound nucleus, suppose you have a bound nucleus  $A$   $X$   $Z$   $N$ , and look for this reaction. I want to remove a neutron from this. I want to remove a neutron.

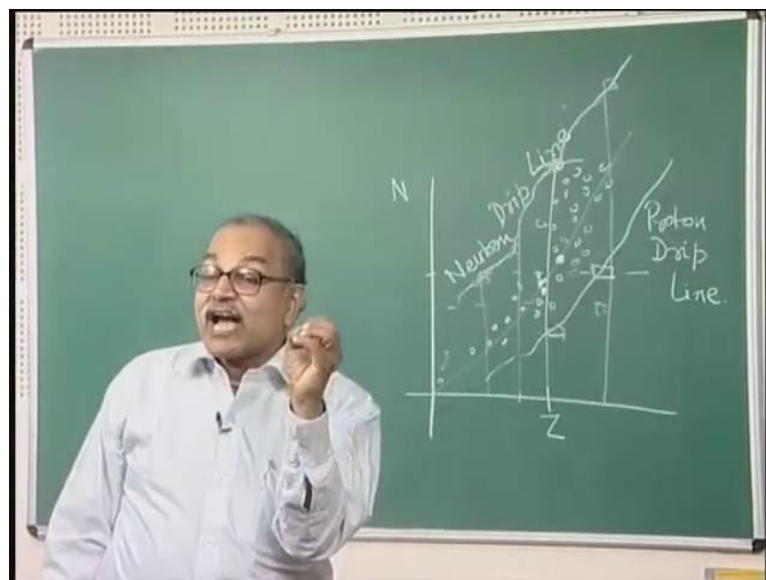
So, the product will be  $A - 1$   $X$ ,  $Z - 1$   $Z$ . And  $N - 1$  and plus neutron. So, from this nucleus I have removed one neutron. So, that neutron number goes down,  $N - 1$  and the total mass number goes down  $A - 1$  and the neutron comes out. So, if it is a bound nucleus, to start with I have to give energy to take this neutron out. And how much will be that energy? Minimum energy that I have to give to this nucleus to take this neutron out that will be again the difference of mass energy here and the difference of mass energy here.

This time I am giving energy and I am getting this. So, this side is larger mass if this side is larger. So, I will have that energy will be mass of  $A - 1$   $X$   $Z$   $N - 1$ . Time  $C$  square plus mass of neutron time  $C$  square. This is the final mass and minus mass of  $A$   $X$   $Z$   $C$  square. That is the minimum energy that I will have to give if I want to take a neutron out of this bound nucleus. So, this nucleus is bound and I am taking neutron out, energy this energy. Let me write. This energy is written as  $S_n$ . What is  $S_n$ ? It is called

neutron separation energy. It is called neutron separation energy. So, this  $S$  is for separation and this  $N$  is for neutron. So, this much energy I have to supply minimum.

So, that neutron can be taken out. Now if I start calculating energies then if I find that this  $S_n$  is negative. This  $S_n$  is negative. That means I do not have to provide any energy. If this neutron comes out, the total energy is decreased. So, the neutron will come out. That means this nucleus will not be forming. This nucleus will not be existing, will somehow try to push that in. No matter how you do, neutron will come out and this nucleus will not fall. So, the message is that with a particular value of  $Z$  you cannot keep on increasing  $N$  and get bound nucleus. Somewhere they will limit. Somewhere they will limit. Let me draw this diagram once again here.

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Some stable and some unstable nuclei. And if I am somewhere here  $Z$  and with this  $Z$  I am trying to increase the number of neutrons increase, increase, increase, increase increase. Somewhere there will be a limit that after this I cannot put more neutrons. Let me show you a calculation on excel sheet where using the first four terms. That means in the binding energy formula, that means the volume term, the surface term, the Coulomb term and what is fourth? A symmetric term. So, I am not going into that pairing term which is up and down. There is a reason for it. I will explain later little bit on that. So, with this first four terms I can calculate masses. So, I will calculate these masses and see



whether this trend comes out from that semi empirical mass formula or not that with particular  $Z$  you cannot put that many neutrons.

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Z	A	1H1	mn	av	as	ac	asym
18	58	1.00783	1.00867	15.5	16.8	0.72	23
18	57	1.00783	1.00867	15.5	16.8	0.72	23

Mass (daughter)+m(neutron)	=	58.0166
Mass parent	=	58.0168

Just look at your excel sheet. So, it is the same sheet. Here is some  $Z$ . At this moment  $Z$  is 18 and I will be putting some neutrons. So,  $A$  will be something and then calculation is you have all those parameters mass of the hydrogen atom mass of neutron then  $av$ ,  $as$ ,  $ac$  asymmetric. All those things are there in the semi empirical mass formula calculations. So, once I have  $Z$  and once I have  $A$  it will calculate the masses as usual. It will calculate all those terms in the formula and finally, it will add to give you some mass.

So, this is the mass and neutron mass I have put here separately. So, this will be mass for this  $Z$  and this  $A$ . When one neutron is less but  $Z$  remains the same. So, this is 18 and here also its 18. Here it is 58 and here it is 1 less 57. And then for 18 57 for these values of  $Z$  and  $A$  same calculations will go on. It will calculate mass of that one less neutron that Nucleus. So, I will add this mass to neutron mass.

So, that will be the mass of daughter plus mass of neutron mass here. This one is obtained by adding this mass of that remaining nucleus, mass of the mass of neutron. So, these two things I add to get this value here, and the mass of that first one without neutron emission this one. This mass is directly written here, Right? Let us say that this  $Z$  is equal to 18 and I put some, let us say start with 40.  $A$  is forty. How many neutrons I have put here?  $Z$  is 18 and  $A$  is 40.

So, how many neutrons I have put? 22, 22 neutrons. And let us see what is the mass of that 18 40 thing. 18 protons and then total mass number is 40, so 22 neutrons. So, mass of this 39.9274. Now if you want to take one neutron out so that  $Z$  is equal to 18 but capital  $A$  becomes 39. So, one neutron has been taken out, mass of that plus mass of neutron that is here. How much is that? More or less? More? More. So, it will not emit neutron, emitting neutron it is increasing its energy, Right? The original energy, mass energy was for 39.974 and if I separate the neutron then the total mass energy becomes 39.9375. So, by itself it will not do that. It will be stable against neutron emission. It will not emit neutron. Let us increase from 40. Let us make it 45, Right?

Now, it is comparing this 45.  $A$  equal to 45 and  $A$  equal to 44 and look at here. 44.933 and here 44.939 will it emit neutron? No. Increase further from 45. Let us make it 50. Now what happens? Now, Let us make it 55, Right? See that? Make it 60. Now see what has happened. Mass of the parent nucleus is 60.0355 and mass of the daughter plus mass of the Neutron. That means after emitting one neutron, the mass has become 60.0347. So, decreased or increased? So, by emitting the neutron, the mass energy is decreased. So, if you keep on increasing the number of neutrons, Right?  $Z$  remained 18 and this neutron number kept on increasing.

So, there is there came a stage after which this neutron emission became energetically favorable. So, you can work out exactly at what stage I went in step or so. But you can put 55, 56, 57, 58, 59 and so on. At what stage it just becomes opposite. So, what have you seen? At a particular value of  $Z$ , we took  $Z$  is equal to 18 and then as we kept increasing the number of neutrons. There was a stage when you found that no more neutrons can go in.

Do not take seriously the numbers. I only wanted to show you that yes our semi empirical mass formula which we developed using some kind of average behavior of nuclei. The Coulomb interactions and the first three terms as you know are from classical type of physics and the consideration. And the fourth term comes from the shell model or some nucleon going on shell and so on.

So, these general considerations which lead to the semi empirical mass formula is able to tell this trend. Is able to explain this strength that we experimentally observed that there are limits in this stability diagram. We cannot go beyond that particular limit. Suppose,

here is the limit and for different  $Z$  you will have different limits. So, for each  $Z$  suppose here is a  $Z$  and the limit is somewhere here. Here is a  $Z$  and the limit is somewhere here. Here is a  $Z$  and the limit is somewhere here.

So, if you join all these, if you join all these. This is known as neutron drip line. Drip line. Beyond this, to the left of this you do not have nuclei, stable or unstable. It will not form because the neutron will drip out. You know dripping? Water drips. If you have a bucket you have water in it and some small hole somewhere. Water drips from there. It comes out of that bucket. Similarly, here neutron drip line means? Beyond this the left of this you will not be able to form the nucleus because the neutron will drip out. The neutron will just come out. This is called neutron drip line. Below this value of stability also you have those terminal points, Right? You can look for proton separation energy.

We talked of neutron separation energy you can similarly talk of proton separation energy. If you have a nucleus, bound nucleus and you want to take a proton out how much energy you have to supply? That is neutron separation energy. You can write an expression similar to this. You can write the reaction where the proton is taken out then you can write the mass differences and you can write proton separation energy. So, if that proton separation energy goes negative we are choosing  $Z$   $N$  combinations where the proton separation energy is become negative and the proton will come out.

So, too many neutrons are not allowed, too many protons are also not allowed. For a particular  $N$  if you go this side. How many protons you can put in? So, you will have some terminal points there. So, similarly, for each neutron you will have some terminal point and then you can join that and that is known as proton drip line. So, what I have shown you in the calculations that the semi empirical mass formula derived on general considerations.

Of course, very good physics is there to write that equation still it is semi empirical and get you the average behavior. Using that formula and by calculating the masses of the different nuclei that we take certain whatever  $Z$   $N$  combinations we take we are able to give this trend. I have purposely not taken that pairing energy because the physics at this drip line in the region close to drip line is very different. Lots of interactions and all those things is there say very active research area.

In the laboratory it is very difficult to create these nuclei, because as you go away from this valley of stability, the thing will be short lived, the lifetime will become shorter and shorter and shorter. And close to drip line, the lifetime will be extremely, extremely small. And to create such nuclei, to study them we will need lots experimental ability, skill and infrastructure. It is not very simple and a lot of research is going on there. And the physics of nuclei in that region could be very different from the physics of nuclei close to the stability or the stable nuclei themselves.

So, there are researches which say that and all the protons hitting, unpaired Protons hitting near that drip line nucleus can increase the stability. We had talked about the protons and the neutrons try to pair up. Pairing the lowest energy, but close to drip line it is observed during experiment that the odd protons sitting there could actually, increase the stability by reducing that gap in the energy level of different shells and so on so forth. So, apart from those details the general behavior that yes, there is a neutron drip line. There is a proton drip line you cannot put more than certain number of neutron with a particular  $Z$  more than certain number of protons for a particular  $N$ . These things do come out from this semi empirical mass formula. So, that is for today.