

**Nuclear Physics Fundamentals and Application**  
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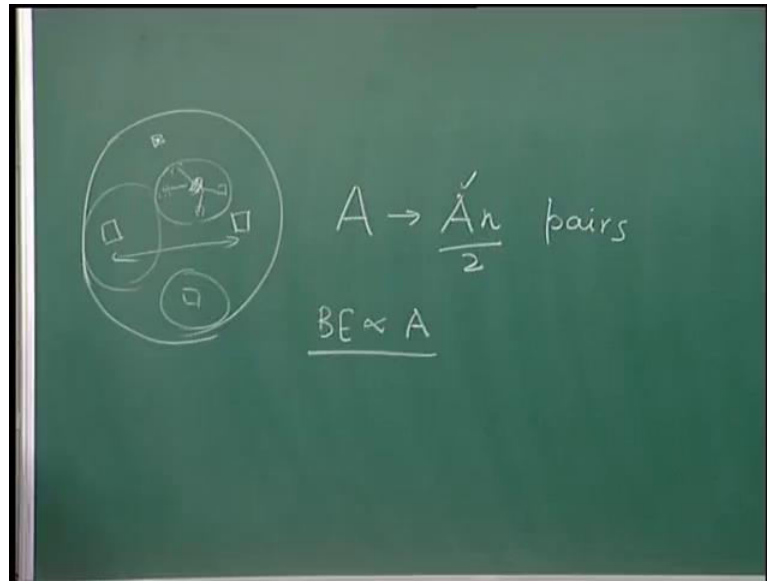
**Lecture - 6**  
**Semi empirical Mass Formula Cont.**

So, previous lecture we discussed something on binding energy, on nucleus binding energy. As I told you the difference between rest mass energy of all the  $z$  protons and the  $z$  neutrons nucleus when they are separated from each other; the total mass energy of nucleus made from both protons and neutrons, when it is nucleus the total mass energy is low. When these are separated protons and neutrons, the total mass energy is somewhat larger. That difference we call binding energy. For that binding energy, we were trying to understand how it depends and so on

What we had seen is binding energy is roughly proportional to total mass number. I had shown you slide on which binding energy per nucleon was on the  $y$  axis side. Total mass number, total number of nucleon was on the  $x$  axis side. Varying some light, very light nuclei, most of the nuclei give you something like 8 MeV of binding energy per nucleon with a slight downward trend as capital  $A$  increases. Now, if I think that nucleon interaction takes place between all the players of nucleon in the nucleus, we should have something like capital  $A$  into capital  $A$  minus 1 by 2 pairs.

One can expect that binding energy will be somewhere somewhat proportional to a square will go like a square. But it does not. Binding energy goes as capital  $A$ , not as capital  $A$  square. Then, we discussed that message of this experimental finding is that not all pairs of nucleons in a nucleus is able to effectively interact through nuclear interactions. So, what does that mean? That means the nuclear forces are short ranged if you have nucleus.

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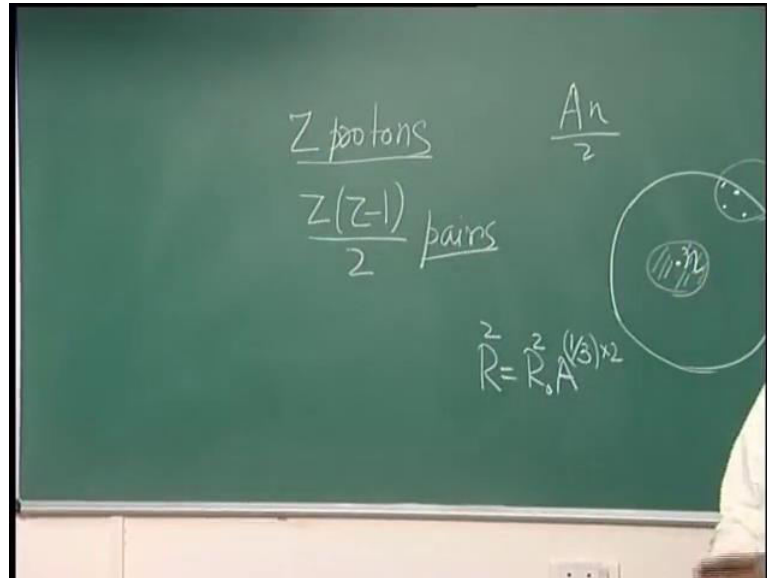
If you have a nucleus of some 6 fathometers of diameter and you have nucleon here, you have nucleon here and the separation 5 fathometers, 6 fathometers. If this pair is not contributing significantly to the binding energy that means, the nuclear force between these two is extremely small. So, the nuclear forces are effective only if the separation is much smaller. So, may be each nucleon interacts with only few nucleons in its own surrounding and nothing outside.

If I consider this nucleon, then possibly whatever nucleons are here, only with those few nucleons, it is effectively communicating through nuclear interactions. Outside nucleons do not form nuclear interactions pairs. So, if you say that each nucleon interacts with small  $n$  nucleons in its own surrounding, then how many pairs will be there with interact through this? That number will be if each nucleon use unitary method, each nucleon interacts with small  $n$  nucleons. So, there is capital  $A$  nucleon. So, you will have  $A$  into  $n$  pairs. But each pairs is counted twice.

So, this divided by 2 also if each nucleon interacts with small  $n$  nucleons around. One by one, either this first nucleon is interacting with these. Small second nucleon is interacting with these  $n$  and so on. I go from first to last capital  $A$ , all capital  $A$  nucleons. It is capital  $A$  into  $n$ . But then remember each pair is been counted twice. Once I was considering this nucleon has the seed nucleon and second time I am considering this nucleon. But any way does not matter. So, it is proportional to the capital  $A$ . This will give you

binding energy is roughly proportional to capital A, which is observed. So, this observation tells us that nuclear forces are short range may be some 1, 2 fathometers, 2.5 fathometers. After that, sharply the magnitude decreases.

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So, as first approximation, we will write that binding energy is some constant time capital A. That constant we write as a and subscript v. What is this v? v is for volume, volume of the nucleus, the entire nucleus. This term itself is called volume term. If you have larger volume, you have larger number of nucleons. Capital A is large. Proportionately, this term goes up that is volume term. Here, if I say proportional to what I am doing for each nucleon, I am saying it is interacting with these small n nucleons.

Of course, I have taken the possibility of counting it twice and all those things. But this is some kind of overestimate. Why? If you have a nucleon here and if you draw a surrounding volume, so that it interacts with all the nucleons here, there are small n nucleons. That is how you count and go for from first to last, all nucleons capital A nucleons and say that there are capital A into n by 2.

The problem is for the nucleons, which are adding the surface, near the surface. These nucleons with the sitting here, this also will interact with nucleons surrounding itself. But then what is the radius? If you draw a sphere a spherical volume and say that it will interact with all nucleons in this volume. So, there are no nucleons here. It only finds

nucleons here. It does not have  $n$  pairs. It cannot form  $n$  pairs because sitting on a surface one, there are no nucleons.

So, if I have used this unitary method, I have overestimated surface nucleons. The nucleons near the surface do not have  $n$  nucleons. During activation, that number is smaller. Therefore, we have subtracted something from this binding energy expression  $v$  into capital  $A$ . Where do I have to subtract to that will be proportional to the number of nucleons that are on the surface. On the surface means once again there is no sharp boundary of, but still close to the surface where this density goes very low proportional to the surface area. It means proportional to  $R$  square, capital  $R$  square.

Surface area is  $4\pi R^2$  for a sphere and proportional to  $R^2$  means proportional to the capital  $A$  to the power  $2/3$  because  $R$ , the radius goes as  $R \propto A^{1/3}$ . So,  $R^2$  will be  $A^{2/3}$ . Then, this into 2 is  $2/3$ . Something proportional to capital  $A$  to the power  $2/3$  that should be subtracted from this volume term because of a surface nuclear. So, we write the second term. That second term is minus a  $s$  capital  $A$  to the power  $2/3$  a  $s$  for the surface. This is the constant. a  $s$  is a constant.

It is referring to the surface contribution or other over estimate that I had done earlier and compensating for this. This is known as surface term. So, a volume term and you have a surface term. This is the nuclear interaction. But in the nucleus, you have protons and these protons interact through coulomb forces also. That will also contribute to binding energy. Whenever there is a interaction, there is corresponding binding energy. Here, it is not attraction; proton, proton through columbic forces will be repulsion.

Repulsion means loss of binding energy, increase in total mass energy. Binding energy is reduced. So, there is a further reaction in binding energy because of this coulomb interaction. How it should depend on the proton number? It will depend on the proton number. It is all proton, proton interactions through coulomb nuclear interaction between proton, proton, proton; neutron, neutron, neutron, all nucleons. But the Coulomb force operates only between the charges electric charges. So, number of protons, if in the nucleus if you have  $Z$  protons, how many pairs can be formed from the  $Z$  protons?

That will be  $Z$  into  $Z$  minus 1 by 2 pairs. Will all pairs contribute towards this coulomb part of the binding energy? It will because Coulomb force will not have short range

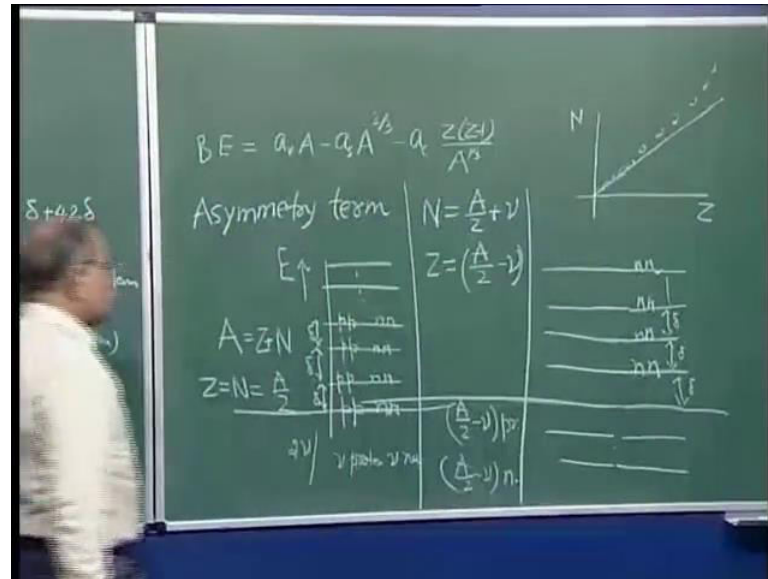
force. Coulomb force is  $1/R^2$  type. It gradually decreases. As  $R$  increases, the force is like  $1/R^2$ . The potential energy pair charged pair goes as  $1/R$ . So, all the pairs will contribute. So, it should be proportional to  $Z(Z-1)/2$ . But the energy potential energy of any charged distribution will be something like  $1/\text{radius}$ .

If you charges given  $q_1, q_2$ , what is the potential energy?  $q_1 q_2 / 4\pi\epsilon_0 R$ .  $R$  is the separation. If you have spherical volume, uniformly charged spherical volume, what is the potential energy? Once again, it is capital  $Q^2 / 8\pi\epsilon_0 R$ , some constant there. Then,  $1/R$  and all that after that  $1/r$ . So, the columbic potential energy or electro static energy will go as  $1/\text{separation}$  between the charges. Average separation between the charges is something of that sort. So, it should be proportional to this  $Z(Z-1)$ . Then, it also be proportional to  $1/\text{radius}$  of the nucleus.

Radius once again is  $R \propto A^{1/3}$ . So, proportional to  $1/\text{radius}$  that means proportional to  $1/A^{1/3}$ . This is to be subtracted from the binding energy because it is repulsion. It is not giving you binding. It is repulsion. It is not binding. It is not trying to keep the nucleus bound. So, from the binding energy, we have to subtract this contribution. Then, you get third term in this formula. We write it as minus a c; again, a constant.  $Z(Z-1) / A^{1/3}$ ; this is for  $1/\text{radius}$  coulomb potential energy.  $Z(Z-1)$  is for number of pairs divided by etcetera is here in this constant a c. So, this third term is known as coulomb term in that binding energy formula.

So, these 3 terms denote the contribution from the volume, the nuclear volume. This denotes contribution from the surface nucleons. This denotes contribution from the proton pairs. So, up to here we can go with what we call classical description. All the language I am using; the volume, the surface, the pairs interaction energy, this is a classical physic language. We can do from here taking this 3 terms alone, properly choosing a  $v$ , a  $s$  and a  $c$ . That general shape of binding energy per nucleon curve can be reproduced. But there are two important quantum mechanical functions or terms which we have to add. One is asymmetry term.

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You remember the set diagram? If I have  $Z$  and  $N$  stable nuclei and nuclei close to them are almost, at least for lighter nuclei, they are almost on this line  $N$  equal to  $Z$ . After that, there is some deviation. So, why the deviation is there? In heavier nuclei, you have more neutrons than protons. For lighter nuclei, you have almost equal numbers of protons and neutrons. This is because nuclear force by itself prefers  $Z$  equal  $N$ . So, in this region where the Coulomb force is not that effective, nuclear force is effective. Nucleus is small. At a smaller range, nuclear force is stronger much stronger than coulomb forces. So, here  $Z$  equal to  $N$  is forced.

But if nucleus becomes big so that the nucleon-nucleon separation, sometimes the grammatically opposite point, this separation is somewhat larger than Coulomb force. It also starts showing its roll moll effectively. Then, to compensate for that repulsion between those protons, you have to bring in more neutrons. That is why  $N$  becomes greater than  $Z$ . So, for the nuclear force itself,  $Z$  is equal to  $N$ . That is preferred composition. Why is that? That is because nuclear force is charge independent. Nuclear force does not distinguish as such between protons and neutrons.

Nuclear force between 2 neutrons at some separation is exactly the same as nuclear force between 2 neutrons as the same separation and other same angular momentum and other things. So, proton and neutron are equivalent as far as the nuclear force goes. But why is  $Z$  equal to  $N$ ? How can nuclear force distinguish between them? So, you have to use

certain degenerate mechanics there also. So, that description we will be doing in more detail later is that each nucleon finds itself in some kind of an average potential. So, if you have a nucleus containing capital  $A$  nucleons, then each nucleon you can think of that it is moving in the potential created by capital  $A$  minus 1 nucleons. Then, you have a potential. Then, you have  $A$  energy levels. You have degenerate states.

This nucleon can occupy one of those degenerate states to get the minimum energy ground state energy. These levels will be filled up below lowest energy to highest energy. So, we will talk it in more detail. We do the shell model roughly. Let us say you have this energy level. So, energy is going this side and this side. This is nothing just energy levels, one dimensional diagram. Now, if you have protons and neutrons and suppose, you have a nucleons which is  $Z$  plus  $N$ . Again, assuming that this proton spins that is intrinsic spin, that can be plus half or minus half. Protons and neutrons are both fermions.

They are  $s$  equal to half particle like electrons  $m_s$  can be plus half  $m_s$  can be minus half. Assuming the energy independent of that, at each level, you can put a proton with spin plus half and a proton with spin minus half; a neutron with spin plus half and a neutron with spin minus half. But not more than that in one particular level can be put. You can put 2 protons and 2 neutrons, 1 with spin plus half, 1 with spin minus half because Pauli exclusion principle will be there. They are fermions. They are spin half particles. Fermions should have the same quantum numbers. So, the  $m_s$  can be different from here and here.

So, putting these 2 protons here in 1 state does not violate Pauli exclusion principle. Similarly, putting 2 neutrons here does not violate Pauli exclusion principle because you can have 1  $m_s$  plus half  $m_s$  minus half. But if you want to put a third proton here or a third neutron here, then the Pauli exclusion principle is violated. So, that should go on the next level. So, in this way it can be filled up. If this a nucleon that I have equally divided,  $Z$  is equal to  $N$ . That is equal to  $A$  by 2. Then, you can go on this scheme. If it is even-even nucleus, if  $Z$  is even and  $N$  is even, then you can just go on filling it is in this style this way. But when  $Z$  and  $N$  are not equal, suppose you have a nucleus in which  $N$  is equal to  $A$  by 2 plus  $\mu$ .  $Z$  is equal to  $A$  by 2 minus  $\mu$ .

This is the situation for all nuclei here and many of the light nuclei. Also, you have more than 1 isotope. So, you can have this  $N$  greater than  $Z$ . This is a very common situation. Suppose that it is  $A$  by 2 plus  $\mu$  here and  $A$  by 2 minus  $\mu$  here. Now, how these levels will be filled up? If you try to fill it, again let me draw it somewhere here. The pole will be there. The levels, the same levels I am trying to draw for protons and neutrons. Just I have separated. So, I have some space to work. So, this  $A$  by 2 minus  $\mu$  protons and  $A$  by 2 minus neutrons you can fill up just like this. That equal number of protons and equal number of neutrons you can fill up.

So, suppose up to here, you have already filled up  $A$  by  $A/2$  minus  $\mu$  protons and same number of protons. How many neutrons are left? You have put all these protons and these many protons, same number of neutrons. But the number of neutrons you have  $A$  by 2 plus  $\mu$ . You have already accommodated  $A$  by 2 minus  $\mu$ . So, how many are left? It is  $2\mu$ . This is the total number of neutrons. This I have already accommodated up to here. So, what is left? The total number minus the number filled up.  $2\mu$  new neutrons are still there to put. Where do I put them? I will have to put them here only  $2$  of them. I cannot put all of them here.

I cannot put in the same energy level, remember same condemned state. So, next set of neutrons will go here, then go here, then go here and so on. You cannot put a third neutron here. You cannot put a third neutron here. You cannot put a third neutron here. So, what is the energy in this case? In the first case, this  $A$  nucleons are divided equally. Then, what is the energy here? Where there are asymmetric numbers more than number of protons, let us calculate that difference first. In this case suppose somewhere here; this is the level here. I have accommodated those  $A$  here.

I have accommodated these with these protons, these neutrons. So, this is same. After that, there are  $2\mu$  new nucleons left here equally divided and there only neutrons. So, here you will go up to how many levels? How many nucleons are left? Still  $2\mu$  new nucleons are left. But you have new protons and new neutrons. In each, you can put  $2$  of them. So, new by 2 levels we are going up. So, what is the energy? Suppose this difference is  $\Delta$ . So, let us calculate the extra energy up to here. It is common here and here it is common after that. So, this will be for this one.



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$$\begin{aligned}
 E_2 &= 2\delta + 2 \cdot 2\delta + 2 \cdot 3\delta + \dots \mu \text{ terms} \\
 &= 2\delta(1 + 2 + 3 + \dots \frac{\mu}{2} \text{ terms}) \\
 &= 2\delta \frac{\frac{\mu}{2}(\frac{\mu}{2} + 1)}{2} = \delta \left( \frac{\mu^2}{2} + \mu \right) \\
 E_2 - E_1 &= \delta \left( \frac{\mu^2}{2} + \mu \right) - \delta \left( \frac{\mu^2}{2} + \mu \right) \\
 &= \delta \frac{\mu^2}{2} = \frac{\delta (N-Z)^2}{8}
 \end{aligned}
 \quad
 \begin{aligned}
 E_1 &= 4\delta + 4 \cdot 2\delta + 4 \cdot 3\delta + \dots \frac{\mu}{2} \text{ terms} \\
 &= 4\delta(1 + 2 + 3 + \dots \frac{\mu}{2} \text{ terms}) \\
 &= 4\delta \frac{\frac{\mu}{2}(\frac{\mu}{2} + 1)}{2} \\
 &= \delta \left( \frac{\mu^2}{2} + \mu \right)
 \end{aligned}$$

Then, there are 4 particles at energy  $\delta$ . Then, there are 4 particles at energy  $2\delta$ . Then, there are 4 particles at energy  $3\delta$ . How many terms will be there? How many terms will be there?  $\mu$  by 2 terms you have new protons and new neutrons. You are putting two in one level. So, how many levels you are going?  $\mu$  by 2, there are  $\mu$  by 2 terms. This is, let us call it as energy  $E_1$ . So, that is equal to what is step to be taken common?  $4\delta$   $1 + 2 + 3$  up to  $\mu$  by 2 terms is  $4\delta$  into  $\mu$  by 2 and  $\mu$  by 2 plus 1 divided by 2. So, this 2, this 2, this 4 can be cancelled.

You have  $\delta$  times  $\mu$  square by 2 and plus  $\mu$ . Now, look at this situation. In this situation, after this these levels are filled up, you have again 2 new particles. But all are neutrons. Therefore, you are filling it in this fashion,  $2 + 2 + 2$ . So, what are the energies? This is  $\delta$ , this is  $\delta$ , and this is  $\delta$  and so on. So, you have for these 2, you have  $E_2$ . Calculate  $E_2$  for this. You have 2 particles at  $\delta$ . So,  $2$  times  $\delta$ , then you have 2 particles at  $2\delta$ .

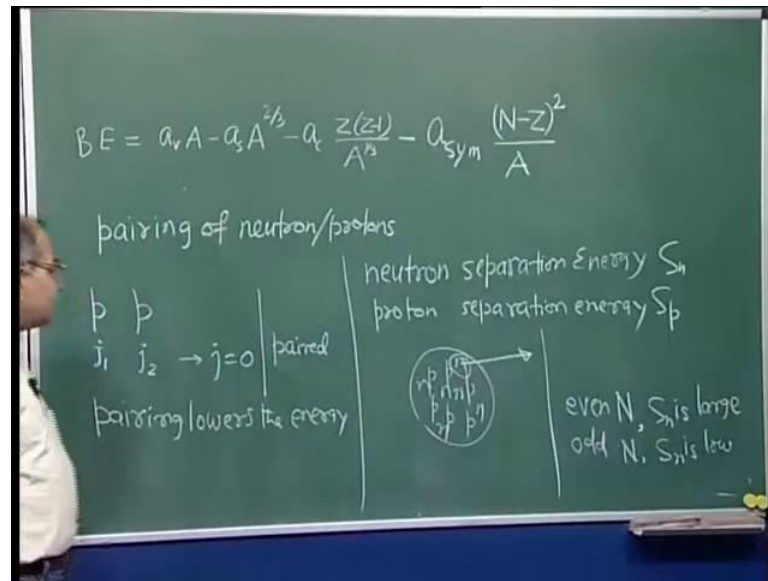
These 2 particles are at  $\delta$  energy. Then, these 2 particles are at  $2\delta$  energy. Then, these 2 particles are at  $3\delta$  energy and so on. So, this is 2 particles at  $2\delta$  energy. Then, 2 particles are at  $3\delta$  energy. How many terms are there  $\mu$  terms? You have 2 new particles to be distributed. So, no such terms will be there. So,  $2\delta$  taken common,  $1 + 2 + 3$   $\mu$  terms is  $2\delta$  times  $\mu$ ,  $\mu$  plus 1, 2 which is  $\delta$  times  $\mu$  square plus  $\mu$ .

So, what is the difference? How much is the difference? Obviously, this is larger than this. You are filling up higher states. So, this is larger. So,  $E_2$  minus  $E_1$ ; what is  $E_2$  minus  $E_1$ ?  $E_2$  minus  $E_1$  is equal to  $\Delta\mu$  square plus  $\mu$  minus  $\Delta\mu$  square by 2 plus  $\mu$  and that is  $\Delta\mu$  square by 2. So, if there is an asymmetry in neutron number and proton number, if they are not equal, then compare with equal division,  $A$  by 2.  $A$  by 2 should be compared from there. The energy of the nucleus will increase by this term. If the same nucleon were divided equally between proton and neutron, then what would have been the energy from there?

The actual energy of the nucleus where  $N$  greater than  $Z$  will be higher this much amount again. This is an estimate. These energy levels are not all equally spaced. All those details are there. But roughly speaking, this is the energy by which the nuclear energy nucleus mass energy has gone up. The nucleus mass energy has gone up. That means the binding energy is decreased. Binding energy is the total rest mass energy of protons plus total mass energy of neutrons minus the binding energy that is the nucleus nuclear energy. If the nuclear energy is increasing that means binding energy decreasing. So, you should have term in the binding energy expression in which this energy is also taken into account.

If  $N$  is not equal to  $Z$ , in terms of  $N$  and  $Z$ , I can write this  $\mu$  from here.  $N$  is equal to  $A$  by 2 plus  $\mu$ .  $Z$  is equal to  $A$  by 2 minus  $\mu$  can subtract  $N$  minus  $Z$  is equal to  $2\mu$ . So,  $\mu$  is  $N$  minus  $Z$  by 2. So, this energy is proportional to  $N$  minus  $Z$  square  $\Delta$   $N$  minus  $Z$  square  $N$  divided by 8. So, it is proportional to  $N$  minus  $Z$  square. So, the term to be added here in the binding energy expression because of this asymmetry between neutron number and proton number will be minus or plus some constant that I write as  $ym$ .

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I have already used here for surface. So, I cannot take one letter. So, that is why I am writing symmetric. This is coming from the symmetric or rather asymmetry. So, this times  $N$  minus  $Z$  square, what is the delta? This delta is there. So, what is the estimate of delta? So, if there is some energy and there are  $A$  nucleons, assume that they are equally divided, it will be as good as 1 by capital  $A$ . So, this divided by capital  $A$  is the energy term to be added here situation apart from volume. From where is this coming? This is coming because in the nuclear volume, you have all those nucleon, which are interacting with each other.

Nuclear attractive force creating this binding energy is positive. But from this you have subtract surface part because the nucleons at that surface do not find those many neighbours. They find less number of neighbours. So, the contribution from here is negative. From this, I am subtracting this. This is coulomb repulsion. This is coming from Pauli exclusion principle. This is coming from discrete energy levels for the nucleons inside the nucleus and all those things. So, this is that asymmetric energy. Apart from that, one more term is important and that is to be concerned with pairing of neutrons and pairing of protons.

What is that pairing business? Once again, contemplated mechanically, the particles have nucleons. They have a own way function. When they combine, when they couple, then 2 protons with opposite angular momentum, if they combine to 0, total angular

momentum, we say that they are paired. So, if you have 2 protons, if this has got some angular momentum, this has got another angular momentum and if these combine to give you a total angular momentum 0, then we say that they are paired. It so happens that is the case. Then, the energy goes down. Energy of this kind of coupling will be lower than any other kind of coupling.

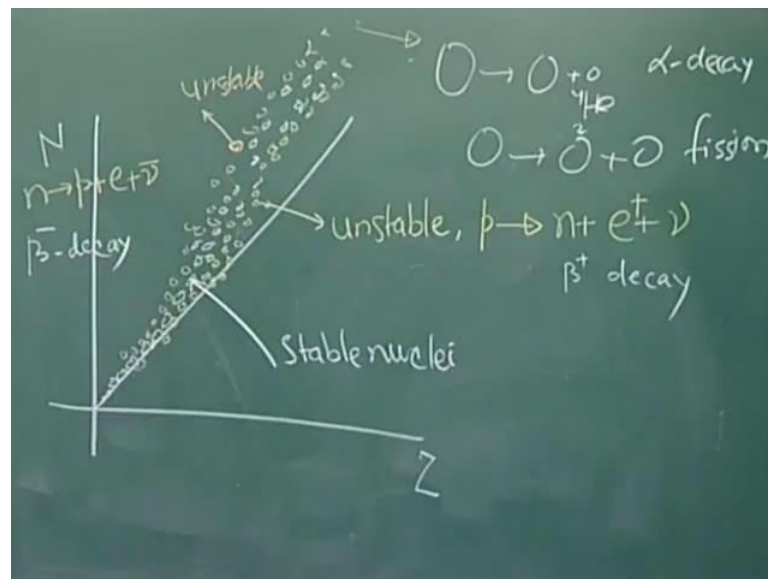
If these couple in a different fashion give a non-zero angular momentum, then the energy will be higher. If it is it gives 0 angular momentum, then it gives lower energy. So, pairing lowers the energy. So, this is called pairing. So, protons and neutrons would like to make pairs of this kind. This kind pairing lowers the energy. Experimentally it can be seen in what we called neutrons separation energy, neutron or proton separation energy. This is something like ionization energy of your atoms. So, if you have a nucleus having certain protons and certain neutrons, you want to take 1 neutron out, so this is a bound system. Everything is bound.

You want to take 1 neutron out. How much energy you have to supply that is known as neutron separation energy. Similarly, if you have nucleus and you want to take 1 proton out, how much energy you have to supply so that 1 proton can come out; that is known as proton separation energy. So, if you take the data for this neutron separation energy and if you look that, you will find that for even-even N, this neutron separation energy, we write it as  $s_n$  and proton separation energy  $p$ . So, if N is even,  $s_n$  is large. If N is odd,  $s_n$  is low. There is one step ahead go for N equal to 8, 6. Then, N is equal to 87. Then, N is equal to 88. N is equal to 89. N is equal to 90. You see the fluctuations. The separation energy is large when it is even.

Then, the separation drops when it is odd. Just add 1 more neutron. It is 7. Let us say 86 to 87. It will drop 87 to 88. Again, it will go up. So, this neutron separation energy oscillates. It is large if N is even and is lower when N is odd. What does that mean? If N is even, all those neutrons are only paired up. They prefer pairing because that lowers the energy of the nucleus. So, once it is paired, it is difficult to break it. You have to supply more energy because the energy had gone down. So, you have to supply that energy also to break the pair. Then, take the neutron out whereas, if this capital N is odd, then surely there is 1 neutron which is not paired to anyone.

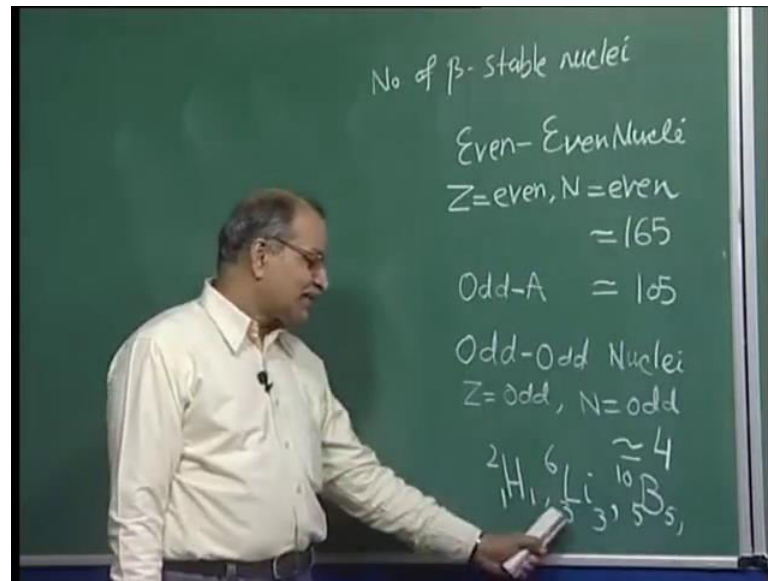
If this capital N is odd, not divisible by 2, then not all neutrons can form pairs. This final neutron, this last neutron which is unpaired, it will consume less energy to go out of the nucleus. When N is even and all the neutrons are already paired up, then it will difficult to take a neutron out because you have first break. For breaking the pair, certain energy is needed. Then, you have to take 1 neutron out so that pairing lowers the energy. Another evidence for that is the number of beta stable nuclei.

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Number beta stable, what does that mean? Beta stable, numbers of beta stable nuclei are the nuclei, which are stable against beta decay, beta plus decay or beta minus decay. So, you remember that figure N verses Z once again. So, in the middle part, you have all stable nuclei. If you are on one side of it proton, proton convert to neutron by beta plus decay. On the other side of stability region, you have neutron rich nuclei. There neutron convert into proton through this beta minus decay. So, beta stable means those stable nuclei, which do not decay by this beta process, neither plus or minus. So, if you look at those numbers, the numbers of even-even nuclei; what does this mean?

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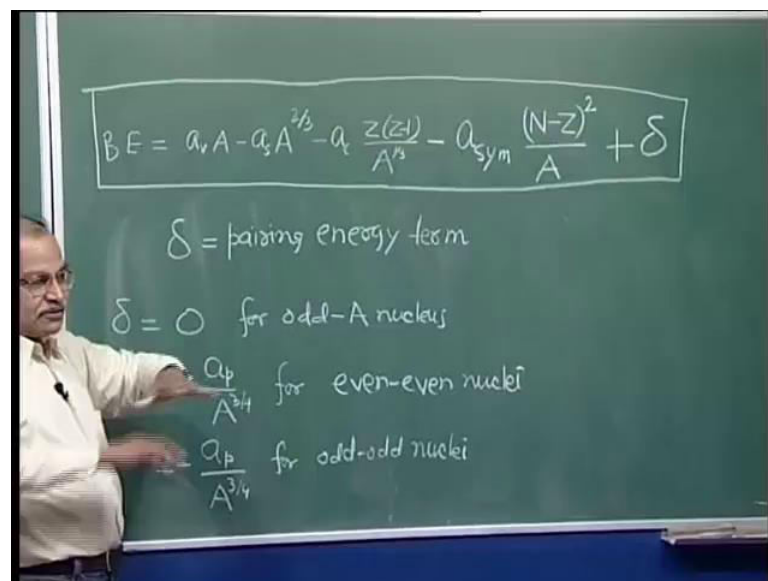


has 5 proton and 5 neutrons. What is fourth next in the series? Z is equal to 1. Z is equal to 3. Z is equal to 5. Z is equal to 7. What element is this? This is nitrogen, nitrogen 14. Nitrogen has 7. These are the only 4 odd-odd nuclei. So, that means somewhere nuclear force prefers even-even thing.

The number of stable nuclei even-even configuration is much larger than odd-odd configuration. Odd-odd is really odd. If you have 1 odd proton sitting here, 1 odd neutron sitting here, then there is a great chance that either this neutron will convert itself into proton and pair up or the proton will convert into neutron and pair up. Depending on the other things, pairing itself would prefer that we have 2 protons or 2 neutrons than 1 proton sitting here and 1 neutron sitting there. So, that means pairing lowers the energy. If it lowers the energy of nucleus that means it is increasing the binding energy for even-even. If you increase the binding energy for odd-odd, you should decrease the binding energy.

We can take reference at the odd A nucleus. We can take the reference point here. We can say that even-even binding energy is increased. If it is odd-odd, then it is negative. Then, it is decreased. So, we had another term here which is delta. We write it delta. It happens to be. It also depends on capital A. So, this delta is known as pairing energy.

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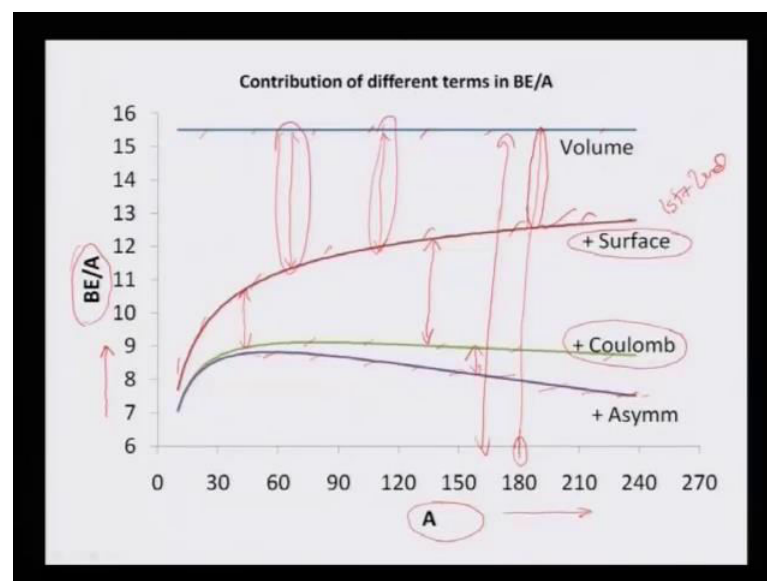


Pairing energy term this you write as delta equal to 0 for odd A nucleus. You write a plus a p A 3 by 4 which is 1. Binding energy is positive and is going up. For even-even

or odd-odd, even-even for even-even nuclei, the whole energy is going down or pairing lowers the energy. The energy of the nucleus going down means binding energy is going up. So, we do plus then minus a  $p A^{-3/4}$  for odd-odd nuclei. So, that finishes the job. Now, these constants; what is a  $v$ ? What is a  $s$ ? What is a  $c$ ? What is this asymmetric? What is this a  $p$ ? These constants are to be determined from the measured message of the nuclei. Atomic masses are very well measured.

The proton mass is known or hydrogen mass is known. Neutron mass is known. So, from these measurements, one can find the binding energy. These are the experimentally available data. Then, you adjust these constants. So, that experimentally measured binding energies can be reproduced. That is how you fix up the values of a  $v$ , a  $s$ , a  $c$ , asymmetry and this a  $p$ . This you can see from text book. What are those values of these constants, which give you a nice fitting of all the experimental data? Now, I will show you a slide where relative importance of these terms will be shown. So, look at this graph.

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So, you have this graph on the screen. On the y axis side here, it is binding energy per nucleon. This side is A, capital A. This line here, top line you are seeing, this is your volume term. The first term, just take the volume term, the first term is just a  $v$  into A. Binding energy is a  $v$  into A. If you look at your expression of binding energy and you calculate binding energy by A taking only the first term, it is just a  $v$  constant. So, that is



this constant line. Then, this is when you had the second term that is the surface term. So,  $A$  is increasing on the  $x$  axis.

So, you subtract from this a  $v$ . This term a  $s$  into  $A$  to the power  $1/3$ . Remember, it is binding energy divided by  $A$  that is plotted here. You have written the expression of binding energy. So, from binding energy of  $A$ , when you take first term and second term together, then you get this term, this curve here. So, this is the surface energy contribution that has been subtracted. This is the surface energy term that you have subtracted from that volume term to get a final curve like this. So, this curve corresponds to both volume and surface term. Then, the third one that you are seeing here, this one is the  $\mu$ . Also, add the coulomb energy.

The third term in expression at coulomb term, this is also subtracting from the total binding energy. So, this much is subtracted. This is the coulomb contribution. This much is further subtracted third term. Then, you get this one. So, see the relative contribution. This graph I am showing to give you an idea of how much is the surface contributing, how much is the coulomb repulsion contributing. So, at this particular  $A$ , this  $A$  will be somewhere around 180. Also, at this point, there is volume energy. This whole thing is volume energy. It is coming from that volume term. Then, this much is surface term which is subtracted to get this.

Then, this much is coulomb to subtract you getting this. From asymmetry, you have a smaller contribution. Then, you get this. Of course, pairing is not shown here. So, you have seen the relative contributions coming from different terms, the volume term, the surface term, the coulomb term, the asymmetric term etcetera. That pairing I have not shown. But one can calculate it. It is close to asymmetric term. It depends on capital  $A$ . How much is capital  $A$ ? Once I have this formula and the values of these constants  $a_v$ ,  $a_s$ ,  $a_c$ ,  $a_{\text{asym}}$  and  $a_p$ , I get tool to calculate the mass of any given  $Z$   $N$  combination nucleus.

Therefore, many of the trend that. We see nuclei for a particular capital  $A$ . How many protons should be there? How many neutrons should be there? You know that for light nuclei, protons and neutrons are roughly equal number of proton and number of neutron are roughly equal. But for heavier nuclei, the neutron number is much larger than the proton number. Similarly, there are many other trends. So, all this trends where the

nuclear mass is involved or the rest mass energy of the nucleus is involved, we have a tool to calculate such masses. Calculate such rest mass energy. Then, see whether those trends you can understand using this or not.

So, in next lecture, I will be using this first. I will give you values of those constants which are obtained from fitting the known masses of nuclei with this formula taking these constants as adjustable constants. So, we adjust the values of this constant so that the masses, the known masses experimentally observed masses can be best reproduced using this empirical mass formula. I will give those values which are obtained from this. Then, using this, we will look for some of the general trends of nuclei. That is for today.