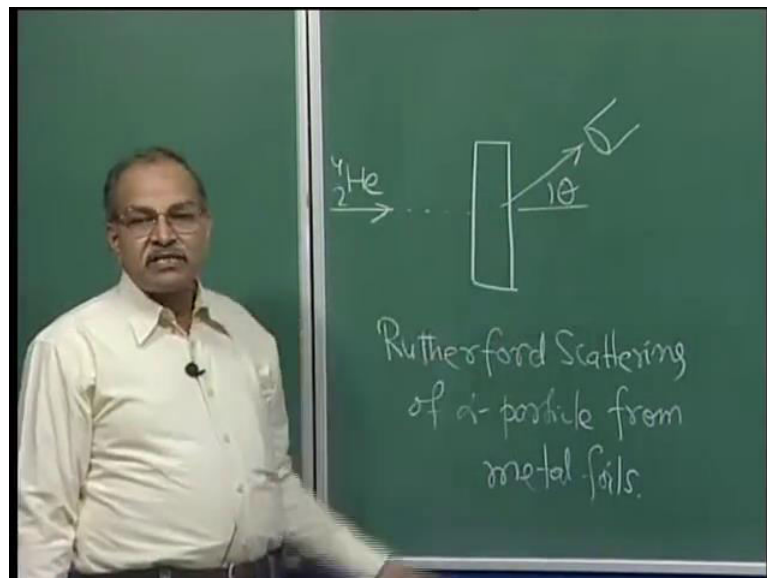


Nuclear Physics Fundamental and Application
Prof. H. C. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 5
Semi empirical Mass Formula

So, nuclear radius size we talked and several methods I described, most of them used this coulomb repulsion, attraction, coulomb interactions. Now, little bit of another method I indicated yesterday, that if you have a set up for say Rutherford scattering.

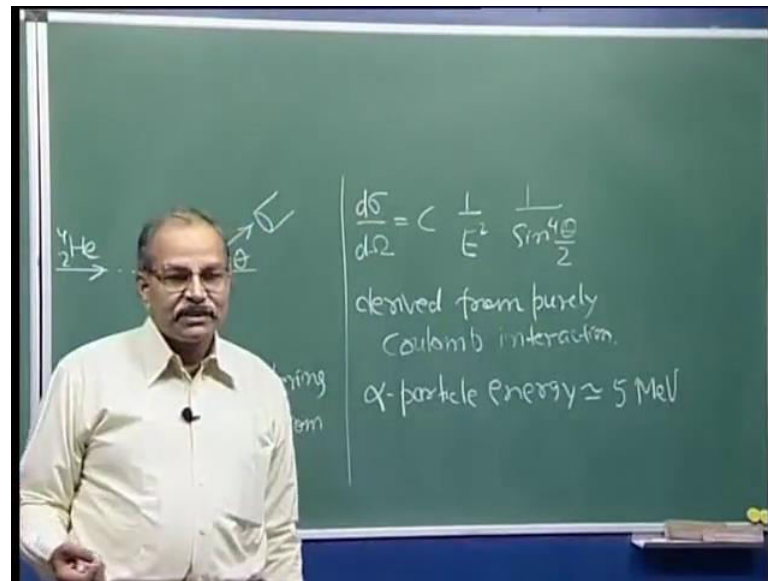
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We have some target material and you send alpha particles, and then these alpha particles scatter in different directions. You put your detector, at some angle θ and see how many alpha particles are coming in this detector, in a given time. This is a setup for what we call, this is how the nuclear physics started of alpha particles from metal foils.

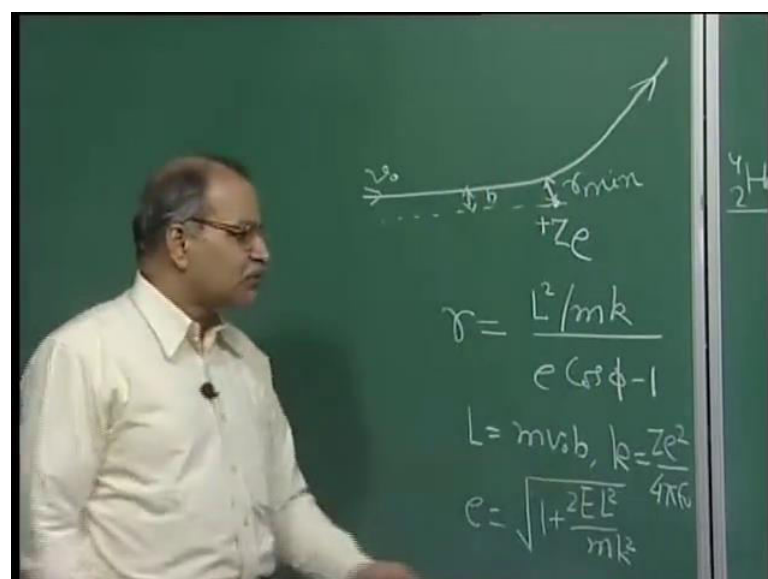
So, the number of particles, which are detected at a particular angle θ , scattering angle θ will depend on the number of particles, that are falling on the target in that time. And the number of target nuclei, with which it is interacting, what is the solid angle subtended by this detector here, all those things. But the interaction part, what deviates it, that interaction part is in, what you call differential cross-section.

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And that is $d\sigma/d\Omega$, you write it $d\sigma/d\Omega$, which turns out to be some constant and then E square in the denominator and angle dependence is here. This is on derived from purely coulumbic interaction. Now, the alpha particle energy that was used in the original Geiger Marsden experiment was around say 5 MeV, used a natural radioactive source. And from the natural radioactive sources, when alpha particles come out, the energies are of the order 4 to 8 mega electron volts.

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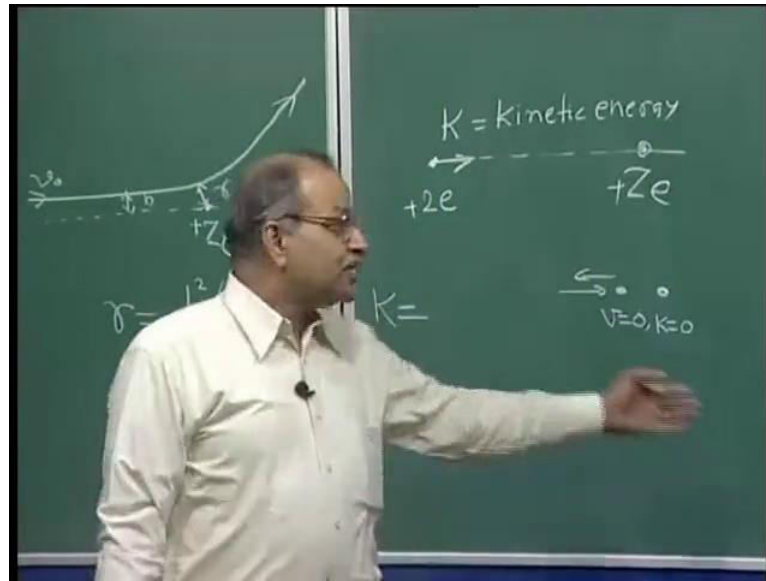
So, if you think of a particular event, where you have a nucleus of charge plus Ze nuclear, a particle is coming here and then it is getting deviated, this is that hyperbolic path. You know all that geometry can be done, here I am interested in finding this minimum separation here, minimum separation. So, if you write the whole equation, I do not need that, but still the equation would be, if I take this as origin and then $R \cos \theta$ this as flexibility axis as R . So, R will be given by, some L^2 by $m k e \cos \phi - 1$.

So these have this angular momentum, that means if I take this distance to be b . Then this L will be $m v b$, where b is not the speed here, the angular momentum k is, that constant Ze^2 by $4\pi \epsilon_0$ is that eccentricity, which will be $1 + 2$, total energy L^2 by $m k^2$. All those things, I do not need that, I would not do any analysis on that, but using these things you can find, what is the minimum value of r , this minimum value of r if this angular momentum on this impact parameter is known to get an estimate of this minimum what I will do, I will make some simplification and just look at the hidden collision.

So, if the alpha particle is, directly heading towards the nucleus what is that minimum distance, it will reach. That is very simple, you do not have to go into that hyperbolic path and all that. Why I am interested in that, I want to see if nuclear force becomes effective. This formula is derived, a Rutherford back scattering formula is derived using only the coulomb interaction.

So, if the distance between the alpha particles and this positively charged nucleus, which is just scattering it. If that distance, is much more than nuclear range, nuclear radius then this formula will be correct. And if the separation is such that, the alpha particles is getting into that bigger nucleus, it is penetrating, it is going inside, it is interacting. Then nuclear interaction between the proton neutron of helium particle that, alpha particle and proton neutron of this big scattered nucleus, that will become operative. Once that become operative the equation will not describe it. So, I am trying to see whether the alpha particle is getting so close to the nucleus or not. So that, one can use this formula or not to explain the data.

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So, let us estimate that, so if you have a nucleus here Ze and suppose the alpha particle is going right on this line with some kinetic energy K . K I am writing for kinetic energy of the alpha particle. So, when it is far away from this nucleus and you know, far away means a millimetre is very, very far away because we are talking of angstrom and centimetres and all that. So when this, so is far from this nucleus then the kinetic energy is this. As it gets closer and closer to this nucleus, because of repulsion here you have 2 times e both positive.

So because of this coulomb repulsion, the kinetic energy will gradually decrease and in a certain distance it will become 0. And after that the alpha particle will go back, repel from here it will be repelled. So, I am looking for that minimum distance, so add that minimum distance, when it reaches that minimum distance and its now returning, it is trying to return from here. So, this is that r minimum and at this point the velocity is 0, kinetic energy is 0. So, you can use usual energy conservation initially, the total energy is only kinetic energy k and when it reaches here, minimum distance then the kinetic energy becomes 0 and all this original kinetic energy is now potential energy.

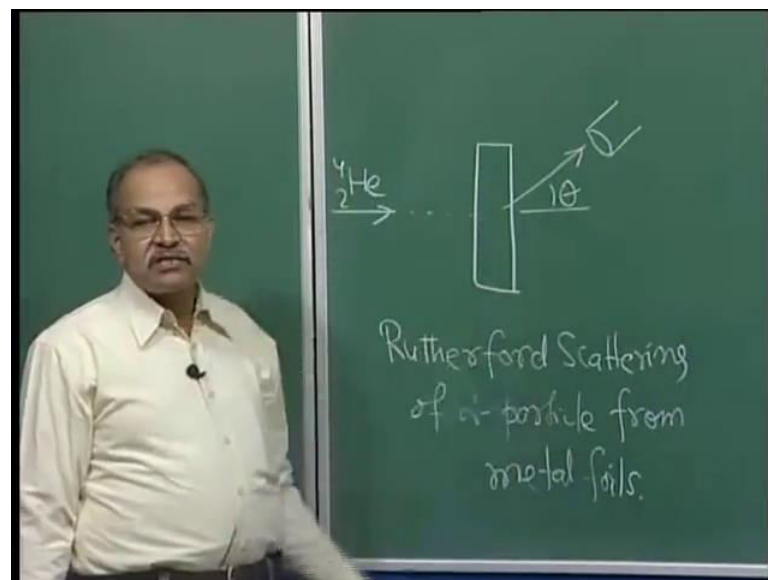
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Yes, this is elastic scattering, nice topic to talk little bit about. What is elastic scattering and what is inelastic scattering, elastic scattering the internal structure of the particles are not changed. So those internal energies are not changed, then whatever you see kinetic

energy, potential energy, so that remains intact. But if during this scattering, internal structure of one of the two particles or both particle changes, internal energy increases or decreases.

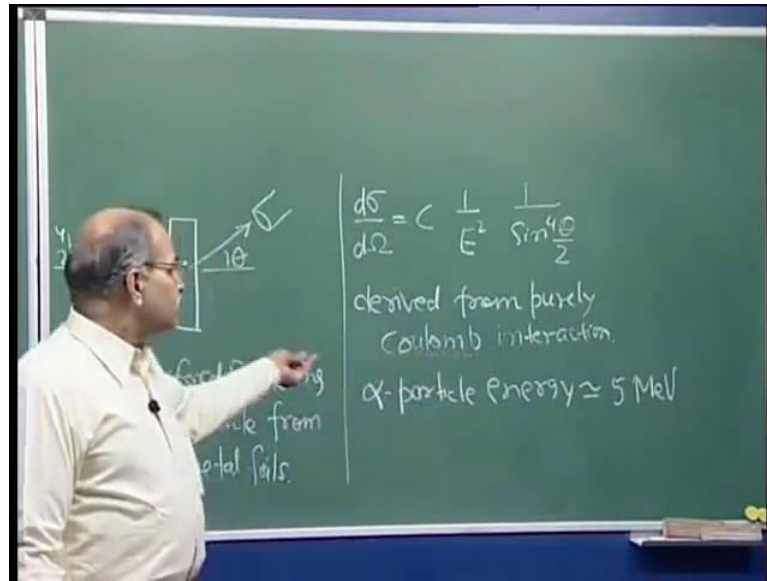
Then you have the visible kinetic energy plus potential energy, not same as the original and it becomes inelastic scattering. Now in this picture, alpha particles remain alpha particles and this nucleus remains this nucleus. So, no internal structure is changing and hence therefore, this is elastic scattering right. So, nuclear radius, size we talked and several methods I described. Most of them used this coulomb repulsion, attraction coulomb interactions, now little bit of another method I indicated yesterday that, if you have a setup for say Rutherford scattering.

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So, you have some target material and you send alpha particles and then these alpha particles scatters in different directions. You put your detector at some angle theta and see how the nuclear physics started of alpha particles from metal foils. So, the number of particles which are detected at a particular angle theta, scattering angle theta will depend on number of particles falling the target in that time. The number of target nuclei with which it is interacting, what is the solid angle subtended by this detector here, all those things. But the interaction part, what deviates it, that interaction part is in what you call differential cross section.

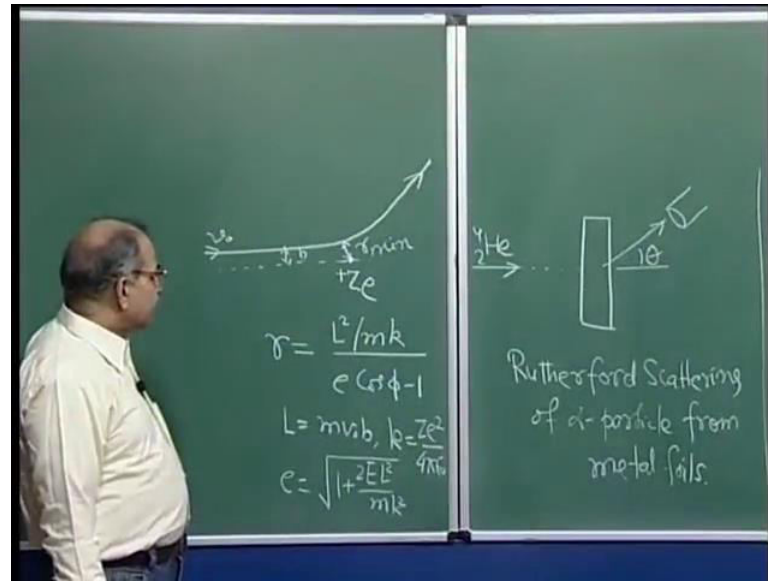
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And that is $d\sigma/d\Omega$, you write it $d\sigma/d\Omega$, which turns out to be some constant and then energy square in denominator and the angle dependence is here. This is on derived from purely coulombic interaction. So, the data where, collector by Geiger and Marsden and the data where there is, a function of theta how many particles are reaching there. Then taking care of the intensity of beam and this and that geometry, the interaction curve here. So, from that data, Rutherford developed this particular model that, in atom you have nucleus with Ze plus charge.

And that is repelling this alpha particle and that is why, alpha particle is going in this hyperbolic path and from there he derived this. Now, the alpha particle energy, that was used in the original Geiger and Marsden experiment was around say 5 MeV . He used, a natural radioactive source and from the natural radioactive sources, when alpha particles come out the energies are of the order 4 to 8 mega electron volts. So, that is the kind of energy, he was using and several metal foils were used in that original experiment.

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So, if you think of a particular event, where you have a nucleus here of charged plus Ze , a particle is coming here and then it is getting deviated. This is that hyperbolic path, you know all that geometry can be done. I am interested in finding, this minimum separation here, minimum separation. So, if you write the whole equation, I do not need that, but still the equation would be as origin and then r theta, this as a symmetrical axis r . So, r will be given by some L square by $m k e \cos \phi$ minus 1. So, these have this angular momentum, that means if I take this distance to be b then this L will be m times v naught times b , where b naught is the speed here.

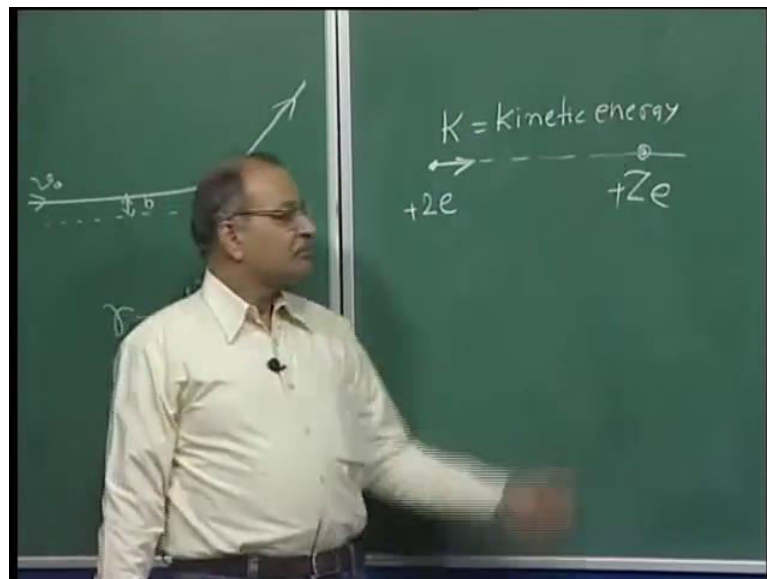
The angular momentum K is that constant Ze square by 4π absolute naught e is that eccentricity which will be 1 plus 2 total energy L square by mk square all those things. I do not need that, I will not do any analysis. But using these things you can find, what is the minimum value of r this minimum value of r , if this angular momentum or this impact parameter is low. To get an estimate of this minimum, what I will do, I will make some simplification and I will just look at the head on collision. So, if the alpha particle is directly heading towards this nucleus, what is that minimum distance it will reach.

That is very simple, you do not have to go into that hyperbolic path and all that. Why I am interested in that, I want to see if nuclear force becomes effective. This formula is derived, a Rutherford back scattering formula is derived using only the coulomb interaction. So, if the distance between the alpha particle and this positively charged

nucleus, which is just scattering it, if that distance is much more than nuclear range, nuclear range then this formula will be correct.

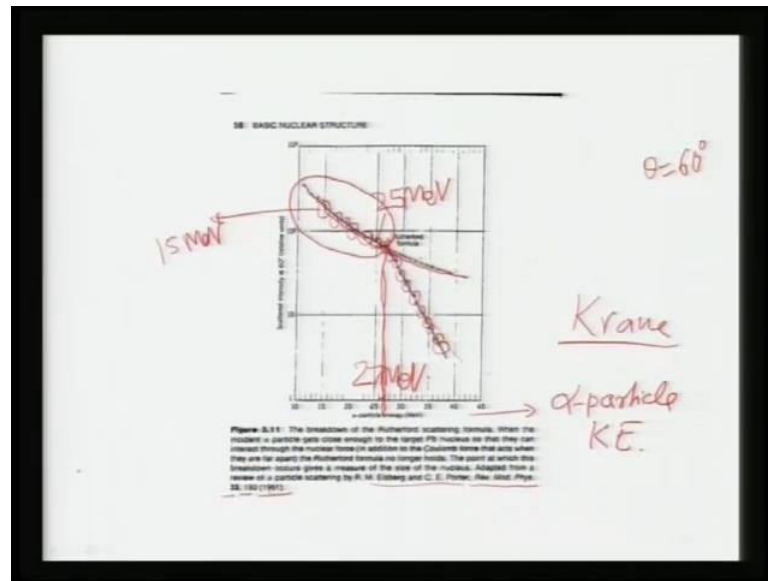
And if the separation is such that, the alpha particle getting into that bigger nucleus, it is penetrating, it is going inside, it is interacting then the nuclear interaction between the proton neutrons of helium particle. That alpha particles and proton neutrons of this big scattered nucleus, that will become operative. And once that becomes operative, this equation will not describe it. So, I am trying to see whether, the alpha particle is getting so closed to the nucleus or not. So that one can use this formula or not to explain the data of course, the Geiger Marsden data were very well explained.

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So let us estimate that, so if you have a nucleus here Ze and suppose the alpha particle is going right on this line, with some kinetic... So, when this is far, far away from this nucleus, then the kinetic energy is this. As it gets closer and closer to this nucleus, because of repulsion here $2e$ both positive, so because of this coulomb repulsion the kinetic energy will gradually decrease. And in certain distance it will become 0, from here it will be repelled. So, I am looking for that minimum distance.

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So, at that min, so now on your screen what you see is a graph on the y axis, you have number of particles which have been received at theta equal to 60 degrees, like scattering angle of 60 degrees. So, the detector is placed to receive particles, alpha particles scattered at 60 degrees from the initial direction, in all these experiments. And on the x axis, you have alpha particles energy, you have alpha energy, kinetic energy. So, each data point on this graph that you are seeing corresponds to one full experiment. So, at certain kinetic energy this is for example, this point here is 15, this point here is 15 MeV, this point corresponds to 15 mega electron volts.

So, at 15 mega electron volts kinetic energy, alpha particles are sent on this target and at 60 degrees how many alpha particles are received in a particular time. Then the experiment is changed, say here it is 25 MeV this corresponds to 25MeV. So, in this experiment, alpha particles 25 MeV are sent on the target and at the same angle 60 degrees, how many particles are received in the same amount of time. So that is plotted here, so this is how all the data points are there. So, we have a data here, you have a point here, you have a point here, you have a point here, here, here, here, here. So, these are experimental points, experimental data points.

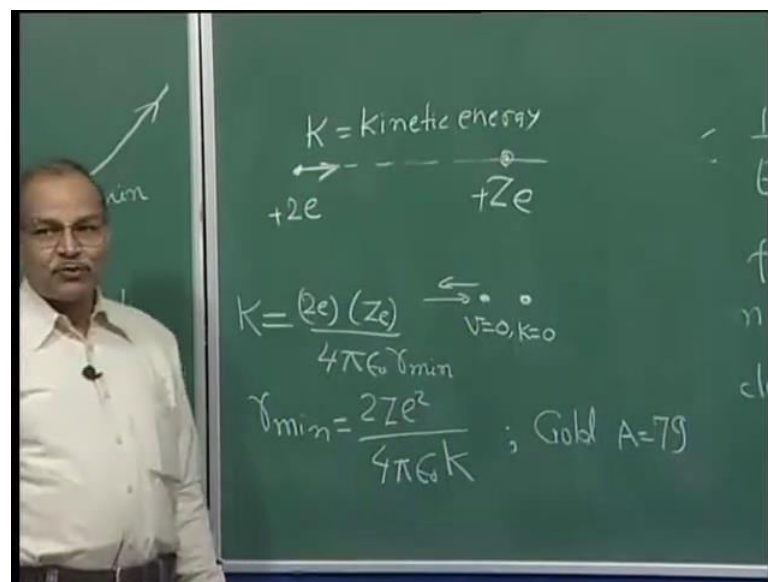
Now, Rutherford scattering formula, if you calculate the number of particles from that formula, at 60 degrees that, in that particular geometry, in that much of time. How many particles should come 60 degrees, so this smooth curve that you are seeing is calculated

from this curve, that you are seeing here, this curve. This curve is calculated from Rutherford formula, so you can see up to here, up to here the calculated values using only coulomb interaction is very nicely matching with like, the data that is obtaining experiment.

But once kinetic energy is more than this, this is somewhere around 27 MeV. So this point is, somewhere around 27 MeV. So, once the kinetic energy is more than this 27 MeV, the Rutherford scattering formula is here and the data points are here, very sharp deviation. So, this is the point, after which the nuclear forces have started operating, that mean the helium nucleus is now touching or penetrating going into that target nucleus.

So, if you calculate that r minimum, corresponding to this 27 MeV, that corresponds to say roughly, some of radius of helium nucleus and the target nucleus. This particular figure has been taken from the book of Krane nuclear physics, adopter from a review of the alpha particle scattering by Iceberg and Porter, reviews of modern physics. This is the name of the journal, reviews of modern physics, volume 33 page 190 and year 1961.

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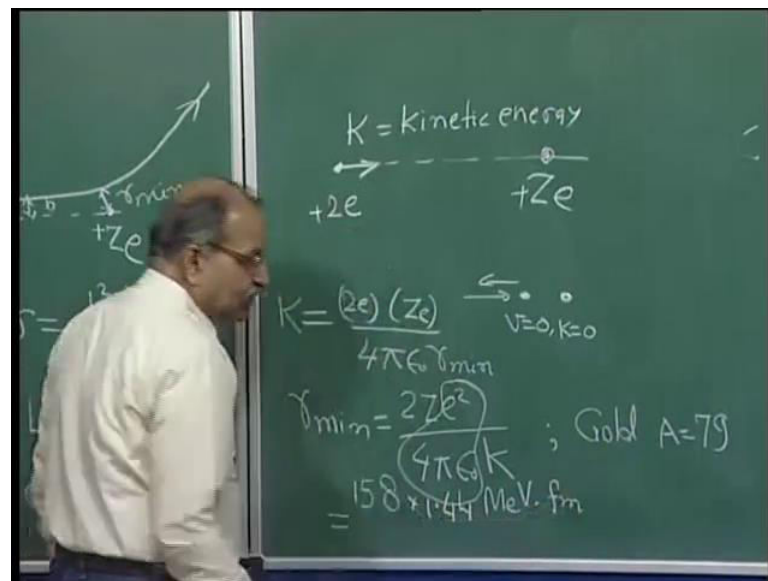


Then, the kinetic energy becomes 0 and all this original kinetic energy is now potential energy.

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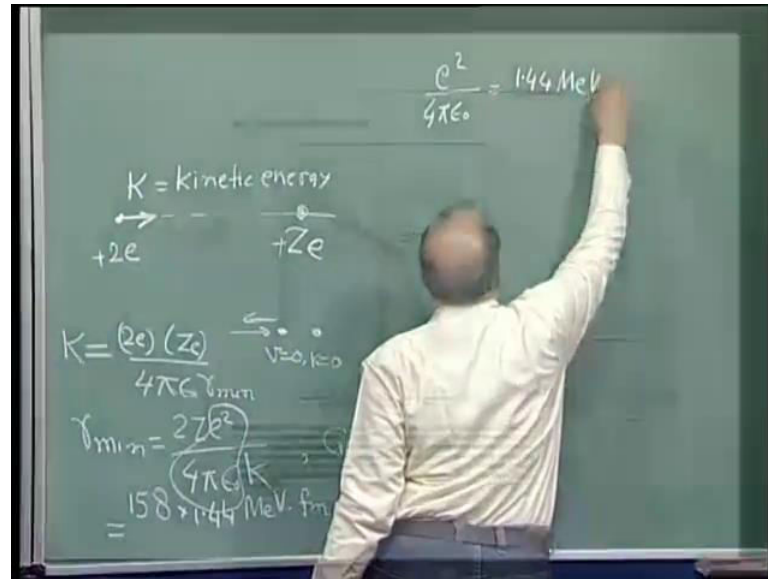
Yes, this is elastic scattering here and a nice topic to talk little bit about. What is elastic scattering and what is inelastic scattering, elastic scattering the internal structure of z times is e square divided by $4\pi\epsilon_0$ times kinetic, initial kinetic energy. So, let us take some numbers for gold A is 79, it is famous gold foil experiment. Although, in that particular expensive paper is gold, is one of several metal foils that has been used by Geiger Marsden.

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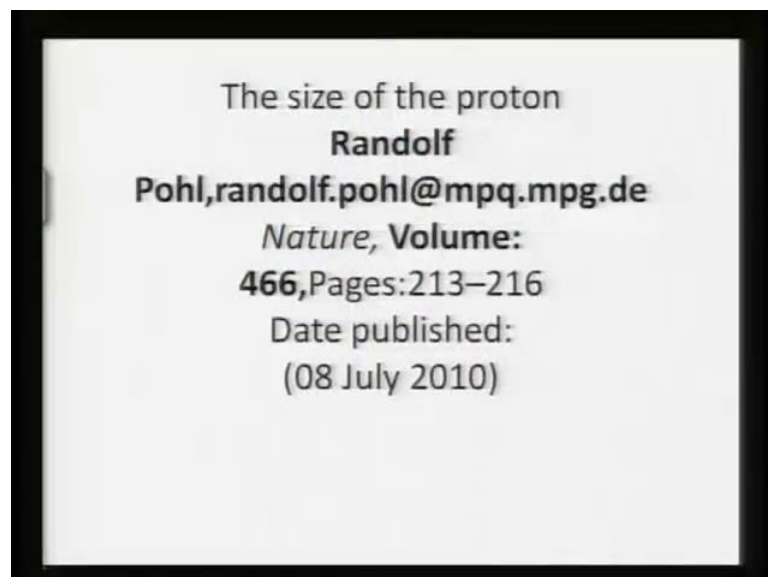
But anyway, so A is this e you know. So, this is equal to, this will be equal to 2 times 79 is 158 and then is e square divided by $4\pi\epsilon_0$ naught. This quantity is again a useful quantity to remember, it is 1.44 mega electron volts and femtometer. This quantity will be quite useful several times, you will encounter this. So, it is good to remember this

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the square over $4\pi\epsilon_0$ naught, this quantity turns out to be 1.44 MeV. Proton size has also been looked at, one single proton. So, it depends, what kind of scale, you are using to look at the nucleus, if you are looking with a very high energy things which can penetrate into the nucleon. For example, then you will start seeing the structure inside the nucleon.

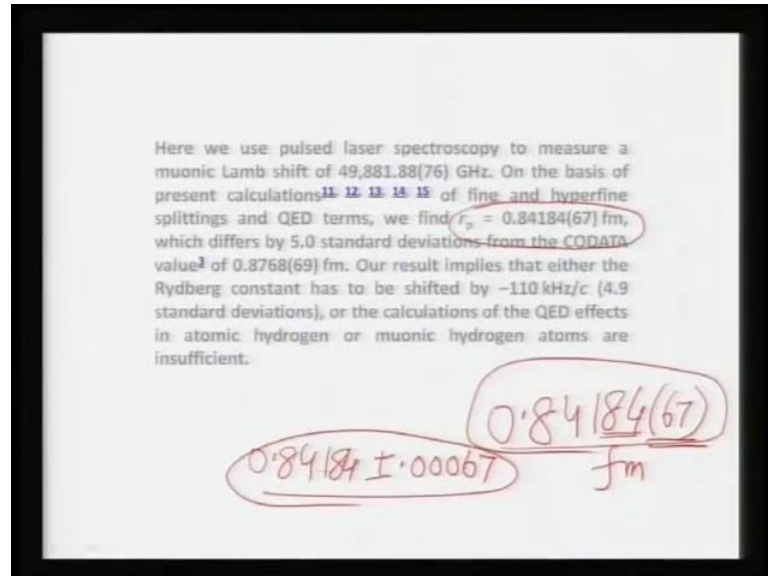
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So, very recent paper once again I will, I will Show you, the just reference size of the proton. This is published in nature and very recent July 2010 and what they have gotten

the results, the method was to send neon inside that nucleus. And neon going into that and then finding the energy difference between the 2 p and 2 s they have done that.

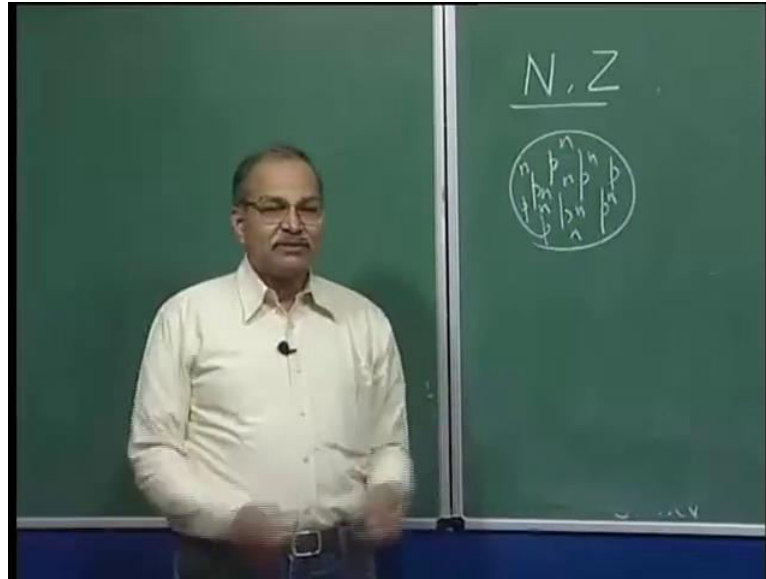
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And the size that they have find, the charge distribution they find is 0.84, can you see that 0.8418467 femtometers, what is this 67 here. This is the way, to write the uncertainty in the result from the experiment they have derived this value 0.84184 femtometer, but then there are some experimental uncertainties involved. Those uncertainties have been estimated, calculated, found out and this 67 says that, the last two digits of the number. They can be different by this amount, so it is essentially another way of writing 0.84184 plus minus 0.00067.

So, if you see this type of data somewhere, you should understand that it is not multiplication by 67, as you see in algebra. It is the uncertainty, in the, in those many figures, in the last two figures. So, proton the charge distributed inside the proton, that has a mean radius of, of point, about 0.84 femtometers enough on size, let us do something else. So, let us look at the number of neutrons and number of protons in a nucleus.

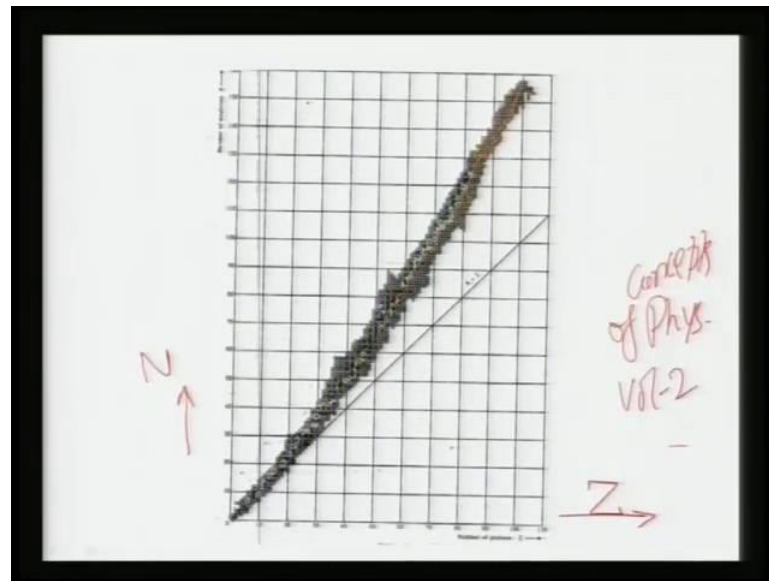
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Nucleus is made from neutrons and protons, so in the nucleus you have certain protons and in this also you have certain neutrons. Can I put any number of protons and any number of neutrons and make a nucleus. No, you do not have a new, a nucleus having 10 protons only, no neutrons, do have a nucleus like that. You do not have that kind of nucleus of 10 protons, 0 neutron. Why, why do not you have that, that kind of nucleus what will prevent it, if I make a nucleus, if I push all those protons together and make a new nucleus of 10 protons what will happen.

Coulomb force will take them apart, but if I take 10 neutrons and try to make nucleus what will happen, there is no pull arm force, but even that is not possible. There are certain combinations of Z and N which give you a nucleus, arbitrary you cannot put any number of neutrons and any number of protons. So, all those nuclei, that are available to us that we have found in nature or which we have created in laboratories, all those nuclei if you look at their neutron number and your proton numbers. And then N verses Z those combinations, if you plot on some x - y graph then the how does that look like let us see.

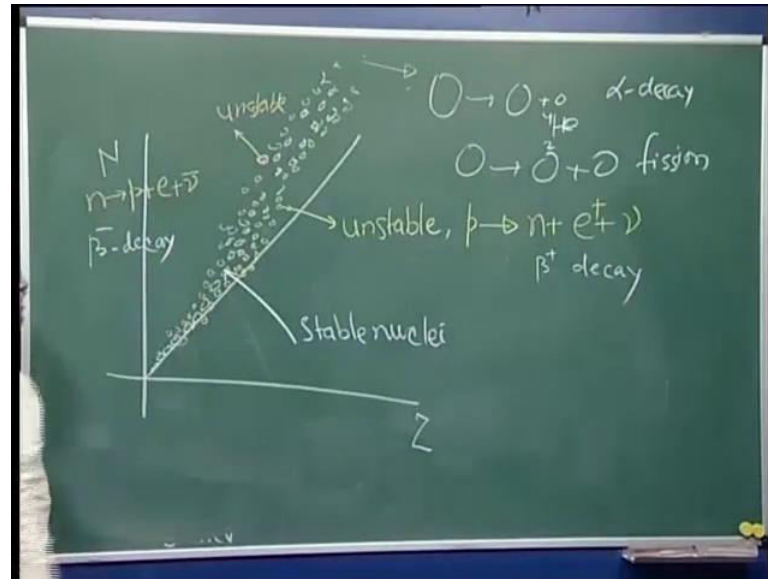
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Look at this diagram, this is on the x-axis side, x-axis side we are plotting Z number of protons and on the y-axis side it is in, this figure is taken from concepts of physics. This figure is taken from the book concepts of physics volume 2 and you can see everything looks very dark there, on this figure you may not able to see. There are certain points, which are more dark than the others here, here yeah these are the points, these are the points. Can you see some white things coming, they are Z N corresponding to stable nuclei.

They are then Z N corresponding to stable nuclei. White things can you see, little bit and then I take another colour, let us say which colour should I take, this pink does it come. Can see these things, I don't know, I will show you on the board rough diagram. So, there are two kinds of things here, one which is, which I have drawn white it is more dark in the whole figure and other where you have grey or whitish. So, the middle one, this corresponds to what you call stable nuclei, this is stable nuclei line and then on the two sides you have unstable nuclei. Let me show it on the board, in any text book you will find it, any textbook you will find it.

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If you have Z on this side and N on this side and this is the line bisecting x-axis and y-axis 45 degrees. So, what happens, you have stable nuclei distributed somewhere like this, something of this sort and then you have nuclei on the two sides of it, like this. You have nuclei like this and you also have nuclei like this, somewhere here. So, this white thing corresponds to this, these corresponds to stable nuclei, stable nuclei means there are certain combinations. These combinations of Z and N for which, once the nucleus is formed, it remains there for long time long, long time and unstable means, that nucleus can changes itself during some course of time, unstable.

The change, so these are unstable, this is unstable and this is unstable in the sense that you have a more number of protons than required, at that particular thing. So, this is unstable, one of the protons or one more protons can change, will change itself to this thing. So that Z N combination will change, even if you make at a certain incident a nucleus with Z and N combination corresponding to this a yellow point here.

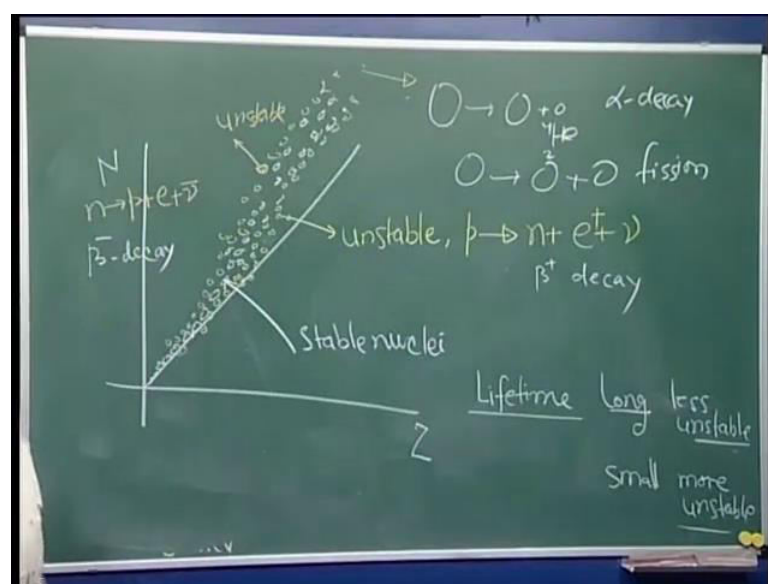
These many number of protons and these many number of neutrons, if you make it, it will not remain there. One of the protons or more protons will change to neutron so that, your Z and N competition gets changed and it comes to this stable region, this white thing is a stable. So it will change similarly, if you have a nucleus you have formed somewhere here, this also on a stable, this is also on a stable.

But here, neutron number is more than what is required at this case. So here, a neutron will change itself into a proton and an electron and an anti-neutrino. So once again, that Z N combination will be, will change. So, not all Z N combinations will make a nucleus, a stable nucleus or if you, if you are here in this valley, in this range, in this the path then yes. But if you are on left of it, then you are unstable, your Z N combination will give you unstable nucleus and the things will change according to this.

If you are to the right of this line, then the nucleus is again unstable and then the proton will change and Z N will finally come here. For very heavy nuclei, for very heavy nuclei there is another instability coming. And that instability is through alpha particles and nothing changes as such, but then the nucleus splits in two parts, this is that alpha particle. So, it is unstable, so many nucleons cannot be held in a nucleus and an alpha particle comes out or it can fission, it can fission in two roughly equal parts, fission.

So this is fission, this is alpha decay, this is beta plus decay and this is beta minus decay. So if it is unstable, anything depending on what nucleus it is, can happen it can alpha decay, it can beta minus decay, it can beta plus decay, it can fission and things like that. Either if you are trying to make a nucleus somewhere here, that nucleus will just not fall stable or unstable. Now degrees of instability, a degree of instability can also be how unstable it is, that is measured from lifetime.

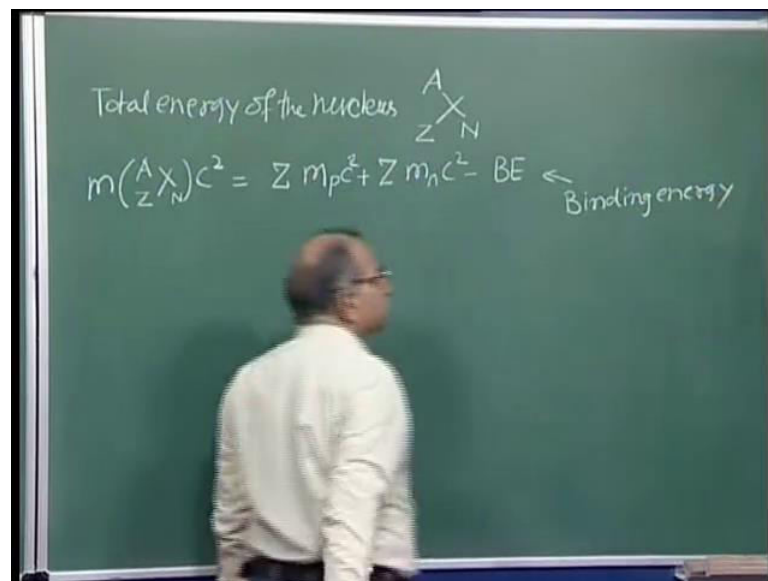
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How much time, average time for how much, what is the lifetime of this nucleus once it is formed it is unstable, it will beta decay or it will do something, some disintegration, so what is the lifetime of that. So if then that lifetime is long that means it is more stable, it is not stable, it is unstable, but then instability is less and if the lifetime is small, these are all relative. So it is, more unstable instead of more stable let me call it less unstable, so that instability can also be little bit quantified, how unstable it is. So that is from lifetime.

Now, to understand why there are only certain Z N combinations which are stable, why there are certain Z N combinations which are unstable against beta plus decay or beta minus decay extra. You need to look at, the total energy of that Z N system if that total energy is minimum then of course it will be stable and if there are more minima available, then it will decay to that combination. So, we have to look at the energy of the, total energy of the nucleus.

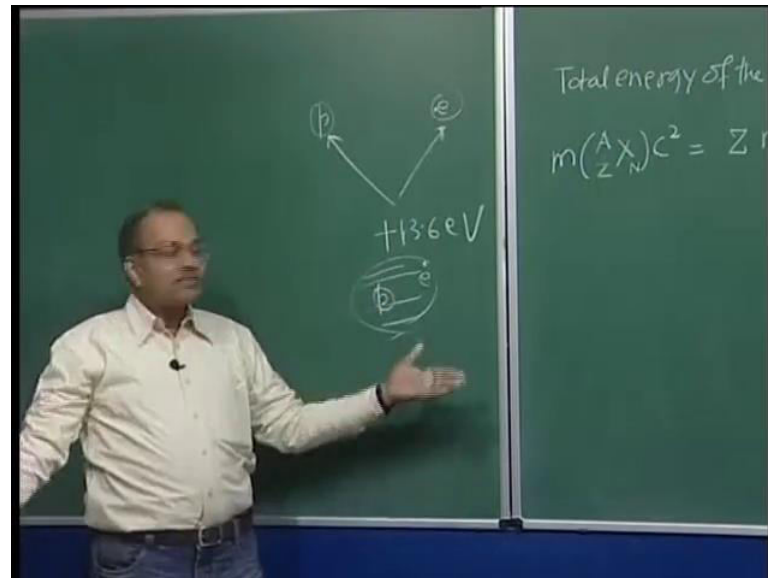
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So, total energy of the nucleus, with Z protons and N neutrons, total nuclear number A and total energy as you know is mass times C square. So, this total energy is mass of this nucleus AX Z N times C square, this you can write as Z time mass of protons C square, mass of neutron C square. But this is not equal, but if you think that there Z protons and N neutrons here are the mass energy, total energy of Z protons and here is the total energy of neutrons. And then added together gives you the total energy of a nucleus that

will be wrong, because it is a bound system and inbound system, this total energy is less than that. And that is binding energy, that is called bind, the difference is called binding energy.

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So, if like hydrogen atom, hydrogen atom you have proton and then you have an electron, a bound system. So the total mass of this hydrogen atom, will be mass of the proton, plus mass of electron, minus something it will be less than that. And that difference in the energy, is the binding energy and how much is the binding energy for hydrogen atom 13.6 electron volt. You give, if it is in ground state, you give 13.6 electron volts then this will be separated, separated means a proton and electron not bound, separate electron is here, proton is here.

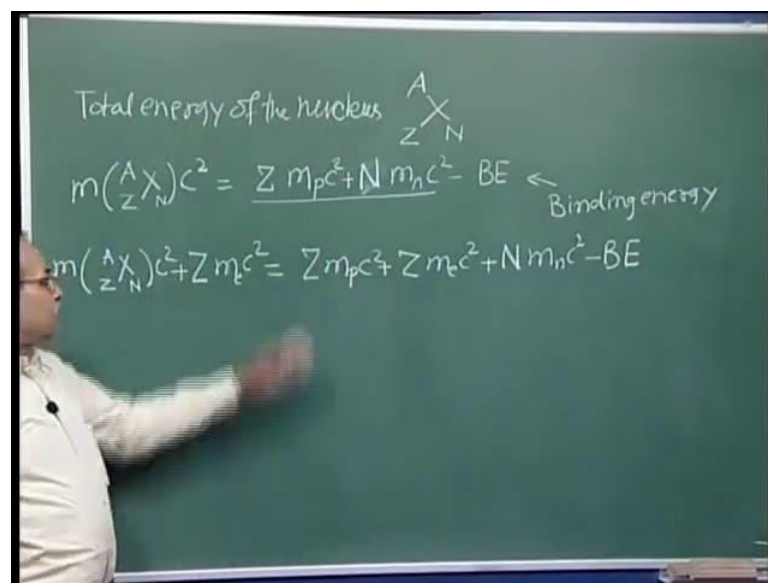
So, this hydrogen atom you have to supply, you have to supply plus 13.6 eV, then you get this and this and anything extra will result in kinetic energy of these things. This is the difference between the mass energy. So, if you have an hydrogen atom in ground state and you give just 13.6 eV you separate the particles and that is it. If you give anything extra more than 13.6 eV, then it is separated and then the rest of energy appears as the kinetic energy.

So, the total mass energy of hydrogen atom is 13.6 eV less than, the mass energy proton plus mass energy of electron. Similarly, here you have Z protons so if they are all separate from each other and Z neutrons all separate from each other, then this is the

energy. And energy of this nucleus, when they are all bound by those nuclear forces and coulomb forces and everything that nucleus, that energy will be smaller than this energy by this much, this binding energy.

Now, if you, if I look and binding energy mass of proton is known, mass of neutron is known so mass of this nucleus is needed. So, if I try to see the pattern in all those nuclei which are available to us, which are naturally available or which we can create in laboratories. If you want to look at the pattern of binding energy then I need all those masses of all those nuclei correct, but then the through experiments it is easier to measure mass of the atom, than mass of the nucleus. So, atomic masses are very accurately measured and tabulated right in place of nuclear masses. So, how do I relate the two it turns out to be that it is very simple, if you add Z electron, rest mass energy on both sides.

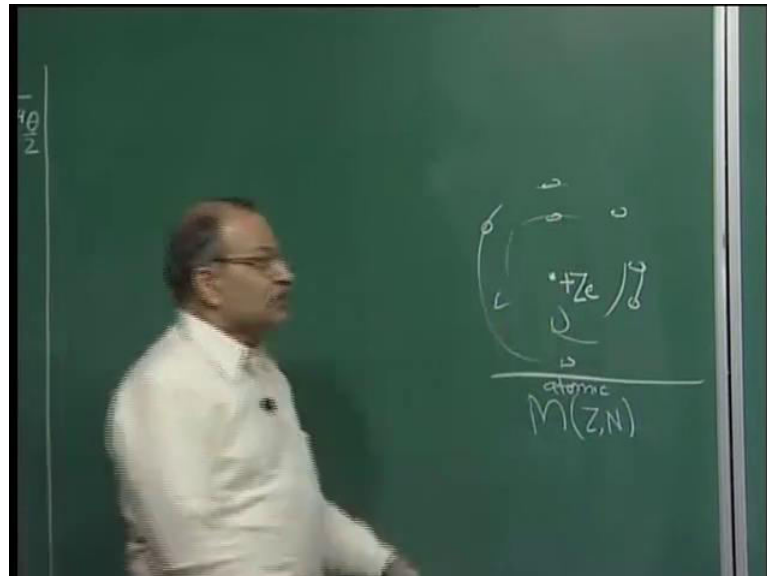
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What you will have, you will have mass of this nucleus and plus let me add Z times $m_e c^2$ on both sides. What I have done in here, number of neutrons $m_n c^2$ and minus binding energy of the nucleus, Z protons and N neutrons. So, I have added this term on right side and left side, now look at the left hand side, you have the nucleus, mass of the nucleus plus mass of the electron, Z electrons. You have full nucleus, Z protons N neutrons you have full nucleus, so nuclear mass separately and Z electrons

mass separately. This can make your atom, with Z protons and Z electrons neutral atom right. Like this, hydrogen that I had discussed.

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Suppose, you have a nucleus with, this is a nucleus with plus Z charge, plus Ze charge and then you have electrons, many electrons. What is the mass of this atom, atomic mass, atomic mass of this atom which has Z protons and N neutrons and Z electrons. This will be mass of the nucleus, plus mass of all these electrons minus the binding energy of this atom, minus the binding energy of this atom.

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$$\begin{aligned}
 &\text{Total energy of the nucleus } {}^A_ZX_N \\
 &m({}^A_ZX_N)c^2 = Z m_p c^2 + N m_n c^2 - BE \quad \leftarrow \text{Binding energy} \\
 &m({}^A_ZX_N)c^2 + Z m_e c^2 = Z m_p c^2 + Z m_e c^2 + N m_n c^2 - BE \\
 &m^{at}({}^A_ZX_N)c^2 + BE^{at}({}^A_ZX_N) = Z m^{at}({}^1_1H)c^2 + N m_n c^2 - BE \\
 &m^{at}({}^A_ZX_N)c^2 = Z m^{at}({}^1_1H)c^2 + N m_n c^2 - BE + [Z BE^{at}({}^1_1H) - BE^{at}({}^A_ZX_N)]
 \end{aligned}$$

So, this is atomic mass let me write 80 here, atomic mass of this atom now this is, atom with Z protons, Z electrons and N neutrons. It is the whole, the proton neutrons is nucleus and plus all those electrons outside, so that atom is here. And this plus, binding energy atomic or a t understand that, the mass of the atom should be mass of the nucleus, plus mass of the electrons, minus the binding energy. Therefore, mass of the nucleus, plus mass of electrons will become mass of the atom plus binding energy.

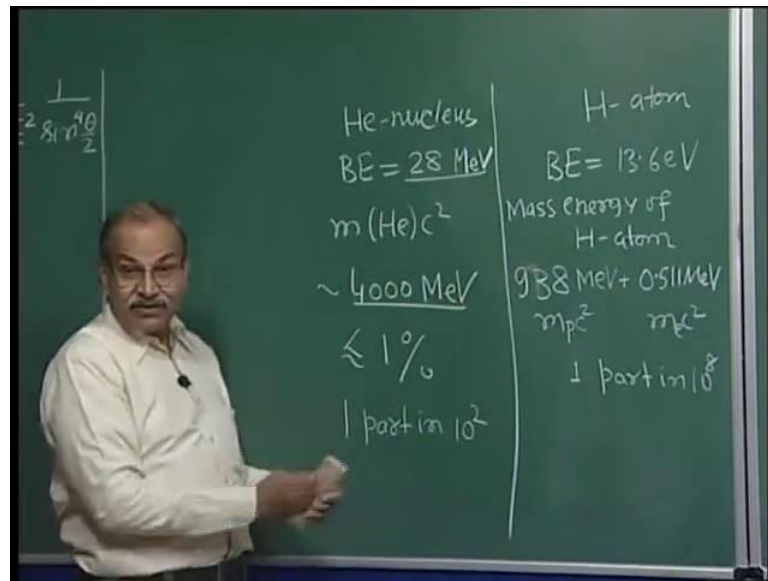
On this side, if you take Z common what it is, if you take Z common what it is, $m_p c^2$ plus $m_e c^2$, look at this one proton, one proton energy mass energy and one electron mass energy. One proton and one electron taken together, will make a hydrogen atom. So, this quantity is mass of the hydrogen atom plus binding energy of the hydrogen atom, because mass of the hydrogen atom is mass of the proton plus mass of the electron minus that atomic binding energy 13.6 eV. So, this mass of the proton c^2 plus mass of the electron, c^2 will be mass of hydrogen plus.

So this is mass, atomic mass, mass of the atom, which atom hydrogen atom and plus binding energy atomic of what, hydrogen 13.6 eV correct. Plus N times $m_n c^2$ and minus binding energy, what is that last binding energy I am writing binding energy of nucleus, in the same BE which I have written to start with. This is all nucleus and I if I put 80 on top, that is atomic. So, this is atomic binding energy of this which atom, which atom, binding energy of which atom it is, of this atom, of the this atom.

So, mass of atomic mass c^2 , put c^2 at proper places. So, atomic mass of this $A - Z$ c^2 is equal to, this I will take on the other side Z times atomic mass of hydrogen, plus N times $m_n c^2$, minus nuclear binding energy, plus binding energy atomic Z times, where is Z so many slips. This, this Z that I had taken common, that Z will also appear here. So Z , so binding energy of Z times, Z binding energy of $1H1$ and minus binding energy of, atomic binding energy of $A - Z - N$.

Let us check one by one that I have written everything, binding energy of this atom is here, the mass of this atom is here, this is atomic binding energy of this bigger atom, that is here, so this part is done. On this part, it is Z times this hydrogen atom, that is Z times hydrogen atom is here and then I have Z times this atomic binding energy. So that, Z time is also here and then N times new neutron mass, that is here and this minus binding energy that is here, everything is here.

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Now, let us take some values for hydrogen atom, how much is binding energy 13.6 electron volts, what is the mass of the hydrogen atom, mass energy you have one proton. So, one proton mass itself is 938 MeV and plus 0.511 MeV. So, this is mass $m_p c^2$ is this and $m_e c^2$ is this. So, compare, compare binding energy with the mass energy, this is coming close to 1000 mega electron volts and this is just 10 electron volts. So, one part in a how much 10^8 , 10 eV and if I take 1000 MeV, 10^9 here and 10^8 here. So one part, one part in 10^8 .

In contrast, look at that helium nucleus 2 protons and 2 neutrons, look at this helium nucleus. The binding energy for this turns out to be, 28 point something point 3. So, MeV look at the numbers, of binding energy it is in mega electron volts for helium and here it was in electron volts and if you look at that fractional thing, the mass of helium c^2 nucleus, you have 2 protons and 2 neutrons. Each proton is 938 MeV, each neutron is slightly heavier something like 939 MeV, so four of them.

So, roughly it is coming close to 4000, 1000 MeV here, 930 and 4 of them, it is almost about say 4000 MeV, this is 28 MeV. So, little less than 1 percent of that order, 1 part in 10^2 to the power 2. Here it was, 1 part in 10^8 , I am doing the order of magnitude calculations just to get an idea that, atomic binding energies are much, much smaller as compared to the nuclear binding energies. So, now look at this last term here, this part, both of them are atomic binding energies.

Here, you have a hydrogen atom, binding energy of hydrogen atom multiplied by Z and here you have atomic binding energy of this whole nucleus, having Z protons. So, atomic binding energies, are themselves small and then here you have 2 atomic binding energies of comparable order. Here also you have, Z hydrogen atoms the first one Z hydrogen atoms and here you have all those things combined together, you get one big atom atomic energy of that.

So, that difference is going to be even much smaller the binding, atomic binding energy themselves are very small and then two binding energies which are comparable, you are subtracting one from the other. So, this whole thing is going to be, on this side you have of all these things and this is nuclear binding energy, remember. So, this whole thing is going to be very, very small as compared to this and unless you have some very sophisticated experiment or calculations you can always neglect that.

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Total energy of the nucleus ${}^A_Z X$

$$m({}^A_Z X)c^2 = Z m_p c^2 + N m_n c^2 - BE \quad \leftarrow \text{Binding energy}$$

$$m^{\text{at}}({}^A_Z X)c^2 = Z m^{\text{at}}(1\text{H})c^2 + N m_n c^2 - BE$$

So, if you neglect that what happens, if you neglect that and write it here. You have mass, atomic mass now you take atomic mass of this atom with Z protons and a total nuclear number. So this atom, atomic mass c square equal to Z atomic mass 1H1 hydrogen atom c square, plus N m n c square and minus BE. Compare, same terms are there instead of nuclear mass now, it is atomic mass, instead of mass of nucleus I am writing, mass of the neutral atom having the same nucleus. This mass is different from this atom mass that means you are putting Z electrons also and all those things.

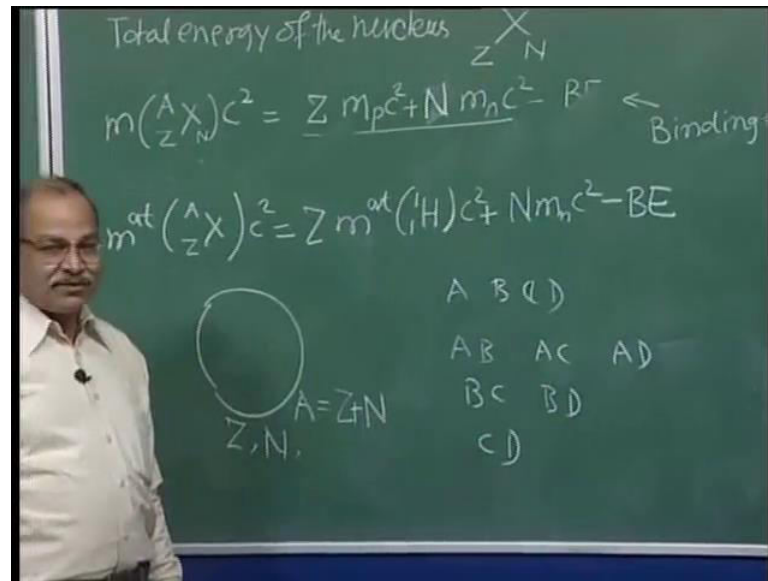
So, instead of this the nuclear mass, if I am writing mass of the atom having that nucleus at the Centre and plus electrons. And similarly, here look this is mass of the proton so instead of mass of proton I am writing mass of the hydrogen atom and neutron is of course, neutron. And this binding energy is same as this binding energy, nuclear binding energy, nuclear binding energy. So, to make a calculation of nuclear binding energy, this term I really, I can work with atomic masses.

I do not need to know the nuclear masses, atomic masses are experimentally easier to measure and because you have atom. And then you strip one electron or two electrons make ions and then deflected in some magnetic field and this and this and that from the curvature and other things. So, that atomic mass is easier to get a bare nucleus strip all the electrons and then measure mass of that is more difficult.

So, this all this calculations I have done to show, that to calculate nuclear binding energy between those protons and neutrons inside the nucleus. You can still work with, the known atomic masses using this atomic mass of hydrogen and then of course, neutron and then the atomic mass of that atom, bigger atom which has this nucleus at its centre. From these atom, atomic mass and neutron mass you can calculate binding energy.

Now, why does this binding energy come, the binding energy comes because of the attractive force. Hydrogen atom, from where that energy coming, the binding energy coming is because of the attraction between the proton and the electron. In the nucleus from where the binding energy is coming, it is coming because the nuclear is bound and it is bound by those attractive nuclear forces. So, whenever you have an attractive force which binds the system, you will have this binding energy business.

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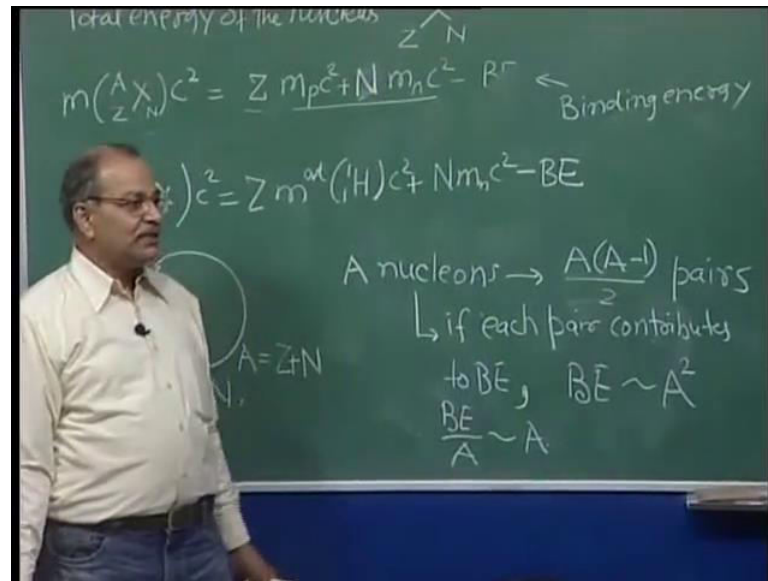


Now, how many pairs are there, suppose you have A nucleus with the Z protons and N neutrons and the total number is Z plus N. The nuclear attractive force acts between two protons, between two neutrons and protons neutrons whatever nucleon is there. Nuclear force, the coulomb force acts only between two protons, one neutron, one proton no coulomb force, two neutrons no coulomb force, two protons yes, but nuclear attractive force it is, it does not look at charge. Charge is taken care of by the coulomb, ok.

Nuclear attractive force is between two protons, same as between two neutrons, same as between a proton and a neutron. So, how many nucleons are there capital A, how many pairs are there. If there are A nucleons how many pairs, because it is pairwise, 2 nucleons attract each other. That attractive force inside the nucleus is, between two nucleons so how many nucleon pairs are there.

If you have three particles, how many pairs can you form A B C, if you have two three particles A B C how many pairs you can form AB, BC and CA three. If there are four particles A B C D, how many pairs you can form 6, 6, 4 C 2 if you have four particles, A B C and D you can form AB, you can form AC, you can form AD. Then you can form BC and you can form BD and you can form CD 6.

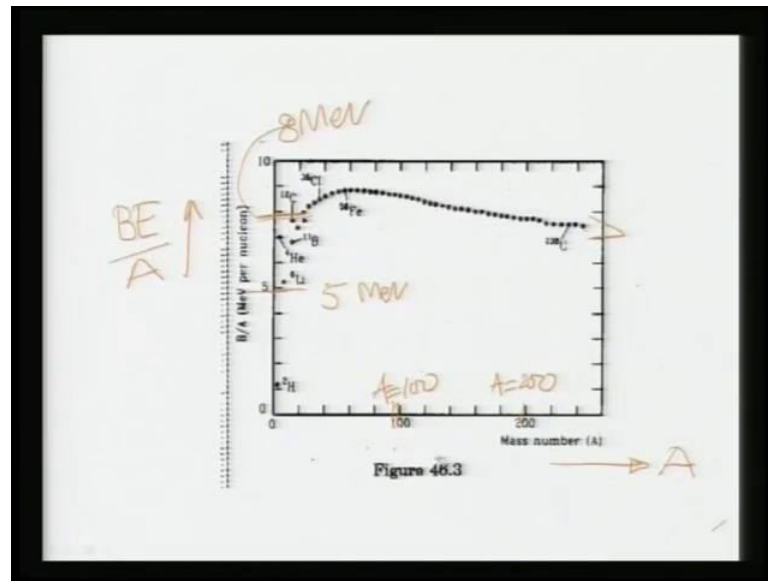
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Similarly, if you have A nucleons, if you have A nucleons, you can form A into A minus 1 by 2 pairs and from each pair, if there is a nuclear attraction and this nuclear attraction is giving you the binding energy. Then the total binding energy, what you think should be proportional to this, if you have 10 pairs of nucleons attracting each other, if you have 20 pairs of nucleons attracting each other this.

And each pair attraction gives you binding energy and binding energies is likely to be proportional to this, but it is not. So if each, if each pair contributes to binding energy, then I should have binding energy roughly proportional to A square. Do not look at A equal to 2, 3 extra go for slightly higher 20, 30, 40, 50. Binding energy should be, around this and if you divide by A binding energy for nucleons, it should be proportional to A, but it is not ok.

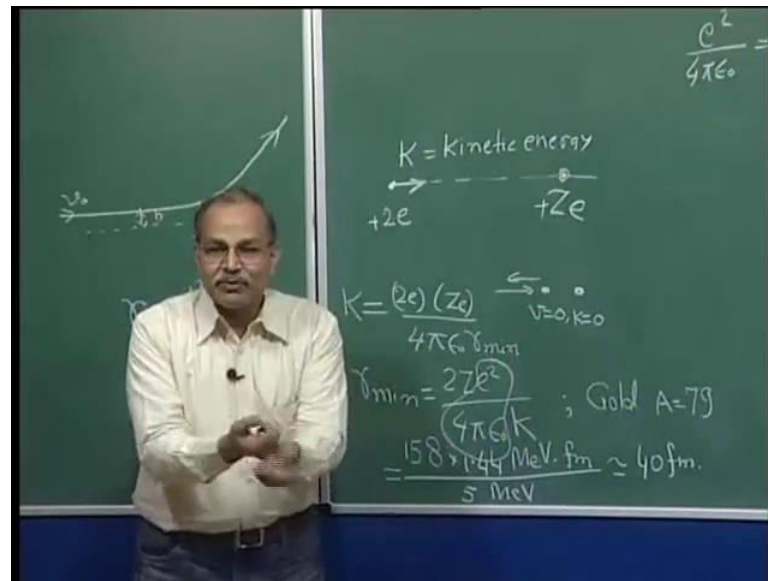
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I will show you the diagram on the slide, look at the figure this is on the y-axis side, it is plotted binding energy per nuclear binding energy divided by A , this is on this y-axis side. And on this side, it is mass number A . Any text files textbook will find you this figure. And buying on some very small light nuclei, for which anyway this result is not true, look at all these. Almost all your slightly heavier nuclei are here, you have nucleon here, lithium here and then you have helium and you have this boron. So, 1, 2, 3, 4, 5 and all rest are here, what you see is, it is almost and the value also, if you cannot see this is 5 MeV, 5 MeV this point and this point is 10. So, 6, 7, 8 this point is 8 MeV, this point is 8 MeV.

If I draw a line from here, look at this everything seems to be around 8 MeV it is not going proportional to capital A , it is not increasing with A . This is A equal to 100, this point is A equal to 100 and this point is A equal to 200. So, your binding energy per nucleon is not proportional to A , it is roughly constant with a small downward trend, small downward see it is decreasing like this. It is decreasing like this, so small downward trend. Got ten all these figures come to the blackboard so what message this curves gives you. This curve gives you the message that, not all pairs are contributing to the nuclear binding energy... So, 150 by 5 will be 30 and then into 1.4 will be 43 or so.

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So, this is around somewhere around 40 to 50 femtometers. Now, 40 to 50 femtometers this is a larger separation, nuclear forces are not operating. Nuclear forces operate only, when you go into 1 femtometer or so. So, it is fine.