Nuclear Physics Fundamentals and Application
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Lecture - 34
Nuclear fission of uranium

So, we talked about fission reactions and we discussed how this liquid drop model can explain the main features of fission reaction. All nuclei with mass number greater than 100 or so energetically favour to split into roughly equal parts. But then the barrier, the this potential energy barrier which is there, that stops many of them from fissioning.

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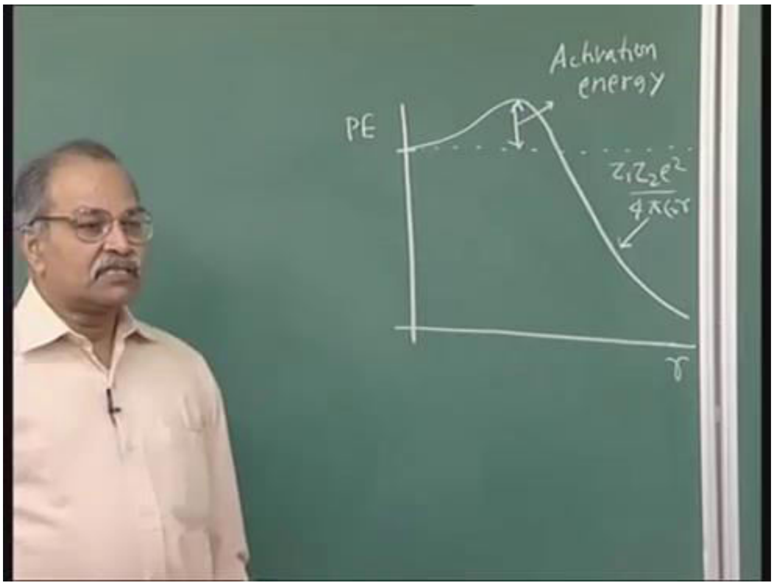

That barrier or the potential energy curve, it is something of this sort, this part is that coulomb part 1 by r part, Z 1 Z 2 e square by 4 pi epsilon r part. This side is your deformation parameter witch later on will become just the separation between the two fragments and this side is the potential energy and this is the energy which is released. When the fission thus take place, then this is the energy because after the fragments are separated the energy is here that we have taken as 0. And, this is that 0 deformation that means initial nucleus energy. So, this is that energy released that we calculate in our using masses and this is the barrier activation energy.

So, for uranium 236 or uranium 239 or plutonium which are some potential elements, potential nuclei which can go through or surmount this kind of barrier, there they become the important fission reactions. Now, this fission can be induced by certain reactions as we discussed and the most important one is neutron induced fission reaction.

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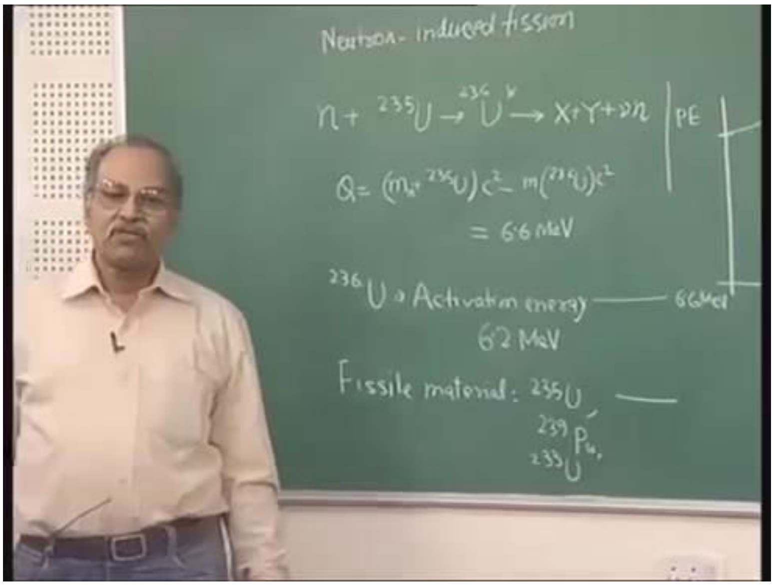

That is the how this whole field was started as we discussed last time, that historically the fission was discovered in some kind of experiment in which neutron was bombarded on natural uranium and then barium another chemical elements were found. So, this is neutron induced fission is one of the very important fission reactions which are used in all nuclear reactors and so on. And, here if I take example of uranium, a neutron going to 235 uranium that makes it uranium 236, in excited state.

And, that decays into the two fragments x plus y and sudden neutrons 1 or 2 neutrons. So, let me write it in neutrons. Now, this element uranium 236, this nucleus which is formed; it is formed an excited state. That is because if you work out the Q value of this reaction; that means, the rest mass of this neutron and plus rest mass of this 235 uranium c square and minus rest mass of this uranium 236 in ground state c square that turns out to be something like 6.6 MeV.

So, if you send neutron with almost 0 kinetic energy and if this neutron get absorbed into this uranium 235 making uranium 236; so that extra energy, that loss or reduction in the rest mass energy, that will appear as excitation energy of this 236. And, this 236 uranium will be prepared in that excited state. So, here is the ground state and here is let us say 6.6 MeV. So, it is not necessary that we write particular level here, but let us say it is there. So, when this uranium 236 is formed because of this increase in binding energy, because of this reduction in rest mass that much energy is there. Where does it go? It does not go into the kinetic energy of this part. Because this is at rest, this we are sending with almost 0 energy. So, the total momentum here, linear momentum here is 0 or almost 0. And, therefore, this product that is there that will also have linear momentum 0 and linear momentum 0 means kinetic energy 0. So, all this energy is goes into internal excitation and is created in excited state.

So, the ground state of uranium 236 is here, but then it is formed in excited state and that energy is 6.6. So, this is at 6.6 MeV above its ground state, here is the ground state and above this ground state. How much is this barrier height? How much is this activation energy? Now, this activation energy can be calculated using liquid drop model and that is what Bohr and Wheeler had done and for different a values, different mass numbers this activation energy is different. And, if you calculate for this particular element for uranium 236, this activation energy; different models will give you different values but typically this is 6.2 MeV. So, this activation energy is 6.2 MeV and the nucleus which is created in this reaction. this come this uranium 236 that is created in this reaction by absorption of neutron in this uranium 235; that is created at 6.6 MeV. So, it is created at higher energy here, higher than this was this highest point of this potential energy function. And, therefore, for this nucleus there is no barrier for fission and so it can readily fission.

So, such materials are called fissile materials. Uranium 235 is a fissile material. Fissile material means, the materials in which very low energy neutrons can get absorbed and then without any potential barrier penetration it can readily fission. So, that is fissile material, uranium 235 is one of the fissile material. There are other fissile materials like plutonium 239, all right. You can have a fissile material like 239 plutonium this also or 233 uranium also these are fissile materials. If you put thermal neutrons, thermal energy neutrons are very low energy neutrons; the Q value is large enough to allow large cross sections of fission. Of course, apart from fission there are other possibilities. One possibility is scattering, scattering of neutron from this nucleus; that is one possible elastic scattering, all right elastic scattering. So, neutron goes, does not get absorbed and scattered; so, that is also possible. Then, radiative capture is very important competing phenomena. The neutron does get absorbed into the nucleus but then it does not fission the extra energy, that higher energy that is lowered by emission of gamma rays.

So, you have a nucleus in excited state so it can always de excited by emitting a gamma ray. So, if it emits a gamma ray and there by lower its energy and then it becomes lower than that activation energy. The remaining excitation energy becomes lower than the activation energy, then the fission will become difficult and more gamma ray emissions it can just come to its ground state. So, you create uranium 236 in its own ground state which will not fission as such. It can decay through alpha decay and the life time of that is something like 24 million years or so. Only as long as this uranium 236 is in excited state, above this barrier it has a chance for fission, if it goes to its ground state it will not fission. So, all those things happen. Now, if I look at the similar reaction with uranium 238, why I am considering uranium 235 and uranium 238? Because these are the main constituents of natural uranium that we get from mines.

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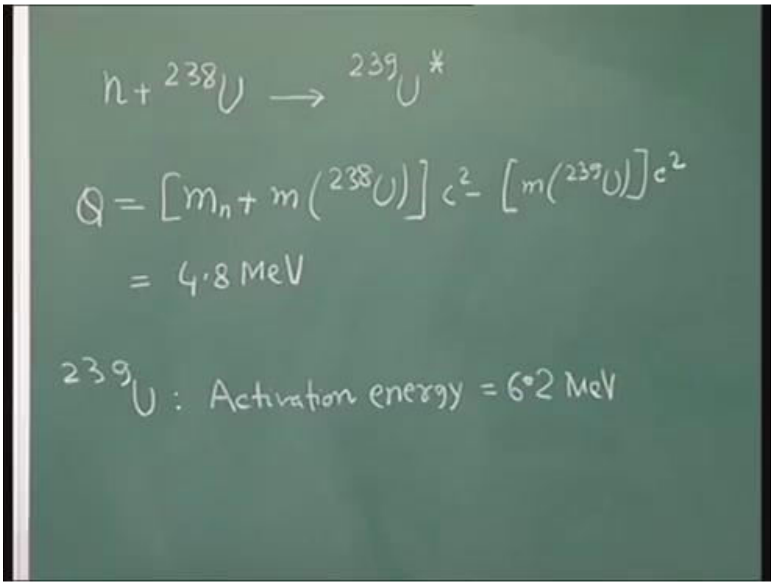

So, uranium 238; if I look at the same reactions, it is n plus uranium 238. So, if this gets absorbed here, it will make uranium 239 in excited state. What is the excitation energy? How much above ground state this uranium 239 is created, if this neutron has almost 0 energy? If it gets absorbed there with the initial kinetic energy 0, then you can calculate the Q value. That Q value here will be mass of neutron and plus mass of this uranium

238 c square, this is the initial rest mass energy. And, then minus mass of 239 uranium in ground state, this is the Q value. And, this Q value comes out to be about 4.8 MeV. So, if this neutron gets absorbed here, this much energy is made available. And, once again if the initial kinetic energy of this neutron is almost 0 and this is at rest, the you do not have any linear momentum here, you do not have any linear momentum here. So, all this energy goes into internal excitation.

So, this uranium 239 is created in an excited state above its own ground state by this much of energy 4.8 MeV. And, how much is the activation energy? This height of this barrier, maximum height of this potential energy curve above the ground state energy for uranium 239 is activation energy, rather this will be higher something like 6.2 MeV. It depends on how, what kind of model you take but it will be higher than this. If I take that as 6.2 this will be something like 5.8. These numbers are not very fixed as such it depends on what model we take. But nevertheless it is still is the this final result is same that the excitation energy is much smaller than the activation energy, in case of uranium 239. Whereas, in case of uranium 236 created by this neutron absorption, the excitation energy is higher than the activation energy.

And, therefore, uranium 239 once it is created, it is created much below this activation energy and it has to cross this big potential barrier. And, therefore, the fission probability will be almost negligible with these low energy neutrons. But one can give these neutrons high enough energy initial kinetic energy, so that the excitation energy is Q value plus the initial energy of the neutron. And, then if you give sufficient energy to this neutron, so that the total energy becomes more than activation energy; then it can fission. So, with low energy neutrons it will not fission, but with somewhat higher energy neutrons; something like more than 1 MeV neutron, so that it can reach that activation energy level 1.4 MeV neutrons, this fission can take place. So, that is difference between uranium 235 fission and uranium 238 fission on absorption of neutrons.

Now, there are some characteristics of all this reaction; let us look at those. One is once this uranium 236 is created in excited state and it fissions; what are the product nuclei? What are those fission fragments? It turns out that you do not have a unique combination in which unique pair of nuclei in which this 236 uranium will split. You can have different possibilities and let me write some of them. Some reactions are as follows.

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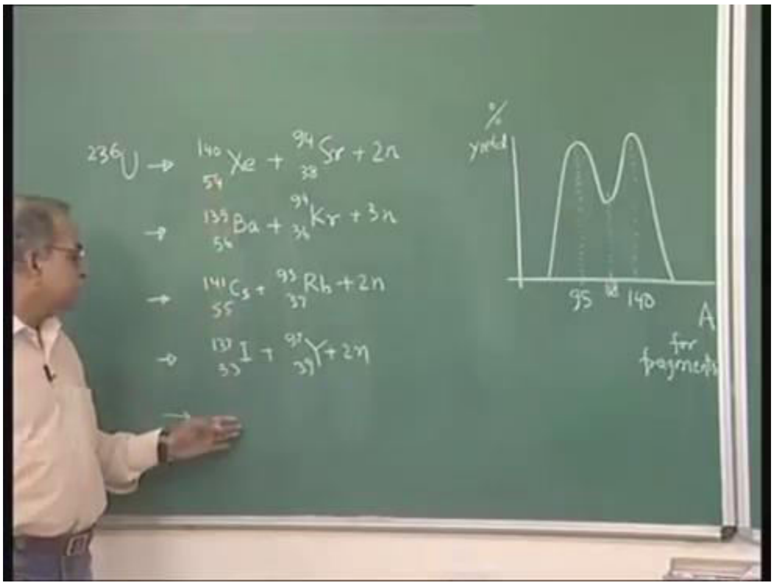

This uranium 236 that splits into 140 xenon Z is 54 and then plus 94 strontium 38 and 2 neutrons are emitted. It can also go through the other reaction where we have 139 barium, barium is 56 Z 56 and then plus krypton which is 94 and 36 and then 3 neutrons possible. Then, you can also have a reaction like 141 Cs 55 and then plus 93 Rb rubidium 37 plus 2 n. You can also have reactions like 137 iodine 53 plus, 97 yttrium 39 plus 2 n and so on, you can add many more. So, what are the reaction products; the reaction products are xenon, strontium, barium, krypton, rubidium, iodine, yttrium and many many things. So, if one looks at the mass numbers a of these products and how much, how was the abundance in that reaction part, what percentage of the reaction products in strontium, what product is how much is iodine and so on that plot can be made.

And, it turns out that this is very asymmetric plot so here I am plotting A of fragments and this side you can say percentage yield in all the material that is here after this fission event how much is this A and how much is that A and so on. And, if you do that it turns out to be that it is like something of this sort and these two humps, these two maximum; they are somewhere around 95 and 140 and the initial one remember is 236. If it is splits into equal parts it would be 118, 118 and that is in fact here. So, the nucleus splitting up in two exactly equal masses that probability is lower and one fragment with mass number around 95 and another around 140; that is the maximum probability reaction that we have written here. You can see that this 140 and 94, 139 and 94, 141 93, 137 97 and so on. So, one fragment is around 95 and another fragment is around 140, those reactions are much more probable than the reaction in which the original nucleus splits in two equal parts. This symmetric distribution is one aspect or one characteristic which liquid drop model is not able to explain if I use liquid drop model and calculate what is the energy yield, how much is the energy liberated in a fission reaction.

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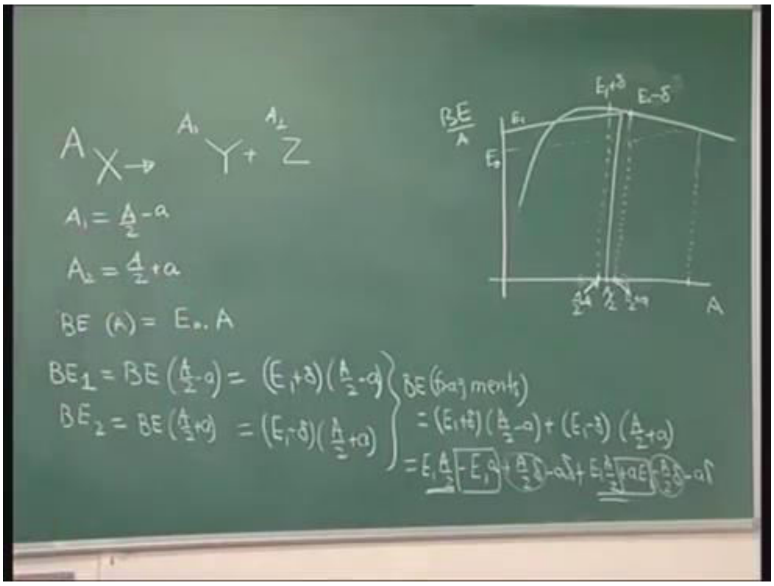

So, if this original nucleus with A and here it is Z and here it is n it splits in two parts one is Y with mass number A 1 another is Z with mass number A 2 and I can calculate how much is the reduction in rest mass energy using semi empirical mass formula or from the binding energy per nucleon curve. So, if I have this binding energy per nucleon curve with me remember something of this sort, this is let us say original A is here and here is the binding energy per nucleon. So, this is E 1 or E naught and then you have A by 2 somewhere here A by 2 and then once fragmented say here A by 2 plus a another fragment is here A by 2 minus a; for the time being I am not considering the neutrons which are emitted.

So, the binding energy my scales are somewhat different so let me put it here, let me repeat what it is. I am finding the binding energy per nucleon value for the initial nucleus A mass number A and then for that two fragment at mass number that is here this is A by 2 minus A and the other is here which is A by 2 plus A. At A by 2 I take the binding energy per nucleon to be this E 1 this E 1 and then for the fragment with slightly larger value A value that is A by 2 plus a this for this; the binding energy per nucleon value will be smaller than E 1, that I am writing E 1 minus delta. And, for the fragment with mass number A by 2 minus a, that is here for this the binding energy per nucleon value will be slightly more than E 1; so, that I am writing E 1 plus delta.

So, the binding energy of the initial nucleus with mass number A; that is how much binding energy per nucleon here is E naught, so it is E naught times A. E naught is the binding energy per nucleon at that mass number and multiplied by the mass number; this is the total binding energy for the initial nucleus. This is not multiplication is binding energy of A binding energy of the initial nucleus. Then, you have the fragments. So, binding energy for the first fragment BE 1 you can write this fragment A 1 is equal to A by 2 minus a it is binding energy at of this nucleus A by 2 minus a. How much is the binding energy per nucleon at this mass number A by 2? Minus a. So, it is on the plot at A by 2 minus a, here the binding energy per nucleon is so E 1 plus delta.

So, this is E 1 plus delta that is binding energy per nucleon and that multiplied by this A by 2 minus a and the binding energy of the second nucleus BE 2 that will be, that means binding energy of this nucleus A by 2 plus a, how much is this? This will be binding energy per nucleon multiplied by the number of nucleons E 1 minus delta and multiplied by A by 2 plus a, all right. So, E 1 minus delta this is the binding energy at this mass number binding energy per nucleon and multiplied by the number of nucleons that is this. So, the total binding energy after the event after that splitting the total binding energy after this splitting is BE of the fragments. You can say, binding energy of the fragments will be this BE 1 plus BE 2, that is E 1 plus delta A by 2 minus a and plus E 1 minus delta and multiplied by A by 2 plus a. This you can expand it is E 1 times A by 2 and minus E 1 times a plus A by 2 delta and minus a delta. And, similarly, the other bracket E 1 times A by 2 and plus a E 1 then minus A by 2 delta and then minus a delta.

Now, this A by 2 delta here, plus A by 2 delta here and minus A by 2 delta here, they will cancel out and the rest of the things you can work out. E 1 times a also I can see here you have a minus E 1 times a and here you have plus E 1 times a, so that will also cancel out and what is left; so, what is left? This binding energy of the fragments is equal to E 1 into A by 2 here and E 1 into A by 2 here. So, this term and this term is just E 1 times A this is E 1 times A.

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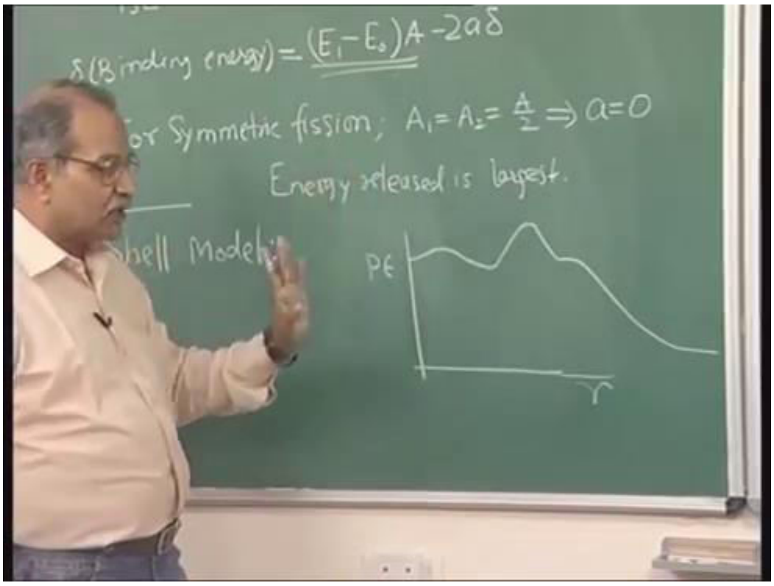

And then, this is out, this is out minus a delta here this I have written these are out and minus a delta. So, minus 2 times a delta initial binding energy binding energy of this nucleus original nucleus A is E naught times a, all right. So, how much is the difference does the energy released? How much is the difference and that difference in delta? This difference in binding energy will be this; E 1 minus E naught a and minus a 2 times a delta.

And, this is what you know in that a equal to 240 region; this e naught is something like this E naught here it something like 7.6 all right, 7.6 MeV and at half the region, this is what? 8.5 MeV. So, that the that is difference we had calculated earlier. So, this is this comes out to be 215, 216 MeV, so that is there. Now, this minus 2 a delta energy released is small if you have this term if you have this a. For symmetric fission, symmetric fission means your the two fragments are just equal A 1 equal to A 2 and equal to A by 2 that is symmetric fission. And, that means a is 0 all right because the two fragments a 1 and a 2 I have taken as A by 2 minus a and A by 2 plus a; symmetric fission means these mass numbers are equal and so, the a is 0.

Then, you have largest energy released and if you have asymmetric fission where one fragment is the smaller other fragment is larger, then the energy released will be less. That means, the final products are at higher energy released is a small that means the initial nucleus is there and then the final fragment energy of that; so that final energy or energy of the final products that is more, all right. If it is symmetric fission then the energy released is more. That means, the final product are going in to a lower state. If you have a lower energy state available and if the, if fission is direct towards that; it should be more likely to go for that lower energy state. That means, the probability should be larger there. But that is not the case the probability of the symmetric fission is smaller by hundreds of time; then, the probability of asymmetric fission one centered around 95 other centered around 140.

So, that is one place where liquid drop model will not work. So, people take all kinds of corrections and calculations involving shell models mixing up shell model with liquid drop model, and other things to create newer theories and still is not very well understood. But one possibility is once you take all these shell model effects that the potential energy curve that we had drawn as a function of deformation parameter. We have shown just one single hump but maybe it is not, it is more complicated by and then this kind of structure can create asymmetric fission; that is one possibility which is being explored. So, this is about one characteristic that the fission fragments they are not of equal mass or nearly equal mass; they are somewhat different, the probability is highest for one fragment around 95 and other fragment around 140.

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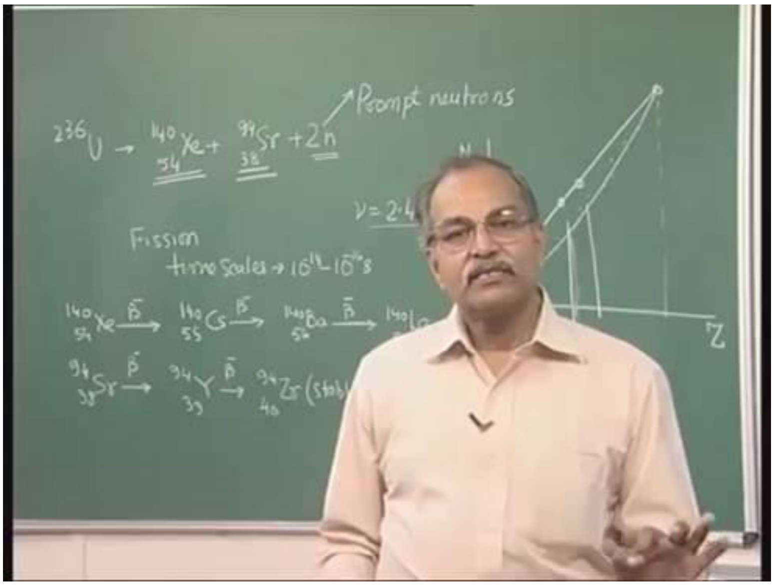

The other thing is neutron access in the final products. So, you remember the sum of the equation that I had written one of that was are any one 236 uranium, going to let us say

140 xenon is 54 and plus strontium 94 that is, 38 and plus 2 neutrons. So, what is this plus 2 neutrons? Why are they here? This breaks into 2 parts; 1 part is this xenon 1, 140 other part is strontium 94. But then, these 236 nucleons here only 234 are here in these two fission fragments and 2 neutrons are emitted together with the fission. What are these?

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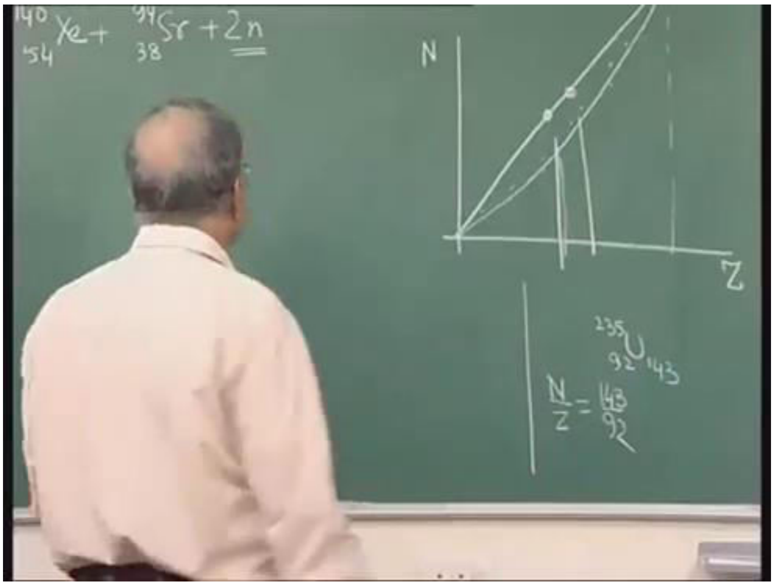

If you look at N versus Z particle at beta stable particle distribution, so if n is this side and Z is this side for beta stable nuclei; remember, at initially for light nuclei n is equal to Z. But, then as the number increases the number of neutrons goes up it is like this. So, these are around this line are the beta stable nuclei for heavier nuclei; you have more number of neutrons than the number of particles. Example, if you look at this uranium say uranium 235 or uranium 238 you have 92 protons and neutron number here will be 143. So, if I look for that N by Z, this N by Z that is 143, 92 that is let us say somewhere here, some here 92 protons and 143 neutrons.

Now, when this nucleus is split in two parts and you assume that, what is the Z and N? They split equally here. So, you have Z total protons in the beginning which gets distributed as Z 1 and Z 2 and n neutrons there that get distributed as N 1 and N 2. So, if that division is in same ratio Z by N; what is there Z by N 1, Z 1 N 1 here, Z 2 N 2 here same ratio, it just split. So, initially it is all uniformly distributed neutrons and protons and then you take one portion here and one portion there. So, in each of the portion the ratio N by Z will remain the same as it is in the original nucleus.

That means, when it is splits here in these two parts close to that A by 2 these two parts the ratio N by Z remains the same. So, if I draw this straight line passing through the origin then the ratios of the fragments will be just the same as the ratio N by Z here, N by Z here, N by Z here. But this is on that stability line beta stability line but this is much above so the required number of neutrons for this Z is only this much but the neutrons which will they, which will be there in the fragment is this much. So, it is highly neutron rich and similarly, here it is highly neutron rich. So, this number of neutrons in the fragments just after the fission will be much higher than what is required for that stable nucleus configuration. So, what will happen? The first thing is that the neutron number is so high that some of the neutrons are just boiled off. So, these two neutrons are those type of neutrons, the fragment is created; the fragment has many more neutrons that it can accommodate some neutrons get evaporated.

So, those neutrons are here. So, in some of the reactions you will have 2 neutrons in some of the reactions he will have 3 neutrons as well in some of the reactions it may be just 1 neutron. So, this number will depend on what kind of fragments it has broken into. So, on the average if you take all those possible reactions for uranium 236, this number comes out to be around 2.45 or so in this reaction it is 2, in some reactions it is 3 and if you take all large number of reactions and average out that average comes out to be this. So, these neutrons are emitted because of that neutron access in the fragments and these are just immediately after the fission. So, the fission time scales are something like 10 to the power minus 14 to 10 to the power minus 16 seconds and as fission takes place almost together these neutrons are coming out.

So, these neutrons are called prompt neutrons, these are called prompt neutrons. So, together with the fragments 2 or 3 neutrons also come out and they are known as prompt neutrons. But even after that, even after emitting those 2 or 3 neutrons the two fragments that are created are still neutron rich, still they are much above in that diagram above the stability line. And, therefore, they are beta active, beta minus active. Because more neutrons neutron wants to convert into proton to reduce that N by Z ratio; so, they are beta minus active and they decay, beta decay ,beta particle emitted and they try to become stable some of those beta chains that I will write.

For example, in this reaction these two elements that are created here, they are the beta active and the they come to stability through these reactions 140 xenon 54 this will beta decay, beta minus decay 140. Then, Cs 55 and this will beta decay to 140 Ba and 56 in beta decay; the proton number will increase by 1. Because a neutron is converted into proton; so, 54 become 55, 55 becomes 56 and so on. The total number of nucleons remains the same 140 neutron is converting into protons; so, the total nuclear number is not effected. So, it this, then this is also radioactive; so, it decays beta decay to 140 La lanthanum 57 and then finally, it decays to 140 Ce; so 58 this is stable. So, now N by Z ratio has come to that that valley of stability.

So, you have one beta decay here, another beta decay here, third beta decay, 4 beta decay, four beta particles are emitted, 4 neutrons are converted into protons, one by one and then you get this is stable final thing. The other element here 94 strontium that is also radioactive, this also have has excess this thing neutrons. So, this beta decays to 94 Y yttrium and that decays beta decays to 94 Zirconia and this is stable. So, the fragments in this fission first they emit few neutrons to reduce their neutron number and after that neutron emission becomes energetically unfavorable and then neutron converts into protons emitting beta particles to get to that stability.

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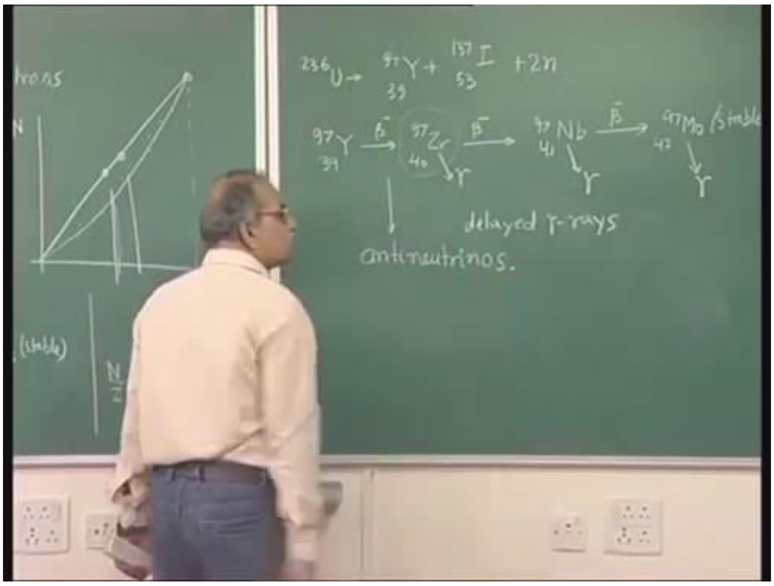

One more reaction I will write and that is let us say 97. So, uranium 236 going to 97 yttrium and plus 137 iodine, iodine has Z is equal to 53 and yttrium has that Z is equal to

39. So, this and plus 2 neutrons these are the prompt neutrons. Then, this 97 Y 39 this decays to 97 Zr 40 zirconium; this is beta minus decay and then this beta minus decay is 297 Nb niobium 49 and that beta decays to 97 molybdenum and that is stable. The other fragment 137 I 53 this fragment this beta decays to 137 xenon Z is equal to 54 and that decays to this is interesting, that decays to or that the reaction is 136 xenon 54 and plus neutron. This is not a beta decay, this last part is not a beta decay so and of course, this is stable. So, you have this is a the regular chain of beta 1 2 3 beta decays and goes to stable. But this one, this iodine first it decays to this xenon 137 and that xenon 137 that goes to xenon 136 and a neutron.

So, again you have got one neutron here. So, you have two neutrons here and then a third neutron is coming here. Now, this neutron has come after this two beta after this beta decay and beta decay has its own life time. And, therefore, this neutron has come after few seconds or few minutes not intended to the minus 16 seconds not at the fission time scale. So, it has come much later than that; that is these type of neutrons are called delayed neutron. Delayed neutrons that means, the fission event has taken place prompt neutrons have been emitted and after long time long time as compared to the time scale of 10 to the power minus 16 seconds of fission; after long time yet again, we get some neutrons. So, those neutrons are known as delayed neutrons and that number of delayed neutrons in overall reaction, if you have some substance few milligrams, a few micrograms of uranium which is fissioning and all sorts of fragments are being created and they are decaying through beta and other things.

So, overall average if you find prompt neutrons are 2.45 or so average number of neutrons in this reaction average number of delayed neutrons is something like 0.2 per fission. All right, so this nu is 2.45, this is for the prompt neutrons per fission all right, per fission sometimes 2, sometimes 3 on the average 2.45. Similarly, if you calculate how many delayed neutrons have come through the total number of fission and then divide per fission, that number of delayed neutrons per fission of uranium 236. We are talking of uranium 236, that is around 0.02. So, say in 50 fissions, one particular combination will give one delayed neutron. This fragments which are created not only the they are neutron rich but they are in a higher energy state also.

So, they immediately give of some gamma rays all right. So, these fragments which are created after this fission, they are at much higher energy and therefore, they also emit gammas and these gammas are in the same spirit are called prompt gamma rays. All right, so these gammas are called prompt gamma rays, gamma rays or gamma photons. Prompt means just immediately after the fission, immediately after is at that time scale of 10 power minus 16 seconds; so that is prompt, the fission fragments are created the fission fragments are neutron rich. They just give off a neutron prompt neutron. The fission fragments are at high energy, so they give off some gamma photons and try to lower their energy. So, that is prompt gamma rays and this energy is prompt gamma energy.

Then, the beta after this beta decay you create this daughter nucleus, this daughter nucleus after the beta decay; they are also created in excited states, regular beta decay process from beta decay the daughter; that is created that is normally created at some high higher excited state. And, from there they will go to their ground state emitting gamma rays. So, from here also you are getting together with this beta, you are also getting this gamma rays. So, this is created in this excited state and that gives me some gamma, after every beta; after every beta you will also get gamma.

These gamma rays are delayed gamma rays, your prompt gamma rays, your delayed gamma rays, prompt gamma rays are gamma rays which have come just after the fission from the fission fragments. And, then, the delayed gamma rays are coming from the products of the beta decays. These fragments they beta decay to convert the neutron into proton and those daughter nuclei which are created through this beta decays, they emit gamma rays; they are known as delayed gamma rays.

So, you have a prompt neutrons and delayed neutrons. Similarly, you have prompt gamma rays and delayed gamma rays. So, the total energy which is made available through this reduction of rest mass in this fission process is distributed among the kinetic energy of the fragments. That takes the major part. So. of about 200 MeV per fission some 160 MeV or so that is taken up by; so, that means about 80 is taken up by the kinetic energy of the main fragments.

And, the rest is distributed in kinetic energy of the prompt neutrons, then kinetic energy of this beta particles, then energy from gamma rays of delayed gamma rays, then delayed neutrons and one more thing anti neutrino. In this beta reactions you also have anti neutrinos. So, some 10, 15 MeV is also taken away by his anti neutrinos which are very difficult to tap. The neutrinos they pass through all kinds of matter without almost without interacting with anything so they just escape. So, that energy whatever is taken away by the this anti neutrinos in beta decay, that is any way goes out that reaction area. So, that is how the total energy is distributed.