Physics Nuclear Physics Fundamentals and Application
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Model - 01

Lecture No- 37
Nuclear Reactions contd...

We had finished our discussion on Nuclear Fission Reactions. We had seen that this an important reaction because it produces energy and lots of our energy requirements do come from fission reactions. Next session that I am going to start is on nuclear fusion reaction. So, let us start with the new chapter, this is nuclear fusion reactions. This chapter is also very important again from applications point of view and same applications.

Nuclear fission is used for energy production electricity production nuclear fusion can also be an is used for energy production in fact everyone knows is a general knowledge, school students Knows children also knows that the energy that we get from sun comes from nuclear fission going on inside the core of the sun, and only because of that energy all life and everything is they are on this earth. And we are also trying to produce nuclear fusion reactors were energy can be produce electricity can be produced from nuclear fission reactions. So, let’s first revise the basics of this fission reaction in fission a nucleus splits in 2 parts, and then if the nucleus is quite heavy the Q value is positive and energy is released in fusion to light nuclei fuse or combine to form a bigger nucleus and from that also if the nuclei are small enough energy is produced Q value is positive.

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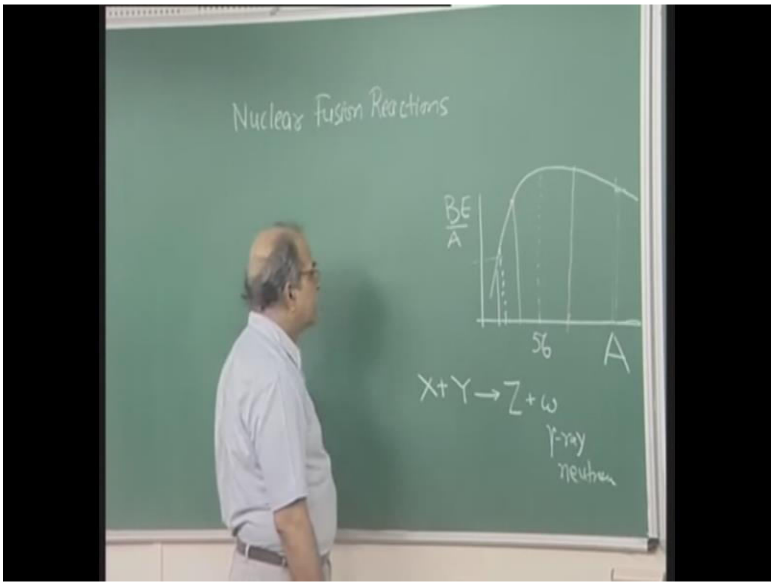

So, if we remember that binding energy per nucleon versus nucleon number mass number curve if this is the mass number of a particular nucleus and then the binding energy divided by this A, binding energy per nucleon is here on the average on the average it increases in the beginning and then it goes to a maximum and then slowly decreases, this is the kind and this maximum occurs somewhere around this 56, 57 that is iron and nickel that region that does not go this is smoothly as know very much different nuclei will have different fluctuations around this but, this is the general average behavior of the binding energy per nucleon versus A. We have used it several times by now in fission the original nucleus the parent nucleus is a heavy nucleus and the binding energy per nucleon is a small, if I shifted this side if it is split in 2 parts, and you have to nuclei in this region the total binding energy per nucleon increases and as the binding energy per nuclear increases from here to here you get this energy released.

Now, the other side of this maximum here if we start with 2 nuclei somewhere here in this region, the 2 nuclei around this may be 2 nucleus I can show slightly different it can be 2 nuclei of the same species, or it can be 2 nuclei of different species but, the binary per nucleon is in this region this much where as if they combine they will go somewhere here and then the binding energy per nucleon will be here. So, if the binding energy per nucleon is more the nucleus is more tightly bound then the rest mass energy decreases and you get kinetic energy and that the energy released. So, from this side also you can get energy from this side also you can get energy, if you get from this side that is nuclear fission and if you get from this side, it is nuclear fusion.

So, the reaction would look like some X and some Y some light nuclei, going to heavier and probably some small particular like neutron, or gamma ray, gamma photon on say neutron are some such light particle may also be there or may be 2 particles but, most of the nucleons which are here in this X and Y most of them are here in this bigger nucleus Z we can take some example.

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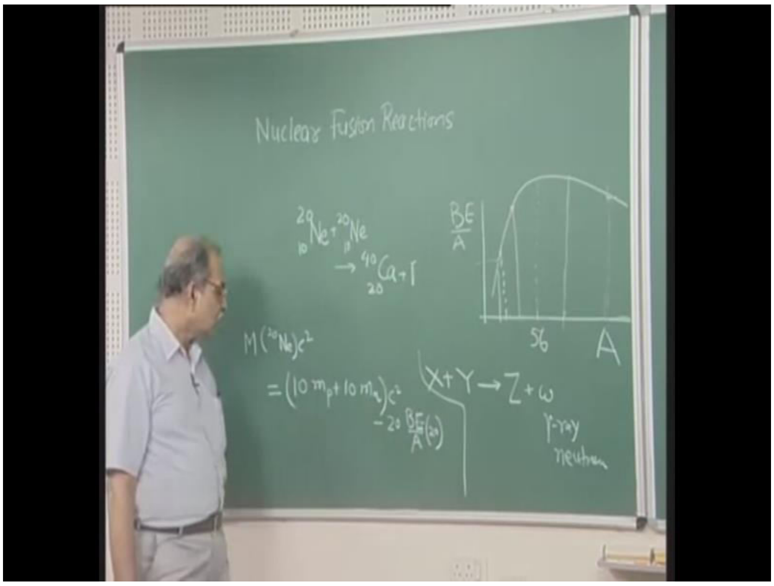

Let us take for example, 20 Neon Z is 10 and Neutron number is also 10, total is 20 supposed 20 Neon and 20 Neon that goes to 40 Calcium plus of course, gamma are something. So, if I look at the Q value how much is the rest mass energy on the left? How much is the rest mass energy on the right? So, if I look at this rest mass energy at the left, it will be mass of this 20 N e C square, this will be you have 10 protons. So, 10 m p plus 10 m n this proton mass and neutron mass.

So, these are the masses of these nucleons separately times C square and minus binding energy, how much is that? Minus binding energy is minus 20 times B E by A at 20, understand the symbols here. I have to write the binding energy. How many nucleons are there? Total nucleons here are 20 neon. I am writing mass of 20 neon nucleolus c square. So, 10 proton mass 10 neutron so, 10 neutron mass rest mass of that minus binding energy. How much is binding energy binding energy per nucleon multiplied by total number of nucleons total number of nucleon is 20. So, this 20 is here and binding energy per nucleon is here B E by A binding energy per nucleon, where because binding energy per nucleon is this curve. So, where here 20 a equal to 20 at a equal to twenty. So that is, this 20 and two of them if I calculate the total rest mass energy on the left two of them.

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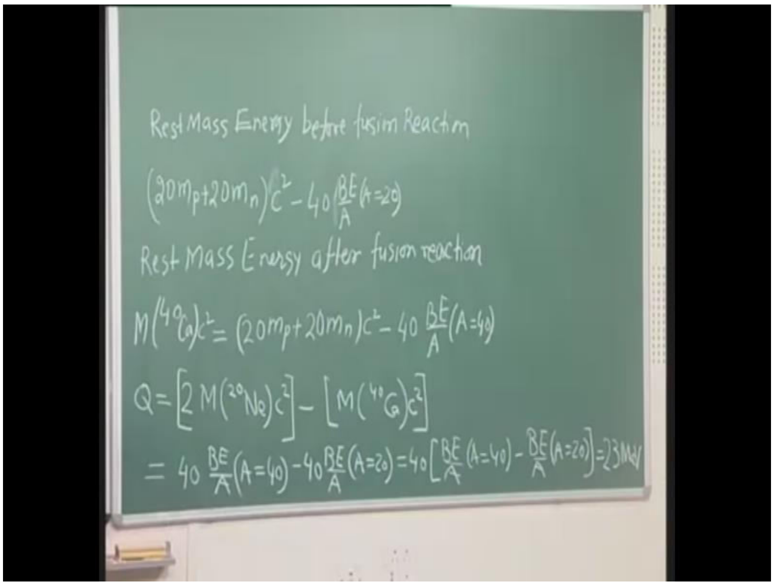

So, it will be before fusion before fusing reaction will be double of that will be say 20 m p plus 20 m n times c square and minus 40 times the B E by binding energy per nucleon B E by A at 20 at A equal to 20. That is the rest mass energy before the fusion reaction. What happens after fusion reaction? After fusion reaction you have single nucleus calcium 40.

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So, you have this calcium 40 Z is 20 here the rest mass energy of this. So that will be now rest mass energy same expression, rest mass energy after this fusion reaction, this will be you have 20 protons and 20 neutrons. So, the rest mass energy of this mass of 40 calcium times c square this will be 20 mass of proton plus 20 mass of neutrons times c square that is the rest mass energy of these nucleons, if they are widely separated from each other but, they are not they are all bound in this calcium nucleus therefore, minus binding energy and that binding energy will be 40 times binding energy per nucleon at a equal to 40, because now you have a nucleus with a equal to 40.

So, the binding energy per nucleon is given by this binding energy per nucleon at a equal to 40 in this binding energy curve, wherever that 40 occurs. So, binding energy per nucleon at a equal to 40 multiplied by 40 because there are 40 nucleons in this. So, what is the Q value? Q value is the initial rest mass energy the rest mass energy before the nuclear reaction and minus the rest mass energy after the nuclear reaction. So, Q value is how much is the rest mass reduced because of this reaction. So, Q value is this minus this 2 times mass of neon c square this is the mass rest mass energy before the reaction and from this we will subtract this mass here.

So, this will be mass of 40 calcium into c square and when you do that when you subtract this final mass from this initial mass. All this 20 m p plus 20 m n c square cancels out and here you will have this minus. So, this will become positive. So, this is equal to 40 times B E per nucleon at a equal to 40 alright, we are subtracting the final rest mass energy from the initial rest mass energy. So, this becomes plus 40 this and here it is minus 40 times b e by a at a equal to twenty. So, 40 also you can take common and 40 multiplied by this is 40 multiplied by binding energy per nucleon at a equal to 40 and minus binding energy per nucleon at a equal to 20.

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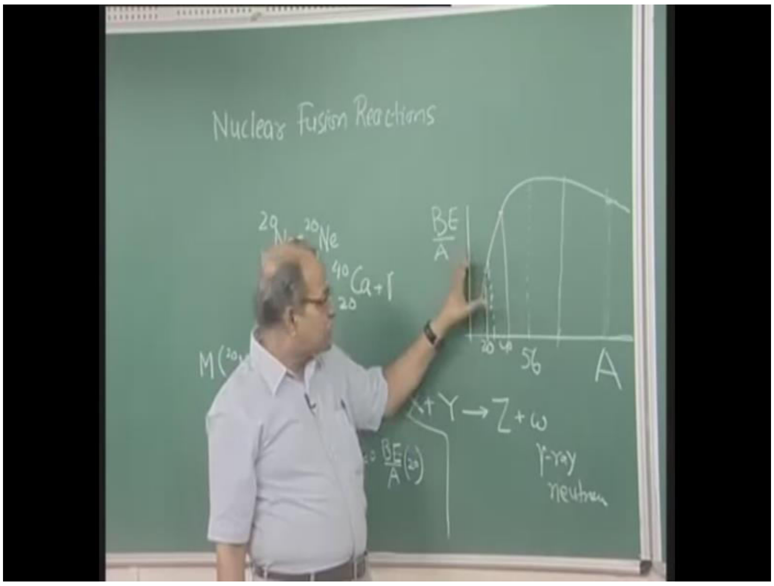

And since binding energy per nucleon at a equal to 40 suppose this is 40 and this is 20 than at a equal to 40 is larger and binding energy per nucleon at a equal to 20 is smaller. So, this difference multiplied by 40 positive and this turns out to be about 23 M e V in this case mega electron volt. So, these 2 neon nuclei they combine and fuse to become 1 single calcium nucleus the rest mass energy is decreased by 23 M e V, and if rest mass energy is decreased by 23 M e V, that should appear as something. So, in this case it can appear in excitation of this calcium and subsequent emission of gamma photon and recoil of calcium. So, this is an example, you combine these nucleons in this joule light nuclei if i plot that energy this side and separation between joule and make a nucleus which is somewhat heavier than that but, do not cross this peak you will get Q value positive.

So, this mode of energy production from nuclear reactions using light nuclei and their fusion has several advantages over what we are currently using that is nuclear fission reactions we had discussed all the radioactivity problems accident problems hazards then availability of nuclear fission fuel. So, those problems will be much lower much lesser if 1 can use this fission reaction to produce energy. So that is 1 motivation for understanding this nuclear fusion reactions another motivation is that if we want to understand what is going on in stars? What is going on in sun? How all these different elements nuclei of different elements where created for the first time astrophysics little bit of astrophysics. So, there also nuclear fusion is the central phenomena and therefore, understanding knowing these fusion reactions physics of it will help us in understanding although rest of physics problem also.

So, all these things we will briefly discuss in this section. Now it looks very simple just take 2 nuclei or 2 species of nuclei with smaller mass numbers light nuclei and put them together to fuse and you get energy but, it is not really that simple nuclear fusion reactors are still not commercially made people are working very hard for decades and decades and we are only hopping that we get this nuclear fission reactors in near future. So, what is the problem with it is that coulomb barrier. We had talked about coulomb barrier during alpha decay discussions, we had talked about coulomb barrier during fission reactions, and once again this coulomb barrier comes because you are working with charge positively charged particles the nuclei which are combining they are all positively charged. So, initially you have 2 different nuclei. So, they are apart from each other nuclear reactions it is interacting through Coulomb force only and therefore, the potential energy will be given by that usual Z 1, Z 2 e square by 4 phi epsilon naught r.

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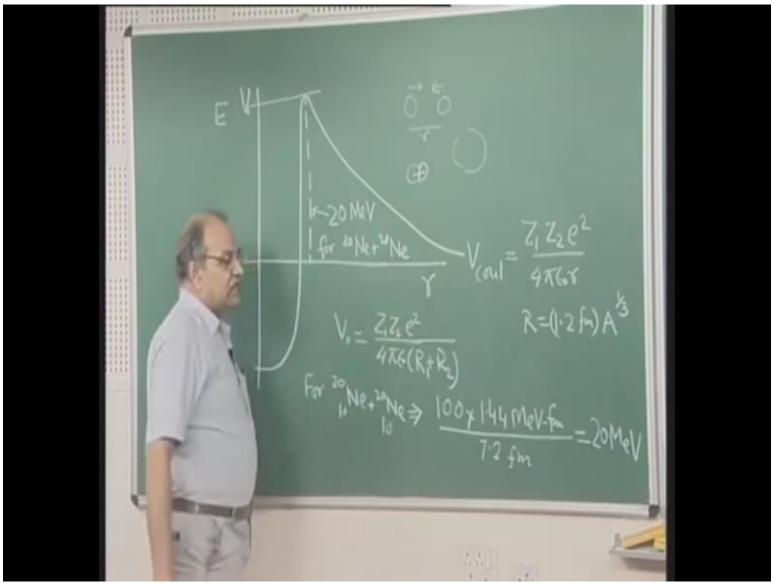

So, if I plot that energy this side and separation between the nuclei this side if you have 2 nuclei which are approaching each other and finally, that they are trying to combine. So, this separation r is what we are plotting on this horizontal side and then the energy is on this side. So, initial energy if they are widely separated in this r is much larger that if I take as 0. So, you have take care of rest mass energy also, when they combine and form one big nucleus when they combine and form one big nucleus, that means now r is very small in the nuclear range the rest mass energy has gone down and therefore, you have to the energy become somewhere here if I take this as 0.

So, as compared to the rest mass energy of this rest mass energy of this is down. So that is the Q value and from here if this r decreases if when they come closer and closer, this coulomb potential energy V coulomb that is zone and Z 2 E square by 4 phi epsilon naught r this keeps increasing as the particles come closer, r degrees and this V increases just as hyperbolic 1 by r pattern. So, it goes like this is the usual coulomb potential nature and then when they come very close to each other you can say when they are touching, they are going now, they are going inside, then the nuclear forces will take over and the potential energy will decrease and that will finally come here, somewhere it goes back and comes like this.

Now what is the initial energy available that is important this height first this height how much is this height. So, if I just take the simplistic model as we had done earlier also that when these 2 just touch each other till that time, we say that it is only the coulomb force which is effective and once they have touched now they are going into each other. That time we say that the nuclear thing has taken over. So, the maximum will correspond to this maximum, which is the barrier height, this V naught will be Z 1, Z 2 e square over 4 phi epsilon naught and r 1 plus r 2 radii of the 2 nuclei which are trying to combine. So that is this situation r 1 and plus r 2 when r is small r is equal r 1 plus r 2 that we take as the maximum potential energy here. So that is the barrier height you can estimate how much it is let us take again the same example for say 20 neon plus 20 neon for this if I make.

So, 10 Z 1 is 10 Z 2 is 10. So, it is hundred Z 1 Z 2 is 100 then we have e square over 4 phi epsilon naught we have use this several times it is 1.4 M e V fermi femtometer and then divided by 2 times r 1 plus r 2 times radius of this now radius of this you can estimate r is equal to take 1.2 femtometer times to the power one-third. So, can put a equal to 20 here cube root of 20 will be little less than 3 cube root of twenty seven is three. So that multiplied by the 1.2 something like 3.5, 3.6 femtometers and double of that will be 7.2 let us take it 7.2 femtometers. So, how much is this will be almost 0.2 here and. So, it will be 20 M e V. So, this height for this particular reaction is 20 M e V for 20 neon plus 20 neon this height will be large if Z 1, Z 2 is large and it will be small Z 1, Z 2 is small and smallest value of Z 1, Z 2. How much that can be when Z 1 is 1 Z 2 is 1 proton in sun or in starts when hydrogen fuses the first the step is proton there are the coulomb barrier will be minimum Z 1, Z 2. So, if I take just that case it is Z 1, Z 2 is 1 and for radius take 2 femtometer the range of nuclear interactions or 2 femtometer here. So, this will be 1.44.

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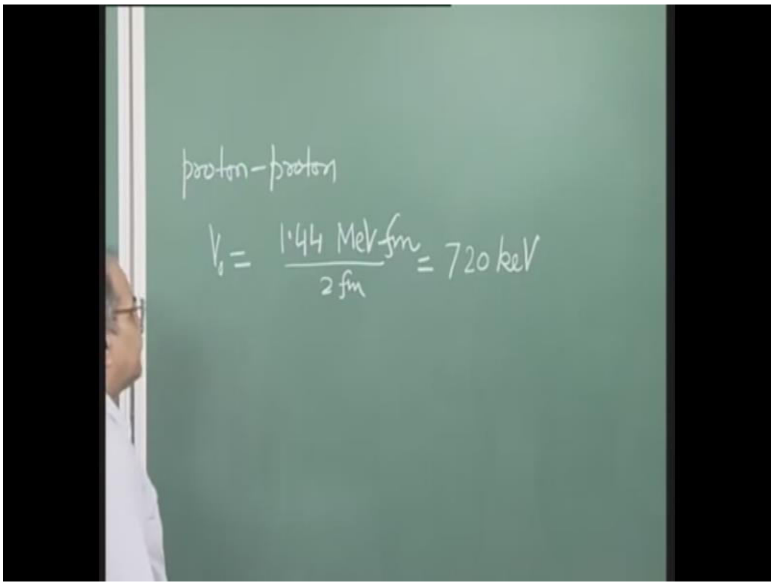

So, for proton this V not will be equal to Z 1, Z 2 is 1 and e square by 4 epsilon naught that will be 1.44 M e V femtometer.

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And for this take 2 femtometers. So, that will be around 700 k e V 720 k e V kilo electron volts. So, for nuclear fusion this will be without minimum barrier of course, one can go for deuteron and tritons higher isotopes of hydrogen in which case Z 1, Z 2 will not change you still have Z 1 is equal to 1 and then the radius will increase if you go for higher isotopes. But, still 100 soft k e V minimum. So that is the kind of coolant barrier is there.

Now, if the world were of classical physics nature fusion would have we possible only if the initial kinetic energy of these 2 nuclei is more than this barrier, and in quantum mechanical world also the probability increases as you go up and if you can surmount that barrier then the probability is very high. Now in our case if we wants to compare the initial kinetic energy is and this barrier height we have to give at least hundreds of k e V minimum, and depending on what nuclei we a trying to fuse if we are trying to fuse something like 20 plus 20 making it 40 remember 56 is our barrier we should not cross that. So, if I use that 20 neon 20 neon type reactions then the energy needed if I want to surmount that not penetrates surmount that should be like 20 mega electron volts. Now giving initial kinetic energy of 10 M e V, 20 M e V, 30 40 M e V, 50 M e V is not a problem.

We have nuclear accelerators, a nuclear accelerators the ions you can strip all the electrons and you can make this nucleus. So that ion is accelerated and 10, 20 M e V is a small energy for most of the accelerators which are there. And one can have some kind of the target and accelerated beam can hit and nuclear fusion can take place this kind of reactions are done in research laboratories to understand the fusion mechanism of these nuclear species but, if you think of producing energy if that Q value is in mind if we are trying to get energy from that reduction in rest mass energy this more of providing initial kinetic energy to increase the barrier penetration probability to a very significant level where the barrier height is comparable to the barrier height is not feasible, because to maintain that accelerator to maintain that machine to that machine lot of electricity lot of power will be needed, and the output that we are going to get from this is negligibly small as compare to what we give to run that accelerator.

Another way to get this initial kinetic energy is through random thermal motion if we think of any system gas the molecules or the particles are in random thermal motion and they have some kinetic energy. So, if we create a some kind of plasma where the electrons we take it to we take particular species, and then we take it to high temperatures. So that the electrons are removed from the atoms and you get a plasma and you have these nuclei and electrons of course, and depending on what the temperature is they will have some kinetic energy the nuclei and electrons they will have some kinetic energy they will be moving in the random directions inside whatever volume you have provided which that is confined.

So, occasionally two nuclei will approach each other with certain kinetic energy, and that kinetic energy can be used to get some significant barrier penetration. When the fusion is nuclear fusion is done using this thermal kinetic energy of the particles of these nuclei in a gaseous type of system confined into volume we call it thermonuclear fusion thermonuclear fusion.

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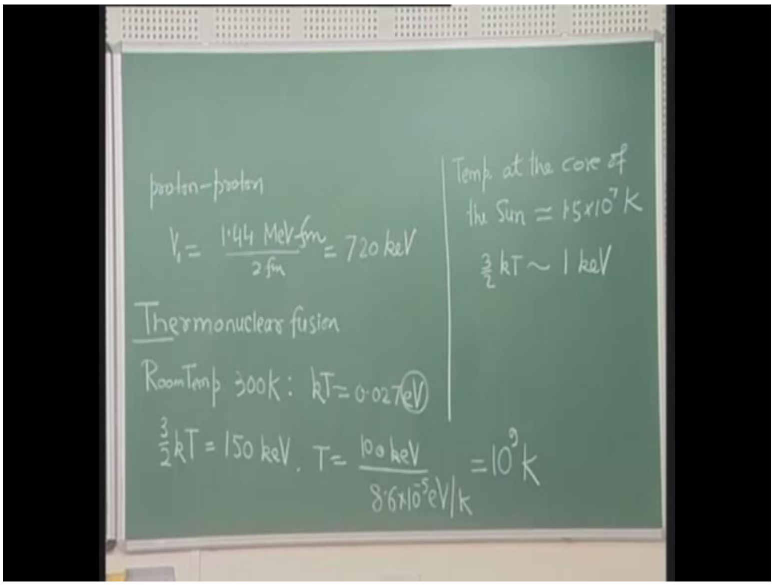

So, thermo, that means we are using thermal kinetic energy and of course, nuclear fusion. So, this is thermo nuclear fusion although reaction that are going on in stars are essentially these thermonuclear fission because they are no one has put any accelerator or any such thing, it is only because of the temperature only because of the thermal kinetic energy that those fusion are taking place and in nuclear reactors that are being planned, or being investigated, or being designed, or being researched that is all based on this thermonuclear.

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So, let us see what kind of temperatures are needed if we have to get some significant barrier penetration probability, what kind of temperatures will be needed. So, let us look at that and b let us let us start with room temperature. So, at room temperature, that 300 kelvin k T is 0.027 electron volts not k e V not M e V electron volts 0.027 that is Boltzmann constant k time the absolute temperature T, and the kinetic energy of the constituent particles at temperature scale is decided by this k T in a thermal system in equilibrium at certain temperature, T different particles will have different kinetic energies there will be a distribution of energies among those particles and you can take that distribution to be Maxwell Boltzmann distribution but, then the what is the average what is the most probable.

So, all this is that scale is decided by k into T three-second k T or 1.5 k T, that is the average kinetic energy all those kinetic energies of different particles if the average that is 1.5 times k T. So that is scale of kinetic energy in thermal motion. So, at room temperature it is 0.27 e v. So, if I take this average three-second k T or. So, this kinetic energy this initial kinetic energy is hopelessly small, when we compared to the barrier height even if I take say 300 k e V of barrier, 300 k e V kilo electron volts, that means 300 thousand electron volts and here it is hardly 0.03, 0.05 type of electron volts. So, the barrier penetration probability vary will be very small. If I want to have a comparable initial kinetic energy comparable to the barrier height let us calculate what should be the temperature? So, let us take three-second k T as this the average kinetic energy.

So, let us say that, this let us take a temperature where it is 100 and 50 k e V kilo electron volts for proton it was 720 kilo electron volts of barrier and if you go for slightly bigger like deuteron or triton or. So, on that will be reduced little bit from 700 little but, because of that radius increasing but, still it will be a say several hundreds of k e V. So, let us say that our temperature should be such that the average kinetic energy is hundred and 50 k e V. So that when and try to combine the kinetic energy is comparable to the barrier height and then the barrier penetration probability it is significant. So, how much this temperature will be T equal to I have taken 150 for 1 more reason this is 1.5.

So, k T is just 100 k e V and this divided by Boltzmann constant and boltzmann constant you can write 8.6 into 10 to the power minus 5 e V per kelvin alright that is 1 unit is in terms of joules and so on but, this is also in terms of electron volt this the value. So, how much is this is kilo electron volt. So, 10 to power 3 will come from there to the power 5 here and 10 to the power 5 from here. So, this will be 10 to the power 10 and divided by 8.6 something like 10 to the power 9 Kelvin 10 to the power 9 Kelvin this is a very high temperature. The temperature at the core of the sun is around 1.5 into 10 to the power 7 Kelvin. So, two orders of magnitude less temperature at the core of the sun 1.5 into 10 to the power seven Kelvin and if you look at that k T 3-second. k T it will be around 1 kilo electron volt. So, compared to 720 kilo electron volts of coulomb barrier. The kinetic energy of these protons which are coming will be around 1 or 2 kilo electron volts very small but, still sun is able to produce. So, many fusions fusion reaction.

So, many protons are combining and giving all these energy, that is because the density is vary that is because of the enormous density. There the pressure of hydrogen there is something like 2 hundred billion times our atmospheric pressure the density is some 100 and 60 times the density of water the density of gas there in the sun the density of hydrogen there in the sun in the core region. It is some 100 and 60 times the density of water. What we have here because of those high concentrations you have regionally good amount of fusion taking place and energy coming out whatever the temperature has to be very high 10 power seven 10 to the power 8 of that range to get a reasonable barrier penetrations.

So, apart from this high concentration of the constituent nuclei that is protons inside the sun another thing which helps is that maxwellian tail of the distribution. The average kinetic energy three-second k T that is around 1 kilo electron volt. So that we discussed and that we are comparing with this barrier height of 700 kilo electron volts. So that is very small fine but, then this is 1 k e V is only the average kinetic energy of those protons but, there are protons which will be having kinetic energies 10 times. This 20 times this 25 times, there will be some protons. So that is the maxwellian tail on the higher kinetic energy side if you look at that Maxwell Boltzmann distribution that means if you have a gas, normally gas let us take normal gas, which has molecules. So, if there is the temperature is capital T then how many of these molecules are moving with the speed say some V to V plus d v?

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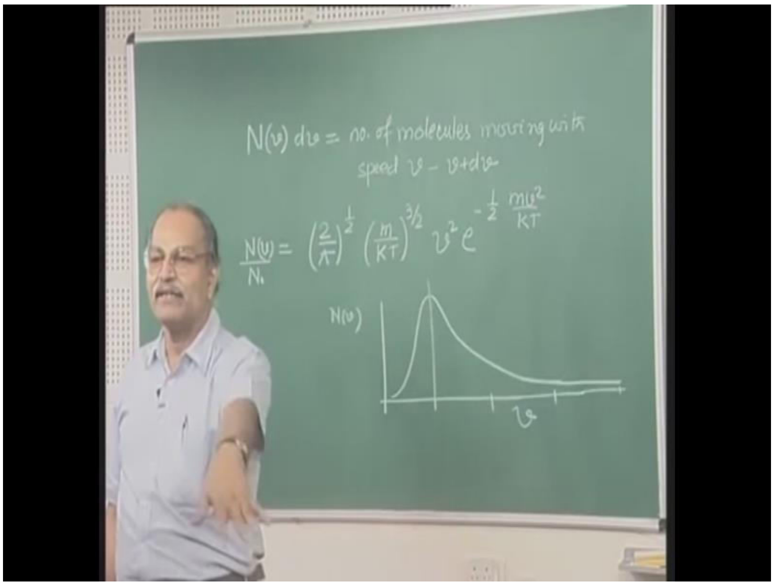

So that number we write as N v d v, number of molecules moving with speed between v and v plus d v. So, the expressions for this is N v. So, the some constant if I remember 2 by phi to the power half, and then m by K T to the power 3 by 2 and then you have v square e to the power minus half m v square by K T and this is actually out of the total and if you make catch of this, how does it look like against V? If you plot this N v that will reach some maximum and then will decay because of that exponential minus half can be square by K T factor will go like this. So, here you have maximum number of molecules at this speed the average will be also around this region that three-second K T will be around this region, little bit shifted from here but, then you also have molecules with this velocity which is much higher than the average speed you also have molecules and so on.

So, you have molecules which kinetic energy of course, that fraction will keep on decreasing most of the molecules are here largest molecules are having this velocity or close to this velocity but,. So, the number will keep on decreasing as you go for higher and higher speed but, still there will be some. So, if the density are height you have a significant number the having much larger kinetic energy than that average three-second K T and for those same happens with this nuclear nuclei as a part of plasma at those high temperatures. Of course at these high temperature 10 power seven and 10 power eight nothing will be in molecular form and or atomic form it will all form a plasma there you have nuclei you have electrons and so on.

So, those nuclei which are moving towards each other with kinetic energy is larger than three-second K T they will have higher barrier penetration probability.

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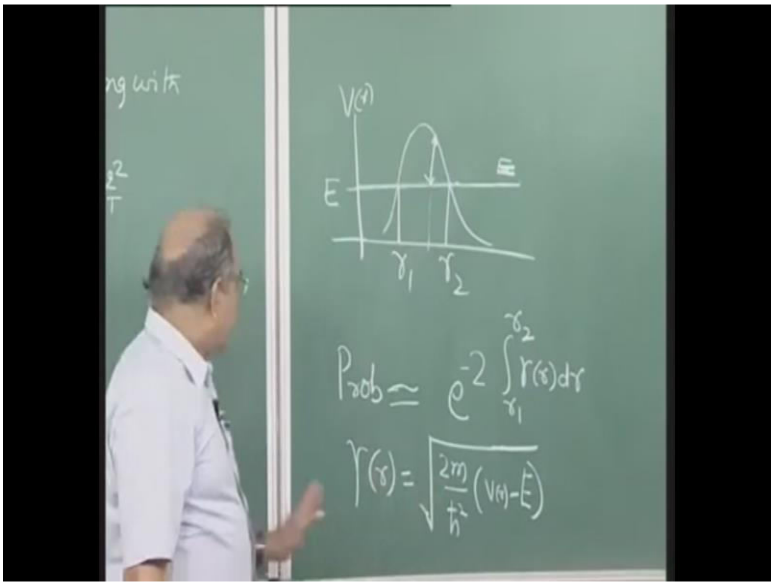

Barrier penetration probability if you remember if you have a barrier and this is let us say r 1and r 2 this is r 1 this is r 2 this is V r here is V r and this is energy, this is energy available this much and this the barrier and 1 has to go from 1 side of the barrier to the other side of the barrier tunnel through this. How much is the probability how do I get that probability specially? if this energy is much smaller than the barrier height not like the figure, I have drawn assume the thermonuclear fusion for example, that case this available energy is always very small as compared to the total height.

So, in that approximation when the barrier height is too big this probability is e to the power minus 2 integration gamma r d r from r 1 to r 2 and what is this gamma r? This gamma r is this gamma r is square root of 2 m by h cross square, V at r and minus e this is gamma r. So, V r minus at this r for example, it is this 1 this is V at r this is V this function is for the potential energy. So, this is V at r and minus e. So, this much and this into d r. So, this into this 2 m by h cross square of course, square root and all that and then d r and this you integrate from here to here to get that probability. So, as the energy goes up this probability increases.

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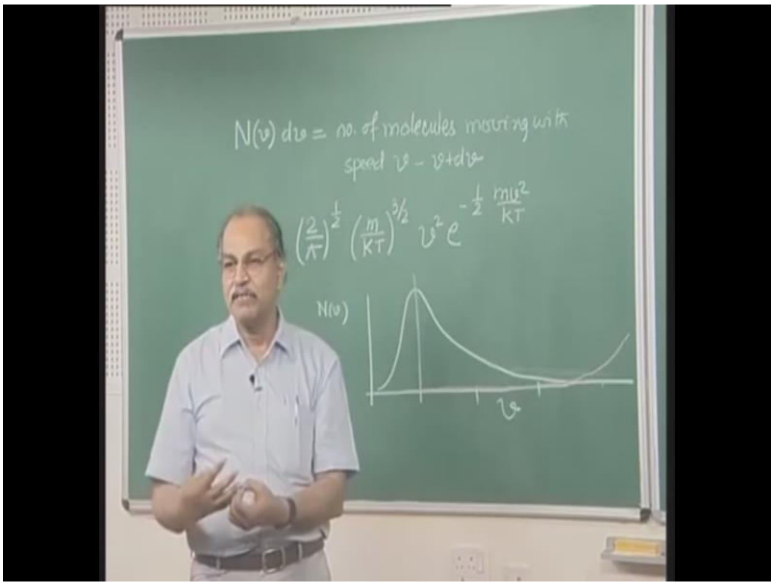

So, if on the same scale if I try to plot that barrier penetration probability probably we would not be able to show it here this in fact false very sharply. So, at the scale of drawing it will fall to almost 0 you would not be able to distinguish, and the barrier penetration probability that will go let us say when you have very small velocities the particles are going with very small velocities almost 0. You are not able to see almost 0 increasing but, still at this scale it is almost 0 almost 0 and then probability will start showing up little bit and it will increase.

So, in these regions the barrier penetration probability is negligible almost 0. So, you have lot of particles colliding with each other but, they are not able to penetrate the barrier and here when the barrier penetration probability becomes somewhat significant the number goes down but, still there is some overlap region where there are some nuclei with certain energies and here there is some barrier penetration probability also, at the end of it you get some fusion reactions.

So that is how it goes now this rate of reactions. How many fusion reactions will take place per unit time? That is an important parameter nuclear astrophysics or it is our fusion reactors the in making whatever this reaction rate how many fusion reactions will occur? Or are occurring in a given condition conditions means, what are the condition? What will decide this nuclear reaction rate though reaction the parameters will be first concentration, how many days constituent nuclei are there? If it is x plus x reaction x plus y how much is the concentration of x how much is the concentration of y per unit volume? How many nuclei are there of x of y? If it is same x plus x going to be a bigger nucleus then concentration of x.

So that is 1 parameter other parameter is the temperature the depending on what temperature it is the reaction rate will be decided. So, in a given volume with a given concentration of these constituent how much is the total output how much is the total number of fusion and from each fusion of course, we know how much energy will be obtained. So that is reaction rate. So that is an important parameter and our next task will be to talk about these reaction rates.

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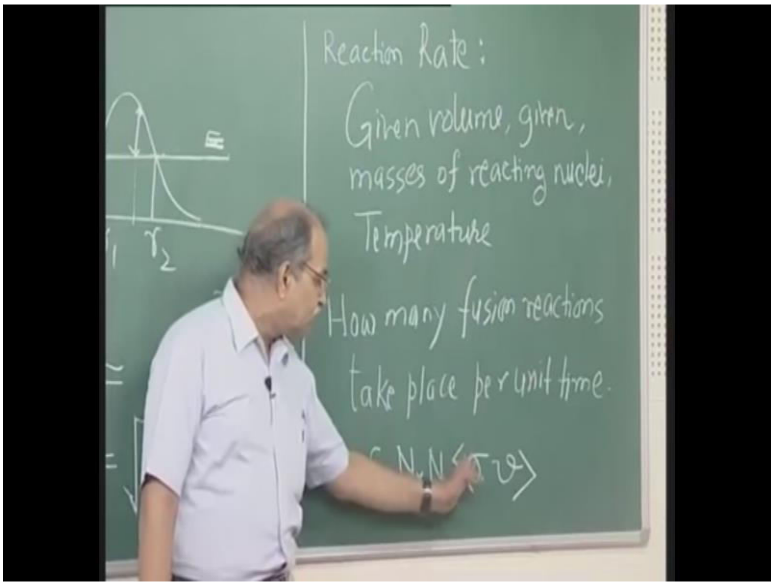

So, volume given mass, or of reacting nuclei and temperature how many fusion reactions take place? Or will take place per unit time. So, will be first it will be proportional to the concentration that almost obvious understandable if you have more concentration of the 2 of the 2 if it is different species x plus y and you put large number large concentration of x and small concentration of y then also it will not help. So, the product of concentration.

So, that is quite obvious then it should be proportional to the cross section for this fusion reaction now cross section itself is defined in such a way that the probability if the 2 nuclei and approaching each other and for that particular pair how much is the probability that they will go for this reaction. So that probability is directly related to the cross section. So that cross section it should be proportional to the cross section another thing which it is proportional to is the relative velocity of these two. So, this reaction rate are this is not radius of the nucleus we are using the same symbol for that also.

It is a Reaction Rate that we are talking off this reaction rate will be proportional to some constant N X concentration of X N Y concentration of Y sigma time's. V I will justify why it is proportional to V sigma time's ? V is the relative velocity of these 2 nuclei which are approaching each other for fusing but, then as I said all these pairs or all these nuclei which are moving in this thermonuclear reaction volume. They are not going with the same velocity the maxwellian distribution you have some particles going with this velocity some particles going with that velocity and. So, on and therefore, 1 has to take average of this whole reaction rate that we are writing for a particular V if we consider a particular velocity relative velocity of a pair then it is this and if it is since it is all distributed it has to be an average but, not average of this velocity because the velocity also comes into this sigma this sigma also depends on velocity. That is also I will justify. So, it is average of this quantity that comes in for reaction rate. So, this expression identify or talk more about it in the next lecture how this sigma depends on V and how this is calculated and those things the reaction rate and all these things will discuss in the next lecture.