Nuclear Physics Fundamentals and Application
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Lecture - 7
Semi empirical Mass Formula Cont

So, semi empirical mass formula, what we call the expression of binding energy; that you have gotten by now. You must be remembering all those five terms. So, can you come here and all those terms? Please come fast. Take chalk.

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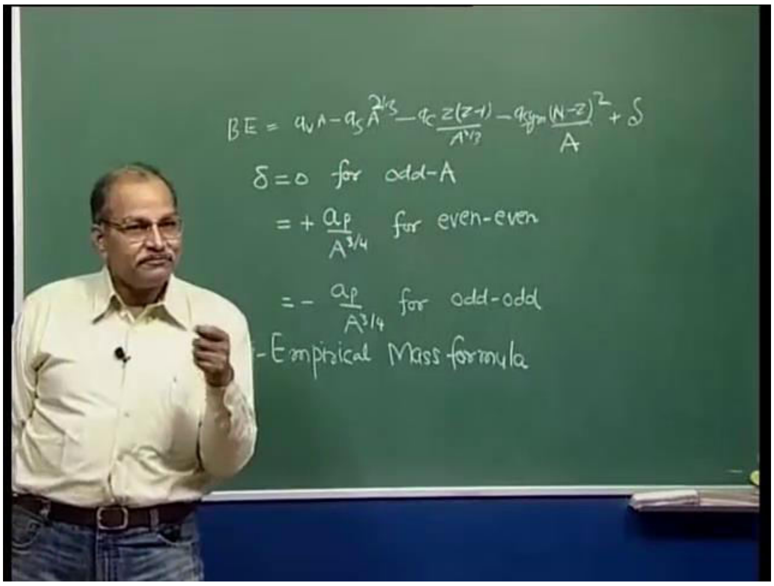

N minus Z, there is the correction, is second term surface term, not that. That is all. The surface term is correct. Good. Thank you. So, this physical picture, you should remember if you try to remember, when it is A to the power 1 by 3 and when it is A power 2 by 3, always there is a chance that you will make mistakes. But physical picture if you remember and from there you write, then it will be fine. This is volume term. This we call volume term because the entire volume you have energies. Therefore, it is proportional to, the volume will be proportional to; more the nucleons, more the volume. It is proportional.

So, that is this is surface. Now, surface is 4 pi r square, surface is 4 pi r square. Your r is r naught A to the power 1 by 3. So, r square will be A to the power 2 by 3. So, if you can connect this physical aspect that is second term is coming. This is because of the nucleons on the surface because they do not have full neighborhood like on one side, you have nucleons. On other side, you have almost no nucleons. So, it is coming from surface. Surface is proportional to r square. Therefore, it is A to the power 2 by 3. Then, you will not make mistake.

Similarly, here this is coulomb energy. Coulomb potential energy is z 1 z 2 e square 4 phi epsilon naught r. So, it is proportional to 1 by r because of the coulomb energy. The coulomb potential between any two charges q 1 q 2 by 4 pi epsilon naught r separation. So, it is that r. r goes by 1 by A to the power 1 by 3. Therefore, it is A to the power 1 by 3. So, if you can connect by this way that this is coming from coulomb potential energy, you will know that it should be A to the power 1 by 3 in the denominator. This is because potential energy is q 1 q 2 by 4 hi epsilon naught r. This is asymmetry.

What is asymmetry? Asymmetry is the number of neutrons is not equal to number protons that are the asymmetry. Therefore, it has to be N minus Z somewhere. Then, your calculation is N minus Z square divided by A. Why A? This is because you had those all those energy levels and if there are capital A nucleons roughly, those separation between the consecutive levels will be proportional to 1by A. A nucleons are to be adjusted in that. That is why 1 by A. Then, this is the pairing term delta. This delta is 0 for odd A nuclei.

This is plus a p A power 3 by 4 for even-even or odd-odd. This is binding energy. So, even-even pairing is done. So, binding energy increases. This is for even-even. Z is even and N is even. This is minus a p A power 3 by 4 for odd-odd. Now, the values; this is some qualitative arguments and some calculations results in this some qualitative arguments. It should be proportional number of pairs. In the neighborhood, there are for this. The surface nucleons are there for this and coulomb. So, there is some qualitative argument, some calculations also. But it is not very rigorous. From very first principle we are driving, it is not like that. So, this is known as semi empirical mass formula.

Empirical is I having the data and then I fit something without any theoretical background. That is called empirical formula. This is semi empirical formula because we are doing some physical arguments. We are coming out with some calculations, some theory, some basic theory, but not very rigorously. So, this is semi empirical mass formula. Now, these values a v as these constants which are coming, how do we get these values? So, this is an expression which is in terms of capital A capital Z, N etcetera.

So, for any N, Z, this will give me a value of binding energy provided I know these constants. These constants are empirically obtained fitted from the known binding energies of the nuclei. A large number of nuclei binding energies one can calculate. One can measure because the masses atomic masses are very accurately measured. From those atomic masses, one can calculate that binding energy. Just take an example. So, I am doing a very basic simple problem. Calculate the binding energy of helium nucleus.

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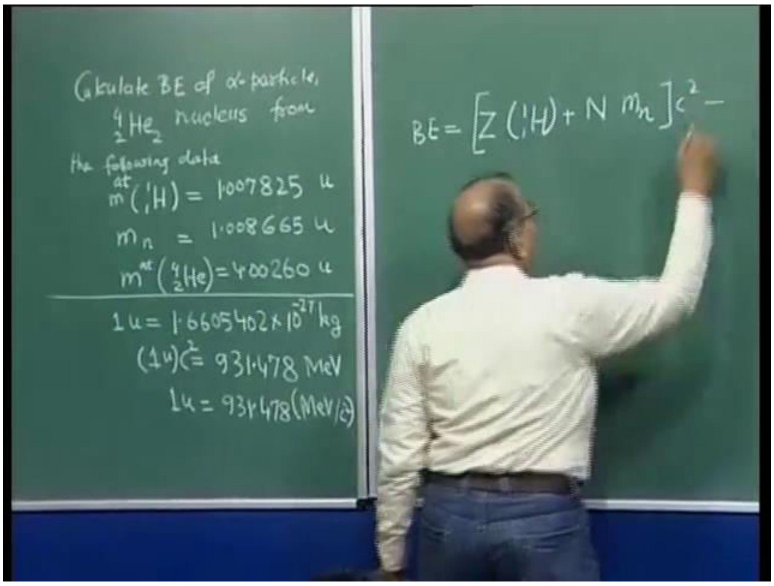

Helium nucleus is alpha particle. That is this nucleus 4 He 2. This is also 2 this nucleus from the following data. This is an example that from the known atomic masses, how one gets binding energy. The data are mass of 1 H 1, these are atomic masses 1 H 1 is hydrogen atom. So, mass of hydrogen atom in its grounded state is 1.007825 u. This is a unit atomic mass unit. I will tell about that. Let me complete the data first. Then, mass of neutron, mass of neutron is 1.008665 u. Then, mass of helium atom, helium atom atomic mass is 4.00260 u.

So, this is the data given. Now, what is u atomic mass unit? This atomic mass unit is mass of what 1 by 12 of mass of neutral carbon atom in its grounded state; 6 protons, 6 neutrons and 6 electrons making an atom in its grounded state. So, that carbon 12 atom mass of that divided by 12 that is 1atomic mass units. If you write in terms of kilograms, these terms have to be 1.6605402 into 10 power minus 27 kilogram. If you write the mass energy corresponding to this mass, that is mass time c square. So, if you take this mass and multiplied by c square, this will be these many kilograms multiplied by 9 into 10 to the power 16 c square is 9 into 10 power 16 meter square per second square.

So, multiply this number by 9 into 10 to the power 16. That will give you joules. How many joules are there; convert that to electron volts. 1 electron volt is 1.6 into 10 to the power minus 19 joules. So, you can do that. That will be 931, 931.478 mega electron volts multiplied by c square. Write in terms of electron volts. When you do this, 1atomic mass unit corresponding mass energy is 931.478 mega electron volts. So, you can write this 1 u as this 931. Generally, for calculations we take 931 depending on what accuracy of calculation is required this MeV by c square. So, another unit of mass, this is mass 1 atomic mass this is mass this many kilograms.

So, 1 u is 931.478 MeV by c square. MeV by c square is another unit of mass, which is quite regularly used in nuclear physics or particle physics. Similarly, Ge by c square, giga electron volt by c square kilo electron volt by c square and so on. The mass of electron will be 511 kilo electron volt by c square. So, energy by c square that is taken by units of mass. Let us come to this calculation. If we do this calculation, let me remove this. If needed, we can write it quickly. So, you have the calculator.

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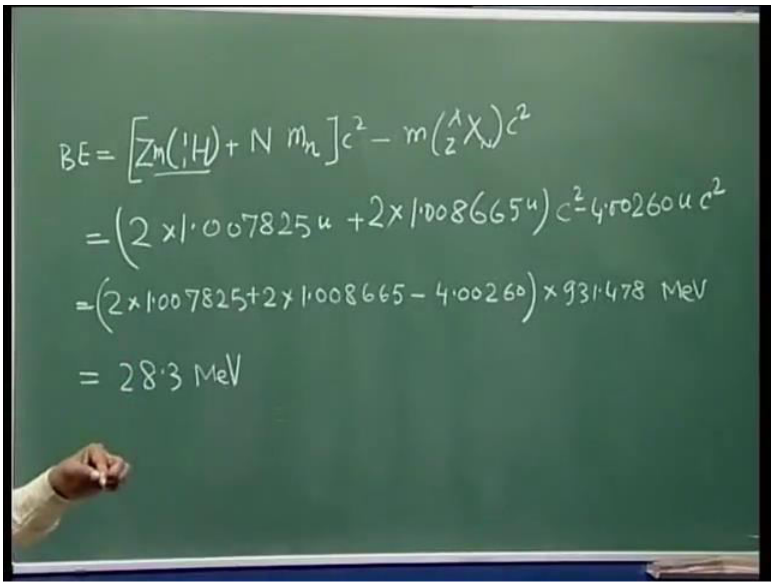

So, binding energy by definition is Z times mass of this hydrogen. Atomic binding energies are small compared to nuclear binding energy. So, we can use atomic masses in place of nuclear masses. So, this plus N times mass of neutron times c square and minus mass of where is mass? Writing mass here Z into mass of this mass of hydrogen atom. So, this takes care of proton part. This takes care of neutron part and minus Z n c square. You can use atomic masses atom; this atom with Z protons and Z neutrons and Z electrons. So, here also when you take hydrogen atom, you take Z electrons.

Here, when you are taking this atom, you are taking Z electrons. So, they will cancel out. The difference in atomic binding energy is very small. So, this gives you this. So, this is by definition. If they are all separate, all protons are separate, all neutrons are separate, and this would have been the mass energy. But when the nucleus as such when it is bound, the energy is small and the difference in binding energy. So, we are subtracting. This is binding energy. Now, put the data. In our case, what is Z? We have to calculate the binding energy for this 4 He 2 2. So, what is Z? Z is equal to 2.

So, this is 2 into mass hydrogen. Mass of hydrogen atom is here. 1.007825 u plus capital N. What is capital N? How many neutrons are there? 2 neutrons are there. So, 2 neutrons 2 into mass of neutron. Mass of neutron is 1.008665 u time c square minus mass of that helium atom. Mass of that helium atom is given here. It is 4.00260 u time c square. So, 1 u into c square; you know it is 931.478. So, this is 2 into 1.007825 plus 2 times 1.008665 minus 4.00260. u into c square is 931.478 MeV. 1 u into c square is 931.4. So, this is it. Now, you calculate on you calculator and tell me the result.

The result is 28.3 MeV. 28.29 is 28.3 MeV. So, this is how one calculates binding energy from the known atomic masses. So, for all different nuclei for which the atomic masses are known one can get the binding energy in this fraction. Then, go for that semi empirical mass formula. Try to put proper values of those constants av, as, ac etcetera, so that all these nuclei, the binding energy of all these nuclei are well produced by the formula. Also, it is a fitting of data. An expression is given with some variable parameters. Adjustable parameters av, as, ac, these things we have to supply.

They are not coming from theory; the format, the form that is coming from our theoretical discussions. But those constants we have to adjust, we have to supply and that is done by looking at that actual data and fitting, trying different values of constants.

What set of constants gives me the best fit to the available data? So, that is how those constants are determined. When people do that, the values that come out, let me state there. In all text books, so that comes out to be av.

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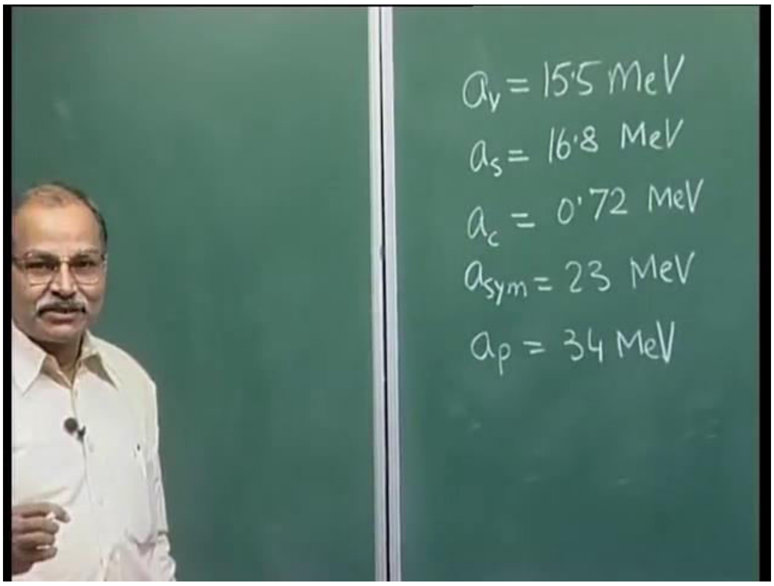

The first volume term is 15.5 MeV. Then, surface term as that constant 16.8 MeV. What is next? ac coulomb term, this is 0.72 MeV. Then, asymmetry constant is 23 MeV and ap is 34 MeV. The units should be in energy units. Binding energy that you are writing, Binding energy is av times capital A. Capital A is dimensionless. It is number of nucleons. This av should be in energy, energy units. So, all these constants are in energy units. So, with this you can fit the things better.

Now, we can understand certain properties of nuclei or certain systematic of nuclei using this. The first which I will be doing is value of Z and N for a given nucleon number. We are discussed not any arbitrary combination of Z and N will give you a stable nucleus. So, if you have say 100 nucleons, so you cannot have 20 protons and 80 neutrons and make a nucleus a stable nucleus out of that or reverse you cannot do. Those are special combinations of Z, N that result in a bound nucleus and a stable nucleus. So, how that Z, N can be obtained? If the total number A is there, so we can do it with the semi empirical mass formula. So, let us see, let us write the mass of the nucleus.

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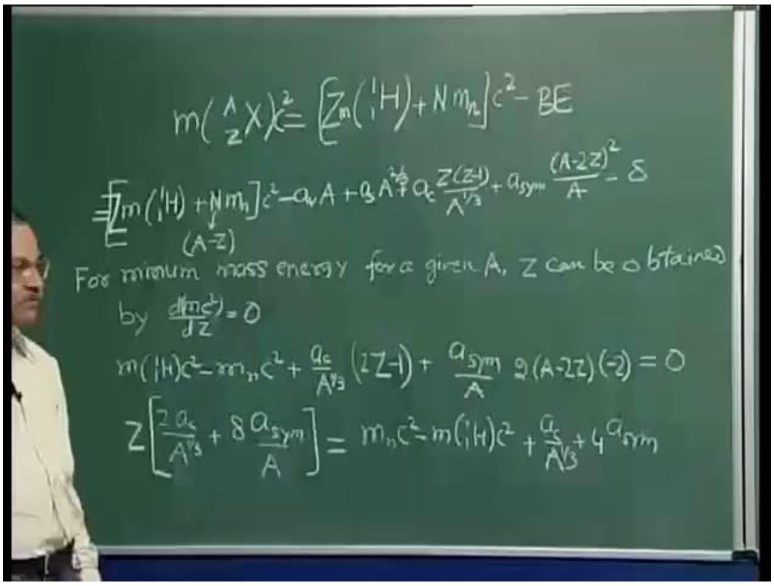

We had written binding energy. Let us write the mass of the nucleus. So, mass of the nucleus A X Z will be Z times mass of this 1 H 1 and plus N times mass of neutron. Let me write c square also energy this and time c square and minus the binding energy. So, this is minus av times A and so on minus binding energy. Again, that is the definition. That is how binding energy is defined. This is the mass when these protons, neutrons are all separate. This is the mass when they are bound to nucleus. So, it is smaller than this by this amount.

So, let me write it once again. It is Z into mass. Let me write it from here. I need longer thing. So, this is equal to mass Z times of 1 H 1 time c square of course, plus N times mass of neutron times c square. Then, minus binding energy remember, it is minus, so minus av times A and plus as times A to the power 2 by 3 and plus ac times Z, Z minus 1 upon A to the power 1 by 3 plus a asymmetric N minus Z whole square. Let me write it A minus 2 Z whole square by A N minus Z N plus Z is capital A. So, N minus Z is A minus 2 Z.

This is N plus Z and minus 2 Z will be N minus Z. Why I am writing in this fashion will be clear. It is plus delta minus delta minus delta and this delta is ap divided by A 3 by 4 plus or minus or 0. So, what I have done? I have written this mass as a function of Z and capital A mass number total mass number and the atomic number. So, the idea is for a given A, what should be the neutron number and the proton number? That is what I want to know. So, for a given A, A is constant. So, that is why I put A. Then, proton number will vary. Neutron number will anyway vary because A is fixed.

So, I will try to see at what proton number, at what value of Z this expression is minimum. This is total mass energy. This is total mass energy of the nucleus. So, at what value of Z corresponding N with a given A, at what value of Z, this mass energy is minimum? So, that will be the stable nucleus. So, that will give me, should give me the proton neutron composition for a given capital A. Now, to get the minimum energy, the minimum value of this, this is mass energy of the nucleus with Z protons and N neutrons.

To get the minimum value of the mass energy as I vary capital Z keeping capital A constant, I should differentiate it and put equal to 0. So, differentiate this and put that equal to 0. So, for minimum mass energy for a given A, Z can be obtained by saying that this dm c square whatever this dm c square over d Z should be equal to 0. Do differentiate with respect to Z. So, here you have A, Z. So, when you differentiate, it is mass of this hydrogen atom times c square. This c square is there. Here you have N. So, let me write this as A minus Z. Then, it will be minus m n c square because I am keeping capital A constant and varying Z.

So, this N also I am writing A minus Z. Then, differentiate with respect to Z, so minus m n c square, no Z here. Capital A is constant. Then, no Z is here. Capital A is constant, Z is here. So, it is plus ac over capital A 1 by 3. Let me write this first. Then, this is Z square minus Z, Z square will give me 2 Z and minus Z will give me this. Then, here plus asymmetric by A and here it is 2 times A minus 2Z multiplied by minus 2Z. You do not have Z in this delta. You do not have Z. So, this is equal to 0. This should be equal to 0. I have to get Z from this expression. So, collect all Z part on one side, without Z on the other side. So, where are the Z parts?

Without Z is here. 2 times ac over A power 1 by 3. 2 times ac over A power 1 by 3. Z, I will be taking common. So, this Z I have taken. Then, where is Z? Here it is. This is 2 into 2, 4 and into 2, 8 plus 8 times this plus A times a symmetric by A overall Z terms I have taken. Now, go to the other side, the terms without Z. So, this will be m n c square mass of this hydrogen atom time c square. So, I have taken this. I have taken this. Here, it is plus ac by A to the power 1 by 3. This has been taken. From here, it is minus 4. So, it is plus 4 a symmetric divided by A. A cancels 2 times A minus 2 Z. A is cancels. So, what is left? That is all 4 times minus 2. That side it becomes plus into 2 is 4. That is all a symmetric.

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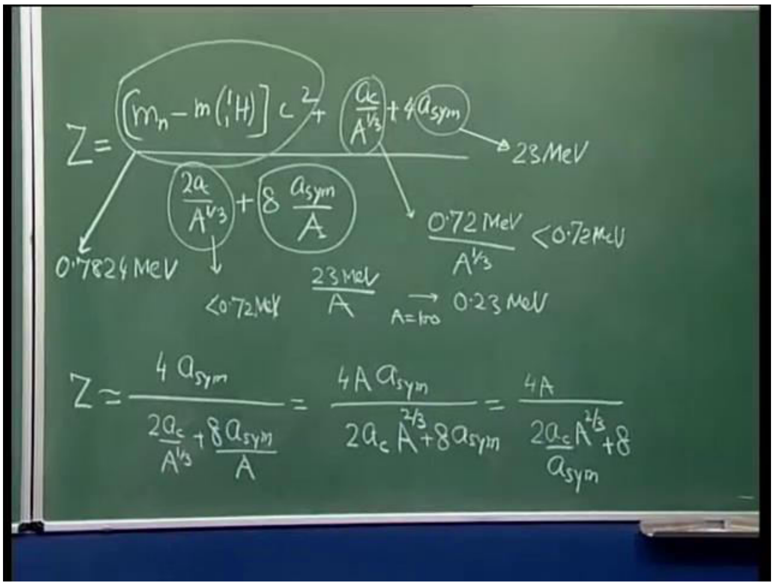

So, Z is equal to this. You have noted those values. So, Z is equal to m n minus m 1 H 1 c square plus ac by A power 1 by 3 plus 4 times a symmetric upon divided by 2 ac by A power 1 by 3 plus 8 a symmetric by A. Now, let us take some numerical values. You have the data for this. Just we solved a problem. You have the data for this mass of this hydrogen atom is there in notebook. The mass of neutron you have, just tell me. How much will be m n minus this H 1 into c square? Make this calculation. You subtract this mass of hydrogen atom from the mass of neutron in atomic mass units.

Multiply this by 931. That will give you the value in mega electron volts. Mass of neutron is 1.008665 minus mass of that hydrogen atom subtract and multiplied by 931.478. So, this portion is 0.7824 MeV. Now, look at this part. How much is ac? It is 72 MeV. This divided by A to the power 1 by 3 is going to be less than 0.72 MeV whatever A you put. How much is this? It is 23. So, it is 23 MeV. So, what do you see? This is 0.78. That is less than 0.72 and this is 23. So, this will dominate. Remember, we are trying to get this Z. Number of protons can vary only in units of 1. So, difference of 0.01, 0.02 and 0.001 will not matter.

The number of protons cannot be 23, 0.7824 etcetera. So, anyway we will have to get the nearest integer. So, all those small variations do not mean much. So, we can neglect this term and this term and this term. We can just keep this one. You can also look at the denominator what the relative things is ac by A 3. So, this is already there. This is already there. This will be around or less than 0.72 MeV. What is this a symmetry? It is is 23 MeV. But it has to be divided by this capital A. We will be talking of capital A equal to 100 or 201, 50, 50 etcetera. So, if I take A equal to say 100, then this becomes 0.23 MeV. So, here a symmetry term was much larger than these. But here this may not be larger than this.

This is only 0.72 or less than 0.72 MeV. So, this is the range. Here, it is 0.23. That is the range. So, here you do not neglect this is in front of this. This can be neglected in front of this. This ac can be neglected in front of a symmetry. But here this ac cannot be neglected in because of factors here, this capital A here. So, with these approximations, what is Z, let me write. Once again, it is 4 times a symmetric. So, I have neglected all other 3 terms here. Here, I am keeping both of them. So, it is 2 ac over A 1 by 3 plus 8 a symmetric over capital A.

Simplify little bit. Multiply both numerator and denominator by capital A. So, you have 4 times capital A a symmetric divided by 2 ac capital A to the power. I am multiplying both numerator and denominator by capital A. So, it is 2 by 3 and plus 8 times a symmetric. Now, divide by this a symmetric first. Let me also take 8 A common from here. So, this is 4 A a symmetric. I am dividing both sides. I am also taking this. Let me do it one by one. So, it is 2 times ac A power 2 by 3 divided by a symmetric plus 8. Is that correct?

I have divided just by a symmetric numerator and denominator. So, numerator is 4 A and denominator becomes 8. This becomes 2 ac A power 2 by 3 divided by this. Now, you divide 4 or take 8 common from here. So, how much is it? Take that 8 common.

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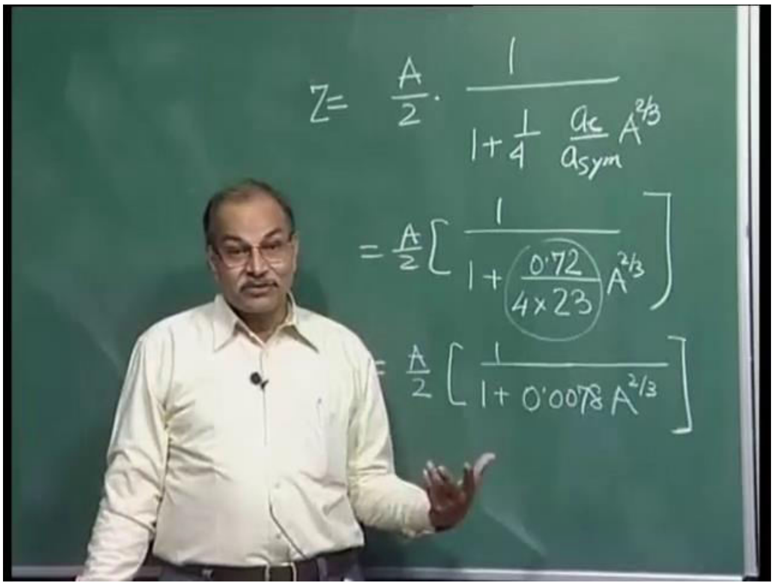

Then, Z becomes capital A by 2 and 1 by 1 plus 1 by 4 ac by A power 1 by 3 ac over a symmetric. Then, it is A power 2 by 3. So, let us put values of ac and a symmetric also. It is A by 2, 1 by 1 plus, what is ac? It is 0.72 MeV and divided by 4 into a symmetric. How much is a symmetric? It is 23. So, this is 23 here. Then, it is A power 2 by 3. Now, tell me how much is this factor, 1 plus 0.0078 and this multiplied by A power 2 by 3. Now, you can see for light nuclei where A is A 10, 12, 6, 8 like that, this factor is going to be very small as compared to 1.

It is 0.0078 into; so, if A is small, small means 10, 12, 6, 8 of that type, then this factor is going to be very small. Z should be equal to A by 2. So, half the nucleons should be protons and half the nucleons should be neutrons. That is what we see in the real world. Stable nuclei oxygen is 8, 8 and carbon 6, 6. So, in light nuclei Z equal to N whereas for heavy nuclei, this factor will start coming up. This will be less than A by 2. This is 1 divided by 1 plus this. So, once this becomes sizable, it is 1 divided by 1 plus something. So, it will be this factor will be less than 1. So, Z will be less than A by 2. So, neutrons will be more and protons will be less. That is what you actually see for heavy, heavier nuclei. When capital A is large, you have more neutrons and fewer protons. Let us do some calculations. So, calculate this factor for say, A equal to 8.

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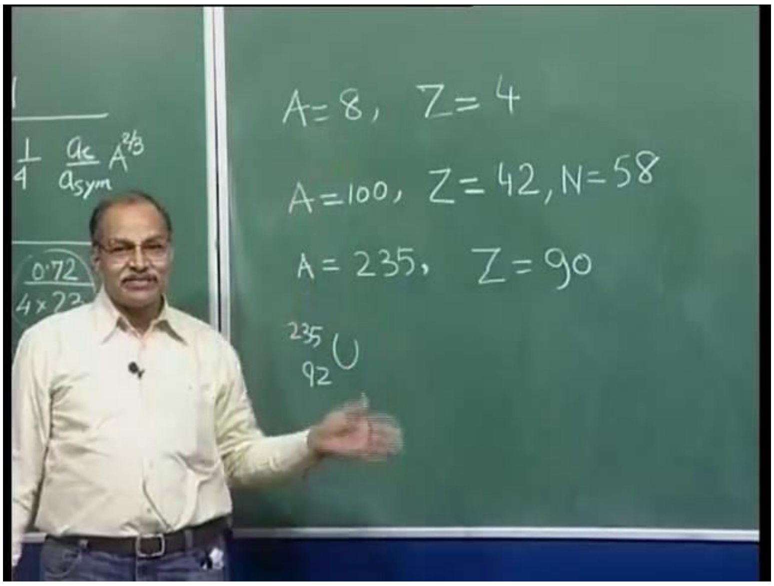

So, A equal to 8. So, tell me what should be Z from this formula? A is 8, Z is 3.93. So, what should I write here? If you are getting 3.93, what should I write Z cannot be 3.9. So, it is 4. So, I have given you a value here. But if you let me talk something about this nucleus, what nucleus this will be? Z is equal to 4. What element it is? Z is equal to 4 thus this is Beryllium. A is 8. So, this nucleus is 8, beryllium, 4, 4. Since, this has just come up; I am telling this is not related to this chapter or topic.

This is highly unstable nucleus. It decays into 2 alpha particles. It decays into 2 alpha particles, 4 He 2, 2 plus 4 He 2, 2. Its lifetime is very small, something like femto seconds, femto minus 15 seconds or so. This is very unstable nucleus. This particular is 4, 4; very unstable. This has good significance for nuclear fusions in star. We will talk about these things later. Now, let us come to the middle weight. A is equal to 100. What should be Z? Make a calculation from this formula. A is equal to 100. So, Z is 42. So, you can see H. If you have 100 nucleons for stable nucleus, this formula predicts that Z should be around 42.

What should be N? N should 58 so that the total is 100. Then, do one more. A is equal to 235. Do it correctly and quickly. A is equal to 235. This is uranium. I am hitting at uranium, around uranium. How much is it? It is ninety. Z is equal to 90. So, you see it is coming very close. You know about uranium 235. Uranium 235 is Z equals 92. You have 238, uranium 235. Uranium is very useful material for all our electricity generation and all that. So, this formula, this semi empirical mass formula is able to tell you what Z, N should be if you have a particular nucleon number?

So, this is one good application of this. Now, look at this mass expression little more closely. Is it there no in terms of Z, mass in terms of Z? The one which you had differentiated, you have that formula that expression. The one which you had differentiated dm by d Z. So, look at that formula m as a function of Z for a particular A. So, what kinds of terms are there in Z? Is it a polynomial or you have something else? So, you have 3 types of terms there. Let me write.

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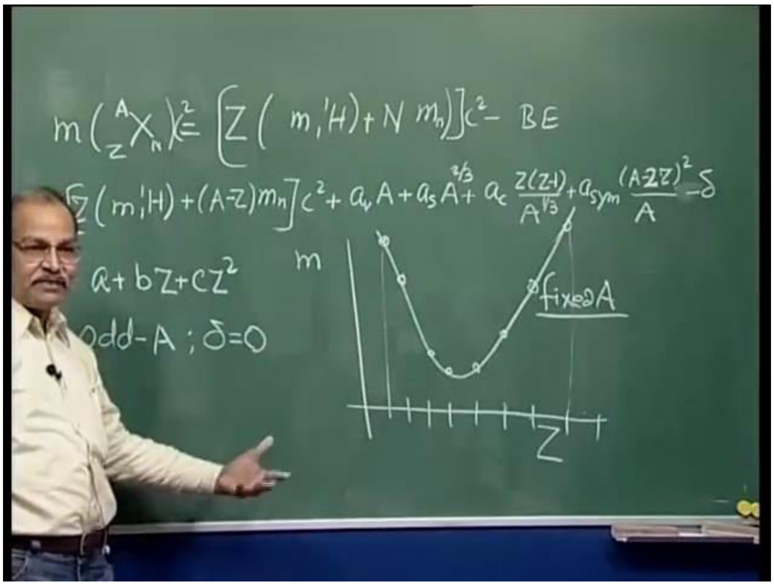

When you write mass of this X A Z N, this will be Z times mass of 1 H 1. Then, let me put c square also plus N times mass of n and then c square, this whole thing c minus the binding energy. So, this will be Z times mass of 1 H 1 plus A minus Z times mass of n and then c square minus binding energy will be plus av capital A plus a s capital A 2 by 3 plus ac Z, Z minus 1 by A power 1 by 3 plus a sym A minus Z, 2 Z, A minus 2 Z whole squared by A and plus delta plus or minus delta. We did write plus there. So, we have to write minus here, minus delta.

So, you have A Z square term here and you have A Z square term here. When you write A minus Z whole square, you will have A Z square here. So, that is the highest power in Z. Then, you have terms proportional to Z that is here. That will come from here minus 2 A Z. So, you will have terms proportional to Z. You will have terms proportional to Z square. You will have terms proportional to Z. All other terms are free of Z. This A times m n, you do not have any Z here A times. You do not have any Z here. You will get that e square.

So, there also you do not have any Z. So, you have 3 varieties Z square terms will be there. Z terms will be there. Constants independent of Z will be there. So, it is like a second order polynomial. So, this is like some a plus some b Z plus some c Z square. You have Z square terms. You have Z terms and constant terms. Now, this is equation of a parabola. So, if you look at the signs of all those things and draw this parabola, draw this mass as a function of Z, you can draw this mass as a function of Z. So, it should be a parabolic shape. Let me take first that delta is 0 odd A case. Odd A although it does not matter, but let us take odd A case first so that I can put delta is equal to 0.

This parabola Z this side and mass this side and this parabola will be something of this sort. This is the shape of a parabola. Of course, the parabola can be in different orientations. But this particular one where you have all those values a b c, parabola comes out something like this. Then, you can, but the points are at integral values of Z. This is for a particular A, a fixed A. This is for a particular value of A. So, Z is different. A is same. You will have a point here. You will have a point here; will have a point here, here, then here and so on.

So, this way you will get the points. So, what are all these points? For a particular value of A, you can have different nuclei with different Z and corresponding N. These are called isobars. You know this word isobar. A is fixed Z is different. Therefore, N is different. So, all these isobars are falling on this parabola. Out of all these isobars, the one which will have lowest energy will be stable. Others will be radioactive beta active this side or this side.

So, stability is another thing. You can have different nuclei with same value of capital N, A, and different value of Z, N. But of these nuclei, one perhaps or two perhaps will be stable. Remaining will be radioactive. Either they will emit beta minus or they will emit beta plus particles or electron capture that nucleus stability. This will be our next topic in our next lecture.