Nuclear Physics Fundamentals and Application
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Lecture - 4
Nuclear Size Cont

So, what we had done last time? We were calculating the change in energy of an electron in an atom taking care of the finite size of the nucleus. So, you take that nucleus as a point charge.

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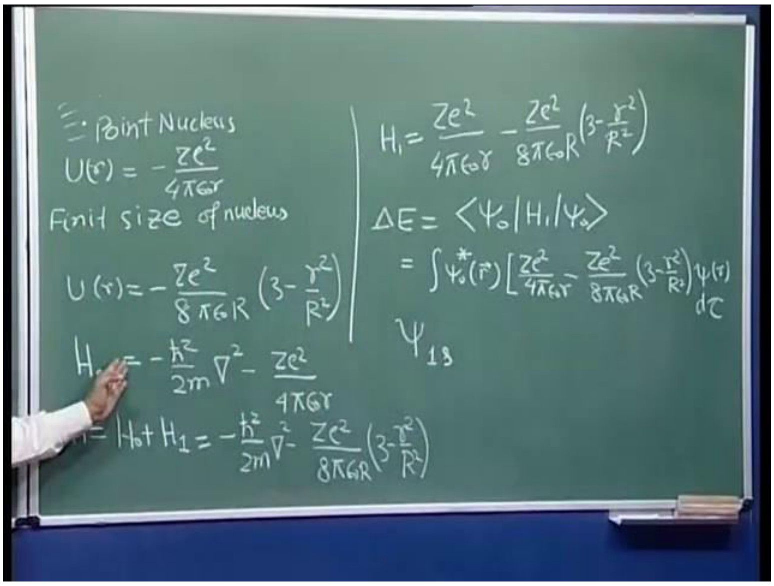

Then, if this is the nucleus point charge and you have electron around, then the potential energy is written as what minus z e square over 4 phi epsilon naught r. If the finite size is taken into account, this is first one, is when you have point nucleus. When this finite size of nucleus is taken into account, then this is different. This becomes what this we had derived earlier. This becomes minus z e square over 8 phi epsilon naught R and 3 minus r square upon R square.

There are several assumptions that you must remember. The nucleus is assumed to be spherical. The nucleus is assumed to be uniformly charged in that spherical volume. So, those assumptions, simplifying assumptions we had taken. Then, we had derived that this is the thing. Now, if I write Hamiltonian with point nucleus, this is the Hamiltonian with point nucleus. This will be minus h cross square over 2 m delta square kinetic energy operator; then, potential energy operator; that will be this. So, it is minus z e square over 4 phi epsilon naught r.

But when you take finite size of nucleus into account, this Hamiltonian is different. This you can write as H equal to H naught plus H 1. That is kinetic energy operator remains the same minus h cross square by 2 m delta square. Potential energy operator is now different. It is minus z e square 8 phi epsilon naught R 3 minus r square over capital R square.

So, this H 1 which is the difference between these two can be taken as perturbation in the Hamiltonian. That H1 you can write is H 1 is this minus this. So, z e square over 4 phi epsilon naught r and minus z e square over 8 phi epsilon naught R 3 minus r square by R square. This H 1 is simply the difference between the Hamiltonian with finite size of nucleus, which is more realistic because nucleus is indeed a finite size and minus the Hamiltonian. If I assume that the nucleus is a point nucleus, so that is this H 1.

We have to get the difference in energy final energy which comes from solving this time independent equation. So, the difference in energy because of this perturbation, you can write that as delta E. That is written as wave function psi naught H 1 psi naught, which is integration of this psi naught star. Then, H 1 and H 1 is simply multiplicative type. So, it is just multiply by that factor 4 phi epsilon naught r minus z e square 8 phi epsilon naught R 3 minus r square by R square. That multiplied by psi r and that multiplied by d tau volume element.

Now, since no differentiations are involved, that psi star and psi can be combined. That makes it psi square. What is this psi function? This is the function of H naught H. If this H 1, if this perturbation is small, then you can take this wave function as the gain function of this original Hamiltonian or unperturbed Hamiltonian. So, that wave function we will write. What is that wave function?

Let us calculate the change in energy of 1 s electron, 1 s electron which is the closest to the nucleus. Therefore, it will have largest overlap with the nucleus. Therefore, the energy shift will be largest. So, if we look at 1 s electron, the wave function of 1 s electron assuming a point nucleus can be worked out in an atom. You have a multi electron atom. You have several electrons. The Hamiltonian should also include in principle the terms corresponding to electron interactions.

But leave that aside. You again assume that all those electrons are outside. They do not give a net electric field inside. 1 s electron is only is its energy is because of the nucleus electron interaction. So, in that case, you can use hydrogen like iron wave function. You have 1 s electron which is orbiting or which is in the influence of this point nucleus having z e positive charge. That is it. So, if you assume that all other electrons are outside and they do not disturb much in that case, you can write this psi for 1 s in point nucleus approximation.

We need wave function of this Hamiltonian H naught. So, that is normalization constant 1 over phi a naught cube phi a naught cube. If it is a naught cube here, it should be z cube. Here, a naught and z, they always come in one combination a naught by z. So, this is normalization constant. Then, you have e to the power minus r by a naught, so z r by a naught.

So, this is hydrogen like ion; 1 electron in front of a nucleus containing z protons and some neutrons. So, this is the wave function. So, we will use this. There are no complex parts. There are no complex parts. So, taking complex conjugate is same as taking the wave function itself. So, it is mod square psi star psi as square of this.

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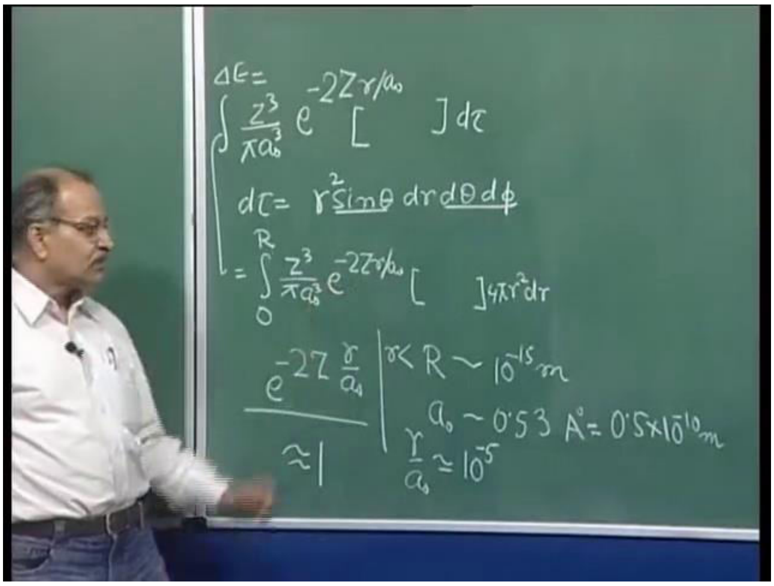

So, your delta E is integration z cube over phi a naught cube e power minus 2 times z r upon a naught, then that whole factor and then, d tau. Now, d tau is; always check dimensions. This is volume element. It has to be length cube 1 length is here no length here. So, it has to be r square. So, that it is volume. Then, the sin theta d theta d phi, this part, that integration will give you 4 pi.

So, this is 4 pi r square d r. So, this integration is integration on r. That is z cube by pi epsilon naught cube, all these things e power minus 2 z r by a naught. Then, this factor z, let me write it this way. Then, 4 pi r square d r where r should go from where to where? Limit of r; r goes from this is to be integrated on whole space. So, r should go from 0 to infinity. But remember this difference of potential energy that we had calculated potential v r for inside the nucleus.

The potential is different outside the nucleus. Even if it is a finite sized nucleus, assume the spherical is symmetric outside electric field. Outside electric potential will be same as the potential due to point charge. Remember that if you have a spherically symmetric charged distribution to calculate outside electric field and outside electric potential, you can take this entire thing as one single point. At the center inside, electric fields are different. So, all that difference that we have calculated potential outside will be the same. So, the difference will be 0.

When I am writing Hamiltonian, I wrote Hamiltonian here H naught. Then, H naught plus H 1, so that H naught plus H 1 expression in which you have all that r, small r square by capital R square. All those things that is valid for the portion inside the nucleus. For outside, it is still the same. So, H 1 is for that small r less than capital R. For r greater than capital R, what is H 1? It is 0. It is 0 because the potential with finite size and potential without finite size, they are same. So, H 1 is non zero only inside that nuclear volume. Hence, the integration is to be done from 0 to capital R.

The formula that is here, this time dependant, independent perturbation; that perturbation you have to take the whole space. But then, here you have to write that H 1. Correct this H 1. This H 1 is 0 for r greater than capital R. Therefore, our integration is to be done only on the nuclear volume theta phi still go in the full range to cover that sphere. But then, small r goes from 0 to capital R only and hence this.

Now, another level of approximation is here. Now, look at this exponential factor. Look at this exponential factor. It is e power minus 2 z r over a naught. Just try to estimate this factor. This small r goes from 0 to capital R. What is this capital R? It is radius of the nucleus. How much this would be typically? Few femtometers, right? So, this r which is always smaller than capital R has to be of this order. A naught, what is A naught? A naught is 0.53 Armstrong. That is 0.53 into 10 power into minus 10 meters.

So, this is of the order of 10 power minus 15 meters. This is of the order of 10 to the power minus 10 meters. So, this r by a naught is of the order 10 power minus 5 into z. z depends on which nucleus you are taking. So, you will put some z here. But that will be less than 100 or so. Uranium is z is 92. So, this is a small number. 10 to the power minus 4, 10 to the power minus 3, 10 to the power minus 5 type; therefore, this factor is almost 1 e to the power 0 is 1 e to the power 0 is 1. This is very small 10 to the power minus 4 or so, 1 by 10,000 or so close to 0. So, this factor will be almost 1. Therefore, we will put this as 1. Then, what happens? Now, let me write the whole thing again. Now, dictate me.

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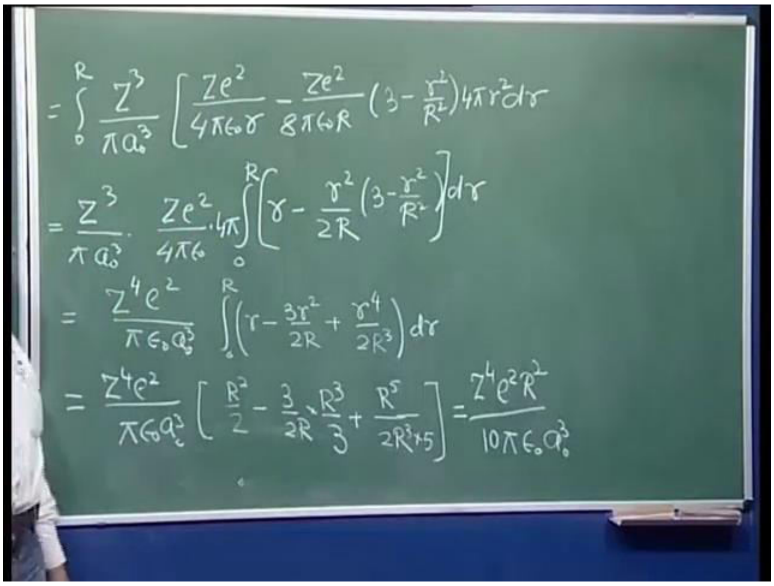

This is equal to integration 0 to capital R z cube pi a naught cube. This factor I am putting 1. What is here? Tell me what is here. z e square z e square upon 4 pi epsilon pi r minus z e square 8 phi capital R 3 minus, then 4 phi r square, 4 pi small r square d r. So, let us integrate. Something I can take common. So, z cube pi a naught cube and from here, what can I take?

Let us take z e square over 4 pi epsilon naught. Let us take this much. Let us take this much out from this bracket and then integration 0 to capital R. I am taking it out of the integration sign constants. So, this I have taken out. So, it is remember 4 pi. So, 4 pi also

I can take out 4 pi. Then, it is only r square. So, how much is this? This I have taken r square is coming from there. So, this will be r minus this. I have taken this 2 capital R will come from here. So, it will be r square by 2 capital R. Then, this multiplied by this. Check if this is all right?

This is z power 4 e square z power 4. e square 4 pi cancels here. Then, you have pi epsilon naught a squares, a cube. So, pi epsilon naught and a cube. This 4 pi cancels. 1 pi remains. a naught cube remains. Epsilon naught remains. z 4 e square and inside now, you have integration 0 to capital R. Then, small r minus 3 r square by 2 capital R and plus r 4 over 2 capital R cube. This is z 4 e square by phi epsilon naught a cube r will become after integration r d r integration r square by 2.

You will put the value 0 to capital R. So, this will be capital R square by 2 minus 3 by 2 R capital R here. Now, this r square will become R cube by 3. So, capital R cube by 3 plus this one by 2 R cube is there. So, 2 capital R cube is here and R 4, R 5 by 5. So, it is capital R 5 here and by 5 here. Look at these first two terms. 3 cancels capital R, r square by 2 same as this. So, these two will cancel out. You will be left with this R 5 R cube. So, it will be capital R square by 10 capital R square by 10. It is z to the power 4 e square divided by 10 pi epsilon naught pi epsilon naught a naught cube. This is all a naught a naught cube and then capital R square. Are the dimensions correct?

This should be energy. What we are calculating is change in energy. So, it should be energy. Check the dimensions. z is dimensionless. Then, it is e square by 4 pi. It is phi epsilon naught and r square here, a naught cube here. So, it is 1 by length. So, it is e square by 4 pi epsilon naught length that type dimensionally right. This is dimensionless. This capital R square by a naught cube 1 by length 1 by r e square here pi epsilon naught. So, it is like dimensionally e square by 4 pi epsilon naught r.

That is the potential energy expression. So, it is energy. So, we have calculated the change in energy of 1 s electron taking account of finite size of the nucleus. If the nucleus were a point charge, then there is some 1 s energy. We know the expressions from that the actual energy is shifted this much is up is going up delta is positive. It is going up because of the finite size the energy of this 1 s electron has gone up by this much of amount. Capital R is the size of the nucleus, radius of the nucleus. So, that radius is coming here.

If I can measure this energy difference, I can get the capital R because this energy difference expression has this capital R here. This is the radius of the nucleus. So, if I can measure this delta E, I can get this capital R. But how do I measure delta E? It would have been, if there is one atom with point nucleus and I measure the energy of 1 s electron, there is another atom with finite size. I measure the energy and subtract. But that is not possible.

Nature has not given us atoms with point nuclei. The nuclei are extended. So, there should be a smart way of getting radius from this expression. You know physicists are always smart. So, they will find solutions how to do that. So, take this. I can remove all these things. Our result is that energy of 1 s electron energy of 1 s electron with finite size.

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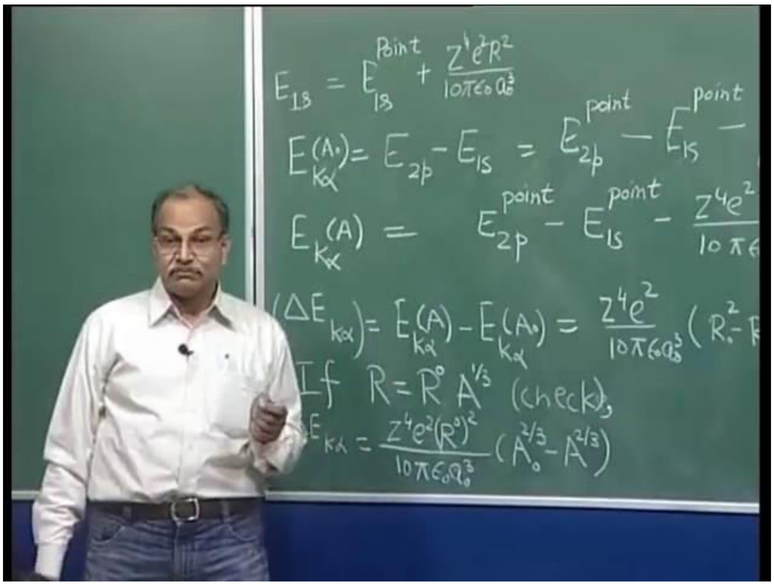

So, let me write this as e 1 s point and plus z 4 e square capital R square 10 pi epsilon naught a naught cube. This is the 1 s energy. It is more than what you would calculate with a point nucleus. Here is the difference. Now, look at what is called k alpha x rays. You know what k alpha x rays are. How do k alpha x rays come out from where they are originated? They are originated again from electron transitions. You have a 1 s energy level. Then, you have 2 s energy level., then you have 2 p energy level very close to this and so on. These are the atomic energy levels in bohr’s model or that orbit like model. If you try to picturize, you have a nucleus. Then, you have some electron some let us say this is 1 s electron.

This is 1 s electron. Then, you will have some 2 s electron at some other radius. Then, you will have some 2 p electron. You know why I am doing this to p electron p x, p y, p z orbital. See this 2 p electron or any p electron or d electron, if you look at the wave function; that goes to 0 at r equal to 0. 1 s electron does not go to 0. So, 1 s electron has overlapped with the nucleus 2 p electron will be 0 at r equal to 0. So, if you are drawing 2 p perhaps, you will be drawing something like this. So, that it is 0 at the center. But 2 s electron will be like this. 1 s electron will be like this, just picturising some geometrical pictures.

So, when this electron somehow if there is a vacant space here in 1 s orbital, you should have 2 electrons. But somehow say 1 electron is knocked out. So, there is a vacancy there. That vacancy is filled up by electrons from higher levels. So, if the vacancy is filled up by this 2 p electron coming down here, an electron in 2 p level that comes to 1 s level. Then, some energy will be emitted. Some energy will be emitted. h c over lambda energy. This energy minus this energy; this is energy of 2 p minus energy of 1 s. That will be the energy. This is x ray. This is k alpha x ray. This photon corresponds to what we call k alpha x ray.

Now, it does not take transition from 2 s because of certain selection rules. It has to come from this so called capital L shell to this capital K shell. These are also atomic physics. Atomic physicists call K L, next will be M capital M N and so on. So, if electron come from L shell to K shell, it come from 2 p to 1 s. Then, the x rays are emitted and energies of x rays are given by this energy difference. These x rays are called k alpha x rays. So, let us see if the nucleus where a point particle, what expression is there for k alpha x ray energy?

The nucleus is finite size. What is the expression? So, the energy of k alpha; energy of k alpha x ray is energy of this 2 p state and minus energy of this 1 s state. Now, as I said, 2 p wave functions are goes to 0 and r equal to 0. So, the 2 p wave function will have almost negligible overlap with the nucleus. Hence, the energy calculated energy of 2 p state calculated with point nucleus or finite sized nucleus should be same because in 2 p state, the wave function is anyway going to 0. So, when you make that integration psi star H 1 psi that psi at r equal to 0 and nucleus is anyway very small femtometers. So, that wave function itself is so small that this delta E is almost negligible. So, this you can write as just e 2 p.

Let me write it here. This is equal to e 2 p point charge. You can take it point charge. No problem, but this e 1 s is e 1 s point and then, minus this delta E. Now, search for an element which has several isotopes. Isotope also you know, isotope means capital Z remains the same. Number of protons remains the same. So, it is the same element. But then, the number of neutrons is different. So, I have several isotopes of an element if I have several nuclei with same Z, but somewhat different neutron number. So, capital A will be different. These are called isotopes. So, if you look for particular element with several isotopes and look at the energy of k alpha x rays from all those different isotopes, what happens?

So, suppose your first isotope has some mass number A naught and others are having different. For A equal to A naught, this is the expression. So, this is the expression. Let me put r naught square here. Then, this is for A naught. This 2 p state is not going to change. If you are taking a different isotope, what will happen if you are taking a different isotope? Then, Z remains the same. Z remains the same, so if it is a point nucleus, it just does not matter if there are 1 or 2 more neutrons or 3 more neutrons because they are not producing electric field electric potential.

Since, 2 p state does not depend on the size, it is only the 1 s whose energy is shifted up. 2 p energy is not shifted up. So, for 2 p, it does not matter which isotope you are taking. So, this remains the same. 2 p remains the same. This is anyway assuming point nucleus. So, this remains the same for all isotopes. The change is only here because a different isotope means different size capital A is now different and the size is different. If the size is different, this term will be different.

So, all these terms are same. Only this term differs. So, I put A 0 here. R 0, R naught and I put this A naught here. So, for this particular isotope, say lowest A isotope, this is the k alpha x ray energy. Now, write it for a different isotope, so e k alpha for A. Now, this number A is different from this A naught. You can take this A naught as the lowest A in that series of isotopes. So, for any other A, this will be written as this same e 2 p e 2 p because this is not going to change. It is same as that assuming point nucleus e 1 s point.

This is anyway point nucleus. So, it remains the same value of capital Z. Here, it will be minus z 4 e square R cube R square by 10 pi epsilon naught a naught cube.

So, k alpha x ray energies for one isotope and another isotope, so same Z. But different values of a, therefore there are different values of radius of the nucleus. So, these are the expressions. Now, take difference shift in energy change in energy of naught 1 s level, naught 2 p level, and naught 2 s level. The shift in the x ray energy, change in the x ray energy, k alpha x ray energy and as you go from A naught to A this isotope to that isotope, how much is the shift in energy delta E in k alpha? It is e k alpha when your isotope is capital A, mass number is capital A and minus e of k alpha. So, I am looking at the x rays energies.

We are going for something which is measurable. The goal was to measure the radius of the nucleus from the experiments. I want to measure it something measureable. Now, k alpha x ray energies, it is measurable. The x rays are coming out of that material. Those x rays can be analyzed in certain detectors and certain encounters their energy can be measured accurately. Now, it is measurable quantity, experimentally measureable quantity. Now, subtract. So, it is this minus this. So, these two will cancel out. This minus that, so it is z 4 e square divided by 10 pi epsilon naught a naught cube and then, R naught square minus R square.

Now, we do not need. What would be the energy if it were point nucleus? So, that point nucleus thing has been subtracted out. But from here also, we do not know how to get that R. This expression is fine. This can be measured. So, I know how many nucleons are in the first isotope. How many nucleons are in the second isotope? That we know; that we have taken the material with this mass number and this mass number, we have gotten this quantity. So, I know the difference in the radius is square. That is fine. But how that radius individually is depending on A? So, that is yet not very clear from here. What we will do is we will assume if let us check if it goes like this.

So, if R is equal to R naught, A naught A cube. This is yet to be checked. If this is the case, if R can be written like this, then this delta E in k alpha will be z 4 e square R. I have already used that symbol R naught. There, I have already used that symbol R naught here. So, let me write something else. Let me write something else. Let me put 0 here. So, R naught square divided by 10 pi epsilon naught a naught cube. This is now A naught to the power 2 by 3 and minus A to the power two by 3. So, if this is true, then the relation should depend on A in this fashion.

If I measure this e k alpha for different isotopes capital A and plot that energy difference of x rays as a function of A to the power 2 by 3, I should get a straight line. So, that is the check. If this delta E k alpha as a function of this capital A happens to be a straight line that means this assumption is true. If it is a straight line, then the slope is here. If it is a straight line, then the slope is here. All other things are known. These are all fundamental constants. You can get this R naught. That means you will get R as a function of capital A. So, that is the trick.

So, I am now going to show you on this power point slide the experimental result, the experiments that have been done with mercury isotope, mercury element. Mercury has several stable isotopes, several some 7 or 8 isotopes. So, you have 7 or 8 data points for different A. What is the shift in x ray as compared to the lowest A isotope? So, I am just going there.

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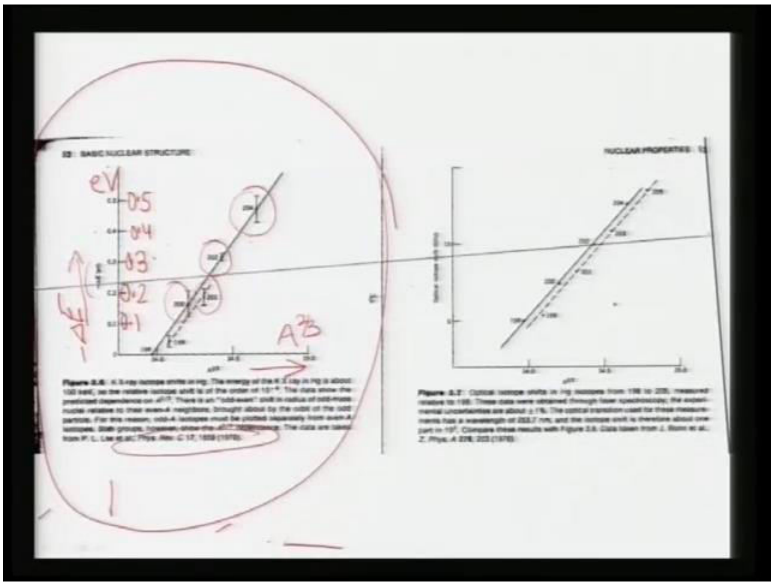

So, look at the screen, look at the graph at the left side. This is k, this is k x ray isotope. So, look at this left side, this graph. This y axis side here is the energy difference this is all for mercury. The lowest one is here mercury 198. Capital A equal to 198. This is the lowest. Then, you have 199 here, 200 equal to 201 here, 202 here and 204 here. So, these are the stable isotopes of mercury. What they are plotting is negative of the change in energy. This is minus delta E. You may not be able to see on the screen.

This is minus delta E here. This side is A to the power 2 by 3. This side is A to the power 2 by 3. Then, 198, 199, 200, 201 and 204; so these are k x ray isotope shifts in mercury. This is taken from this reference physical review 17, 18, 5, 9 page number 18, 5, 9. The year is 1978. So, from this reference, I have taken this. In fact, this for, this figure is taken from the book of Kenneth Krane. So, this is from Kenneth Krane. Now, what do you what do? You see is it a straight line. Can you see there is a straight line here?

But you can see there are two straight lines. Why do you have two straight lines? One is dotted line. You can see two points, 199 and 201; they are on a different line. 198, 200, 201, 204 are on a different line. So, this is what we call even odd effect. Odd nuclei have slightly different behavior. Even nuclei have slightly different behavior. So, the two odd isotopes 190, 191, 201, they are falling on slightly shifted. That line is slightly shifted although if you join these two, the slope is almost the same. The slope is almost the same.

The even ones, 190, 198, 200, 202, 204; they are falling on a nicer straight line. Then, look at the values also a little bit. This point here is 0.1 electron volt, this is all in electron volt. This point here is 0.2 electron volt, this is 0.3 electron volt, this is 0.4 electron volt, this is 0.5 electron volt. So, that also is important thing to note from this figure. So, keep this in mind. Coming to board, I will draw it and discuss.

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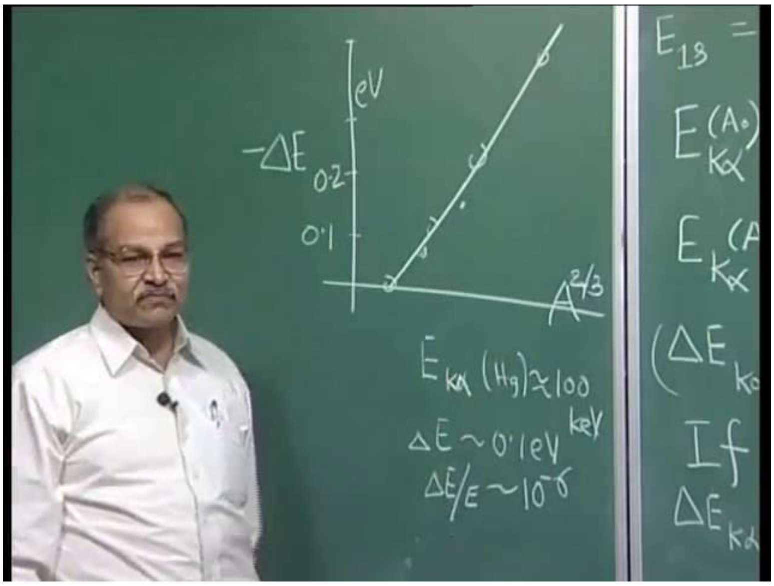

So, what you have? You have something like this. This minus delta E was plotted here like this. You had a straight line behavior like this. Of course, there were two points here. This is A to the power 2 by 3. So, this straight line thing confirms that the radius of the nuclei they follow this rule. Then, from this slope, you can get the value of this, which is around 2.1, 0.2 femtometers or so. However, the values are very small; the value the shift is very small. This is in electron volts whereas, the energy of the k alpha x rays itself for mercury it will be around 100 kilo electron volts.

So, energy of k alpha for mercury isotope will be around 100 k e V kilo electron volts. The shift delta E is around 0.1 electron volts. So, that means you have to measure change of delta E by E of the order 10 to the power minus 6. So, a very small fractional change, but experiments are sensitive enough to pick up this particular change. So, this situation can be improved by using what we called muonic x rays. Have you heard of this muon, you know what muon is?

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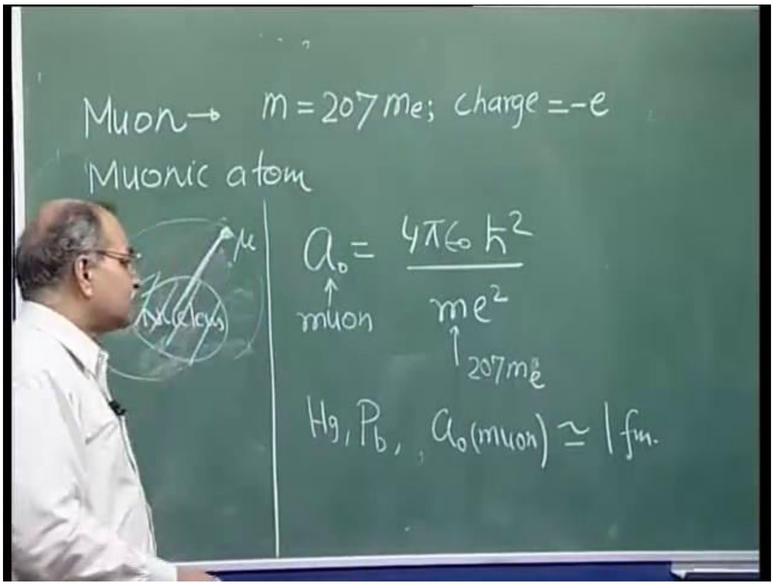

Muon and muonic atoms muon is a particle of the electron family. Muon is a particle of electron family. It has mass which is about 207 mass of the electron and charge same as that of on electron. There are other things left on number; this and that. So, there it belongs to electron family. It is like electron. It is almost like electron with a heavy mass 207 the mass of the electron not freely available, not a stable particle. They are produced in upper atmosphere because of these cosmic rays. But then, these decay in some something like 2 micro seconds or so in their own rest frame.

But these can be produced in laboratory also. You need large accelerators. You need large accelerators nuclear reactions. Those nuclear reactions produced pion. Those pions decay to muons. Then finally, muon also decays in electrons and so on. So, in place of electron, if a muon is captured by nucleus, what you get is muonic atoms. So, you have a nucleus. This nucleus instead of electron, if it catches a muon; this muon has its own wave function and orbit and this and that.

That is a muonic atom. So, only the mass is different otherwise coulomb interaction and everything is same. So, it will have 1 s, 2 s, 2 p all those energy levels. Therefore, again you will have k alpha x rays is for this muonic transitions, muon going from 2 p to 1 s you will have k alpha rays. So, all this can be done with these muonic atoms. You have to produce this muonic in the laboratory with accelerators and that too for a very short time. But if you do that, there is a great benefit. What is that? You do you remember the expression for a naught.

a naught is 4 phi epsilon naught h cross square over mass times e square k. That is the expression. So, this is for electron. If this is muon then, that will be 207 times m e. So, that radius decreases by a factor of 207. So, if the electron is at an average distance of some value, it will be 200 times closer to the nucleus. Hence, the overlap with the nucleus will be much higher because it is anyway close to the nucleus. In fact, you can work out values also. For bigger nucleus like H g or lead or these bigger nuclei, the maximum probability will be inside the nucleus itself. The nuclear size at this level, what will be the nuclear size radius for this heavy nuclei?

It is something like 6 femtometers. We worked out the previous lecture. Some 5, 6, 7 femtometers for these heavy nuclei where the wave function that psi square radial probability density maximizes; that turns out to be of the order of a femtometer or 2 femtometer or 3 femtometers. It is inside the nucleus. Most of the time, the muon is inside the nucleus. Hence, that shift, that change in x ray energy is because of that finite size will be very high.

Therefore, observation will not be that difficult for electronic x rays. You have to measure 1 part in 10 to the power 6 or in absolute sense 10 to the something like, 0.11 eV in 100 keV. If you are using muonic x rays, then since the energy shift is much higher, most of the time the muon is inside the nucleus.

The effect is much higher. The shift is of the order kilo electron volts. So, that is the benefit. There are more methods isotopes of shifts also. One kind we have looked at it. It is visible spectrum. So, you have a nucleus. Then, you have these so called orbits. This energy is 100 keV. You see 2 p 2 1 s was 100 k eV for this k alpha of mercury.

For visible photon, what should be the energy? It should be around 100 keV. This is definitely x rays. For visible photons, visible light; how much should be the energy of the photon? It should be of the order kilo electron volt or electron volt or milli electron volt. What it is around electron volts, few electron volts you can have wavelength. You remember for visible light, what is the wavelength? For visible light, what is the wavelength?

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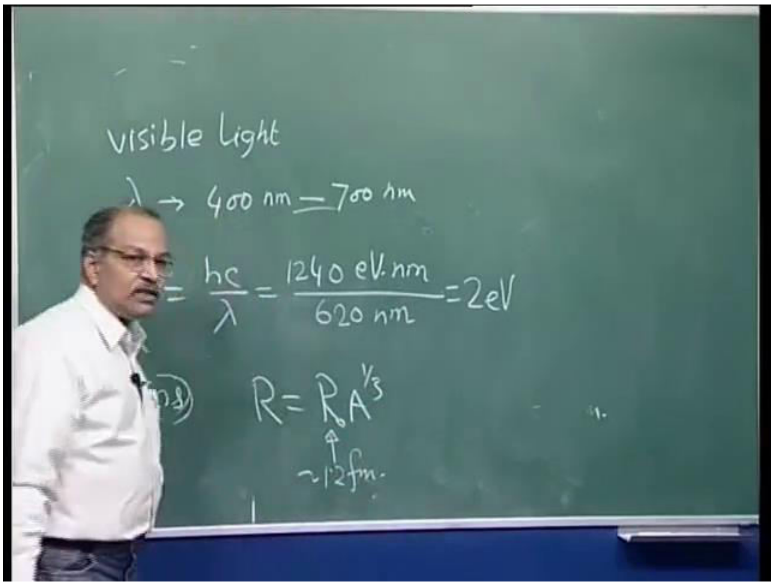

It is of range 400 nanometer to 700 nanometers. The energy is h c by lambda and h c is 1240 electron volt nanometers. Good thing to remember h into c is 1240 eV nm. Lambda takes something here. So, take 620 nm. So, this will be 2 eV. So, visible light photons will be around electron volts. So, you have to go for much higher transition not an equal to 2, 2 and not equal to 1.

Even for hydrogen an equal to 2, 2 and equal to 1 will be ultraviolet. But if there is 1 s level involved in this transition, if you have some s naught 1 s some s involved here in this optical transition, then there will be some isotope shift although very small. 2 s 3 s 4 s are farther away from the nucleus although there is some overlap with the nucleus anyway of s electrons. But much effect has been measured.

So, that is much for today. So, let me summarize what did I, what did we do today. We today elaborately discussed how we get finite radius of the nucleus from the interaction of 1 s electron within the nucleus. This 1 s electron spends part of its time inside the nucleus. Therefore, it experiences a different potential. By hitting to that difference and by very cleverly manipulating to measure that difference, we could verify that radius does go as 1 by 3 r equal to some r naught a power 1 by 3. This is something like 1.2 femtometers.

However, I caution you. I am reminding you that nucleus is not a classical hardball type system. Nucleus is a quantum system. Therefore, it does not have a sharp boundary.

When we are saying radius of the nucleus, keep that in mind we are only giving some kind of n average radius or a typical radius. It is not a plastic ball. So, you can measure the diameter. You can measure the radius. The wave function of the nucleus itself is gradually it decreases its magnitude. You do not have a sharp boundary. You have diffused boundary. But still you can talk that this is the volume in which most of the charge is there 99 percent of charge is there or 90 percent of charge is there or 95 percent of charge is there.

What we have described two methods? One is electron scattering, high energy electron scattering. Then, this isotope shifts. They are only looking at the charges. So essentially, it is giving me the charge distribution. But then, I assume that the way protons are distributed, in the same way neutrons are also distributed. So, the entire size or radius is determined by the neutrons as well as protons, the total number of nucleons. So, that also goes in the similar fashion as the charge goes. So, that is also assumption. But then, there are methods which directly measure the mass or the mass distribution of the nucleus.

It does not depend on the coulomb interaction. There are methods where the probe is directly interacting with the nucleus. Nuclear forces like alpha particle scattering where alpha particle gets scattered from nucleus, Rutherford scattering. It is the Coulomb force which acts, but if the energy of the alpha particle is high enough so that it goes into the nuclear range, it goes so close to the nucleus that some femtometer distance remains.

Then, this Rutherford scattering formula, which is derived on the basis of coulomb interaction, will fail. The experimental data will deviate. So, the energy at which this deviation starts gives you a measurement of size. So, there it is not coulomb when the nuclear forces start operating and nuclear forces operate for neutron also for proton also. So, you are directly getting into the whole mass distribution. So, all those methods are there. But at the end, the final conclusion is that radius does go as A 1 by 3. The nuclear density in most of the nucleus apart from a small surface layer is almost uniform.

So, you have a uniform distribution of particles not only in one nucleus but across the nuclei, all the nuclei taken together. The central density is same for all those nuclei heavy or milli weight light weight. The central distance density is almost the same. Then, after that surface is opposed something like us something like 2 femtometers or so then, this nuclear density suddenly decreases. So, I am trying to familiarize you with the nucleus.

So, start getting geometrical pictures, start getting feel of nucleus not only in terms of wave functions and in terms of expressions, but also in terms of objects. So, that is for today.